An isospin-dependent global elastic nucleon-nucleus potential

… in an exotic beam context

- Theoretical overview of the potential
- Comparison to other potentials
- Asymmetry term analysis – charge exchange

Stephen Weppner Eckerd College

Motivations

Building a completely phenemological potential

- A desire to build a potential for the present and future exotic beam accelerators
  - -- include non-spherical
  - -- concentrate on lighter nuclei
  - -- consistent across chains of isotopes

- One which will complement and augment my earlier microscopic work

- an re-examination of the traditional terms

Makeup of the phenemological potential

- A Woods-Saxon Basis
- Each parameter in the Woods-Saxon basis is fit to a quadratic or cubic polynomial
- Complete separation of the “E” and “A” variables
- No additional theory: no added coulomb corrections or dispersion relationships
- A standard format: terms include:
  volume, surface, spin orbit, and coulomb

$$
\begin{aligned}
\mathcal{U}(r,E,A,N,Z,\mathcal{P},MN) = \\
&-\mathcal{V}_V(E,A,N,Z,\mathcal{P},MN) f_{WS}(r,\mathcal{R}_V,\mathcal{A}_V) \\
&-i\mathcal{W}_V(E,A,N,Z,\mathcal{P},MN) f_{WS}(r,\mathcal{R}_V,\mathcal{A}_V) \\
&+4\mathcal{A}_S\mathcal{V}_D(E,A)\frac{d}{dr} f_{WS}(r,\mathcal{R}_S,\mathcal{A}_S) \\
&+i4\mathcal{A}_S\mathcal{W}_D(E,A,N,Z,\mathcal{P})\frac{d}{dr} f_{WS}(r,\mathcal{R}_S,\mathcal{A}_S) \\
&+\frac{2}{r}\mathcal{V}_{SO}(E,A,N,Z,\mathcal{P})\frac{d}{dr} f_{WS}(r,\mathcal{R}_{SO},\mathcal{A}_{SO})(\mathbf{l}\cdot\sigma) \\
&+i\frac{2}{r}\mathcal{W}_{SO}(E,A,N,Z,\mathcal{P})\frac{d}{dr} f_{WS}(r,\mathcal{R}_{SO},\mathcal{A}_{SO})(\mathbf{l}\cdot\sigma) \\
&+f_{coul}(r,\mathcal{R}_C,A,N,Z,\mathcal{P})
\end{aligned}
$$

volume

surface

spin-orbit

The complete volume term

Real:

$$\mathcal{V}_V = V_{V_0} + V_{V_1}A + V_{V_2}A^2 + V_{V_3}A^3 + V_{V_5}E + V_{V_6}E^2 + V_{V_7}E^3$$
$$+ \mathcal{P}(N-Z)\left(V_{V_{i0}} + V_{V_{i1}}A + V_{V_{i2}}A^2 + V_{V_{i3}}A^3 + V_{V_{i4}}A^4 + V_{V_{i5}}E + V_{V_{i6}}E^2\right)$$
$$+ MN\left(V_{V_{m0}} + V_{V_{m1}}A + V_{V_{m2}}A^2 + V_{V_{m3}}A^3 + V_{V_{m5}}E + V_{V_{m6}}E^2\right),$$

Imaginary:

$$\mathcal{W}_V = W_{V_0} + W_{V_1}A + W_{V_2}A^2 + W_{V_3}A^3 + W_{V_5}E + W_{V_6}E^2 + W_{V_7}E^3$$
$$+ \mathcal{P}(N-Z)\left(W_{V_{i0}} + W_{V_{i1}}A + W_{V_{i2}}A^2 + W_{V_{i3}}A^3 + W_{V_{i4}}A^4 + W_{V_{i5}}E + W_{V_{i6}}E^2\right)$$
$$+ MN\left(W_{V_{m0}} + W_{V_{m1}}A + W_{V_{m2}}A^2 + W_{V_{m3}}A^3 + W_{V_{m5}}E + W_{V_{m6}}E^2\right).$$

Antisymmetric　　Magic Number

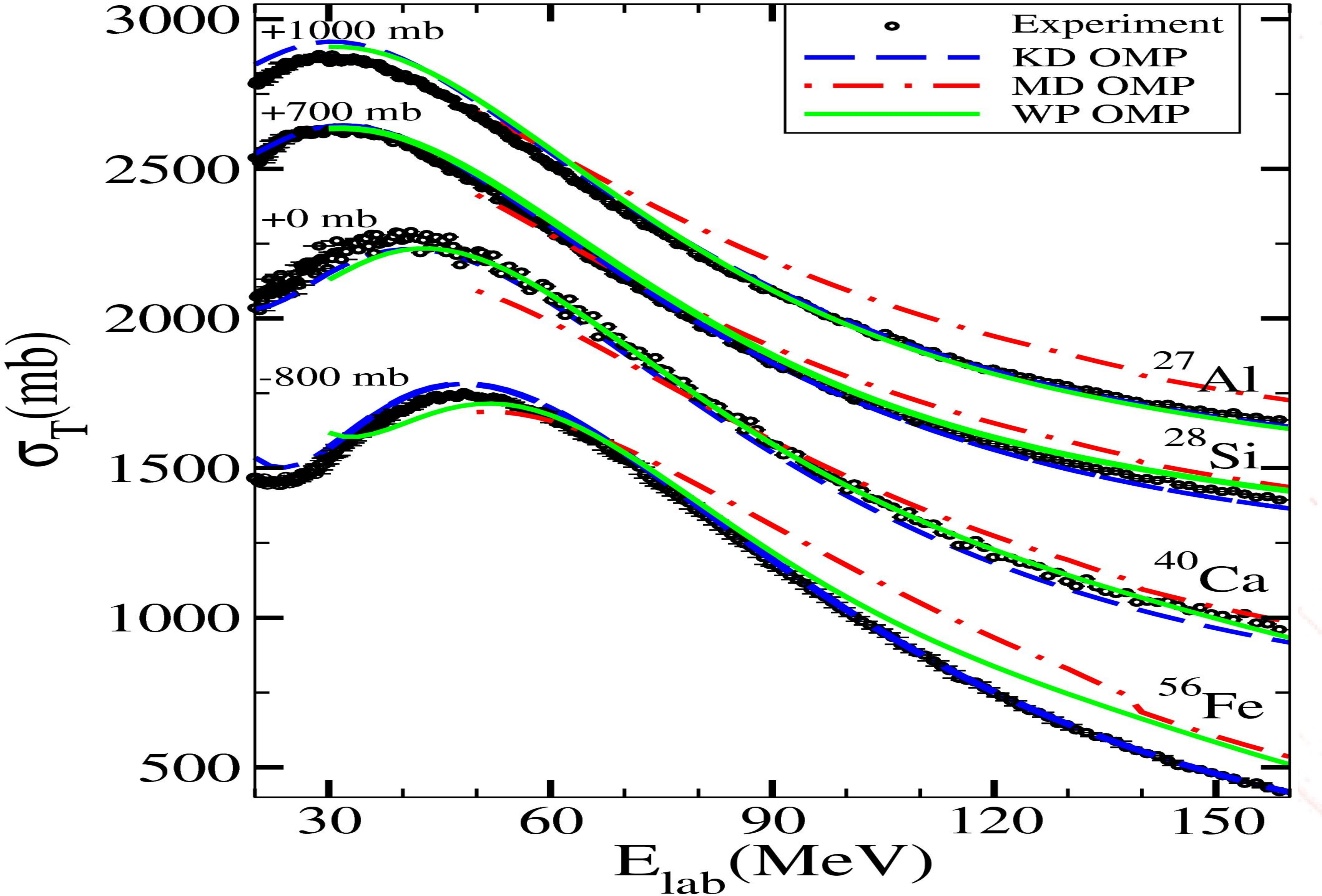
Computational Aspects

- Fit over 500 different experiments
- The searching space was often over 20 dimensions
- The fit tried to find a minimum in the weighted χ^2 parameter space
- Run on a parallel Linux cluster, each run took ~80 computational hours

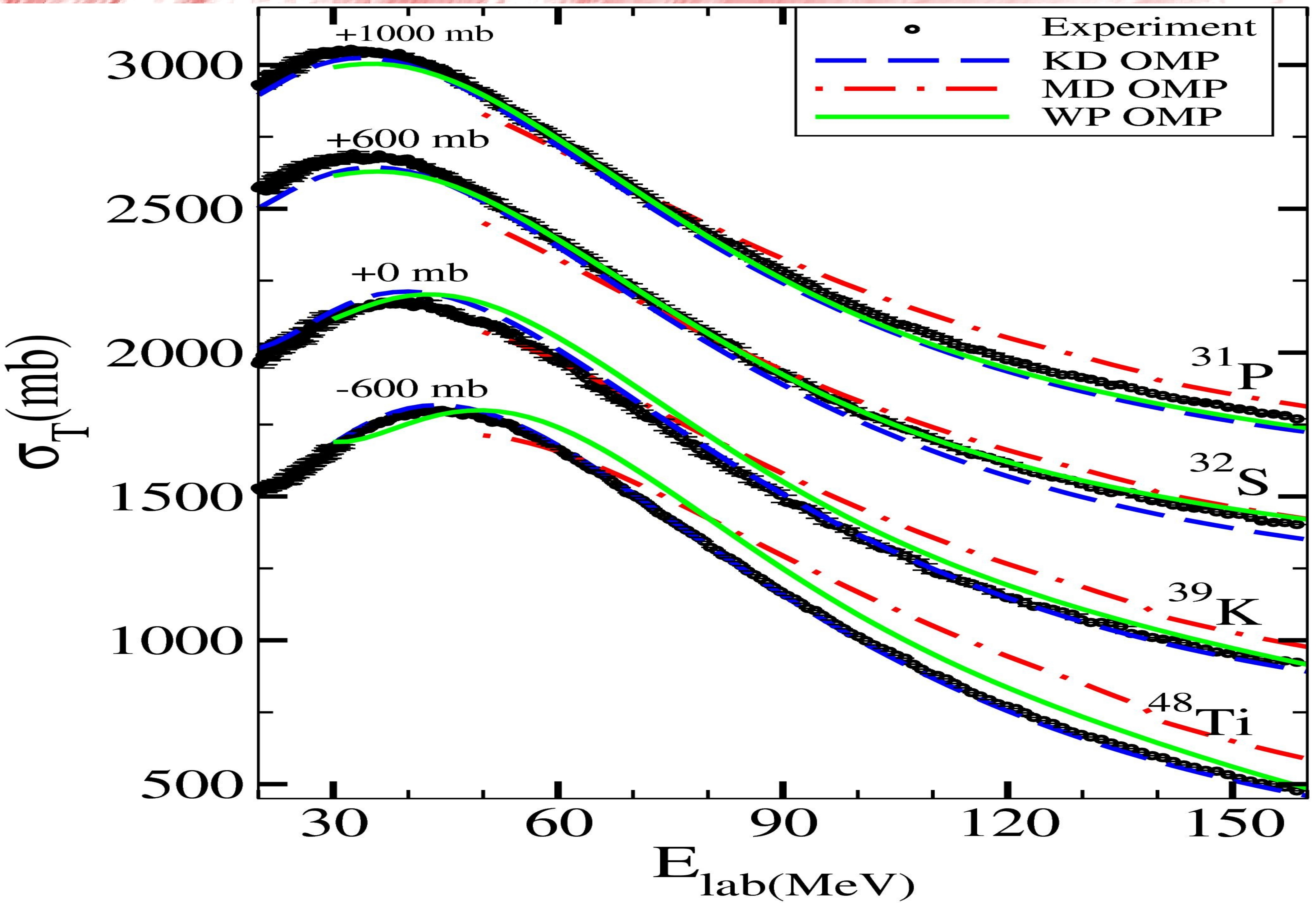
A.J. Koning and J. P. Delaroche,
Nucl. Phys. A 713, 231 (2003)

D. G. Madland
arXiv:nucl-th/9702035v1 (1997)

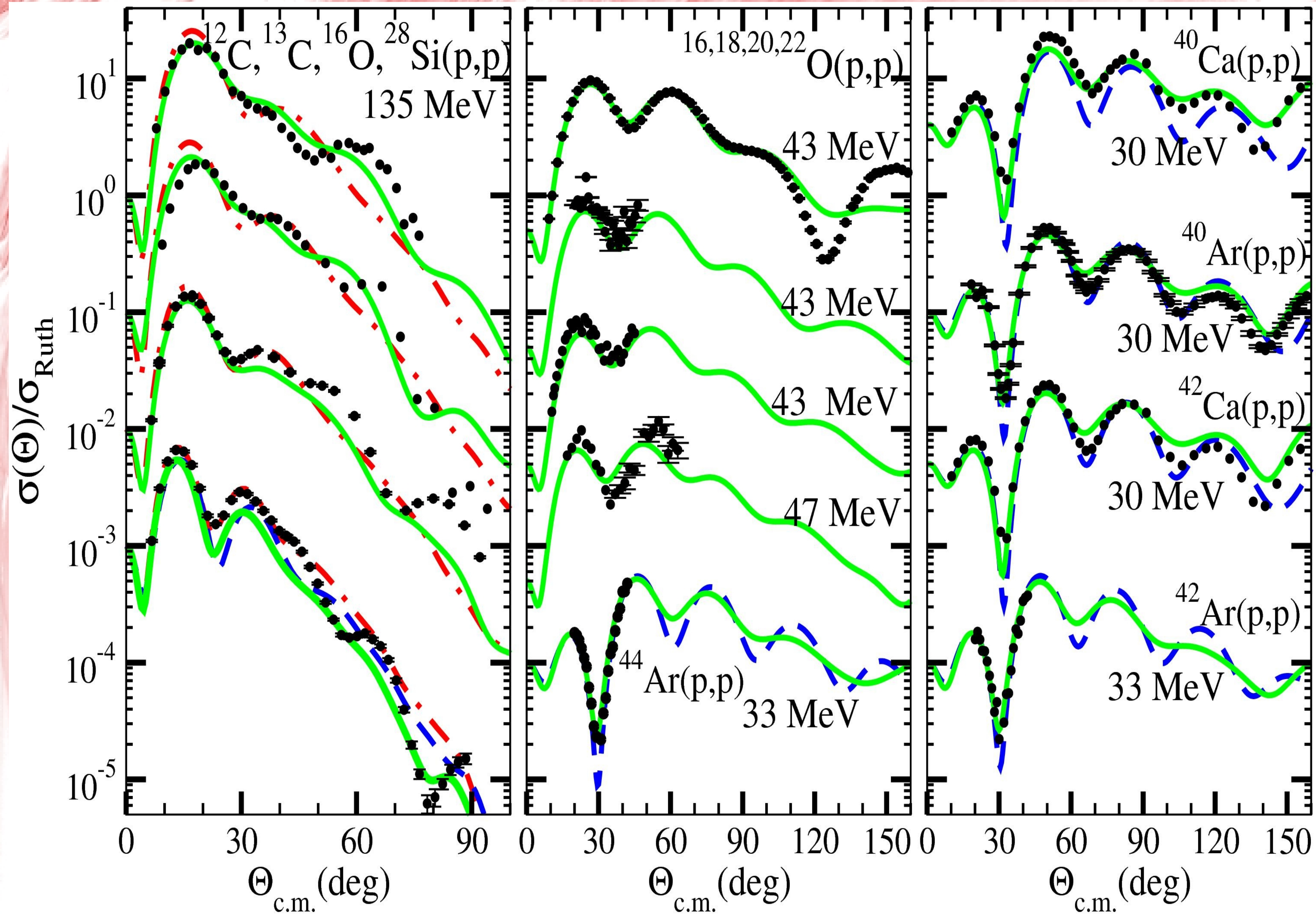
S. P. Weppner et. al.
Phys. Rev. C80, 034608 (2009)



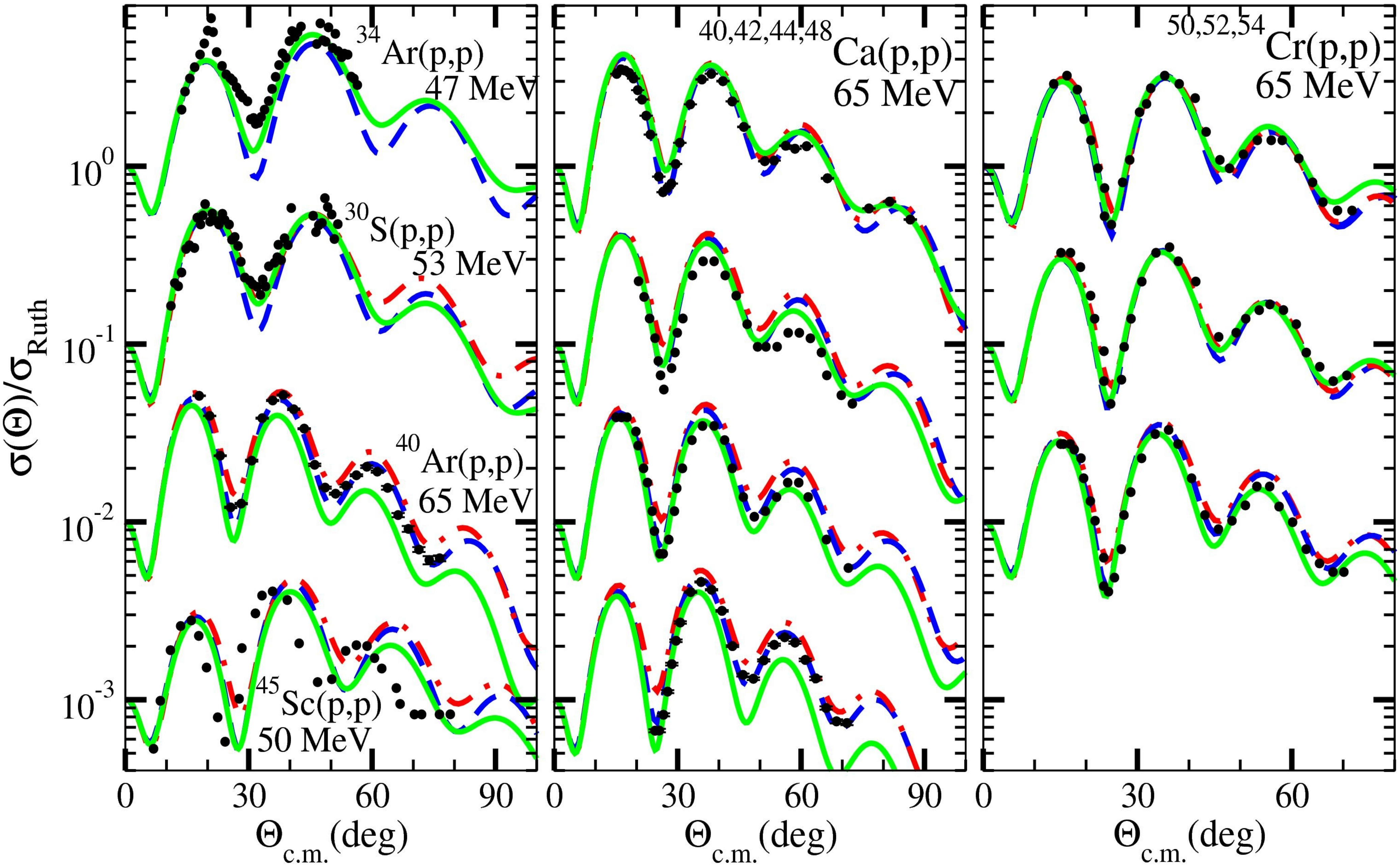
Experiment
KD OMP
MD OMP
WP OMP
+1000 mb
+700 mb
+0 mb
-800 mb
27Al
28Si
40Ca
56Fe
σT(mb)
Elab(MeV)
3000
2500
2000
1500
1000
500
30
60
90
120
150



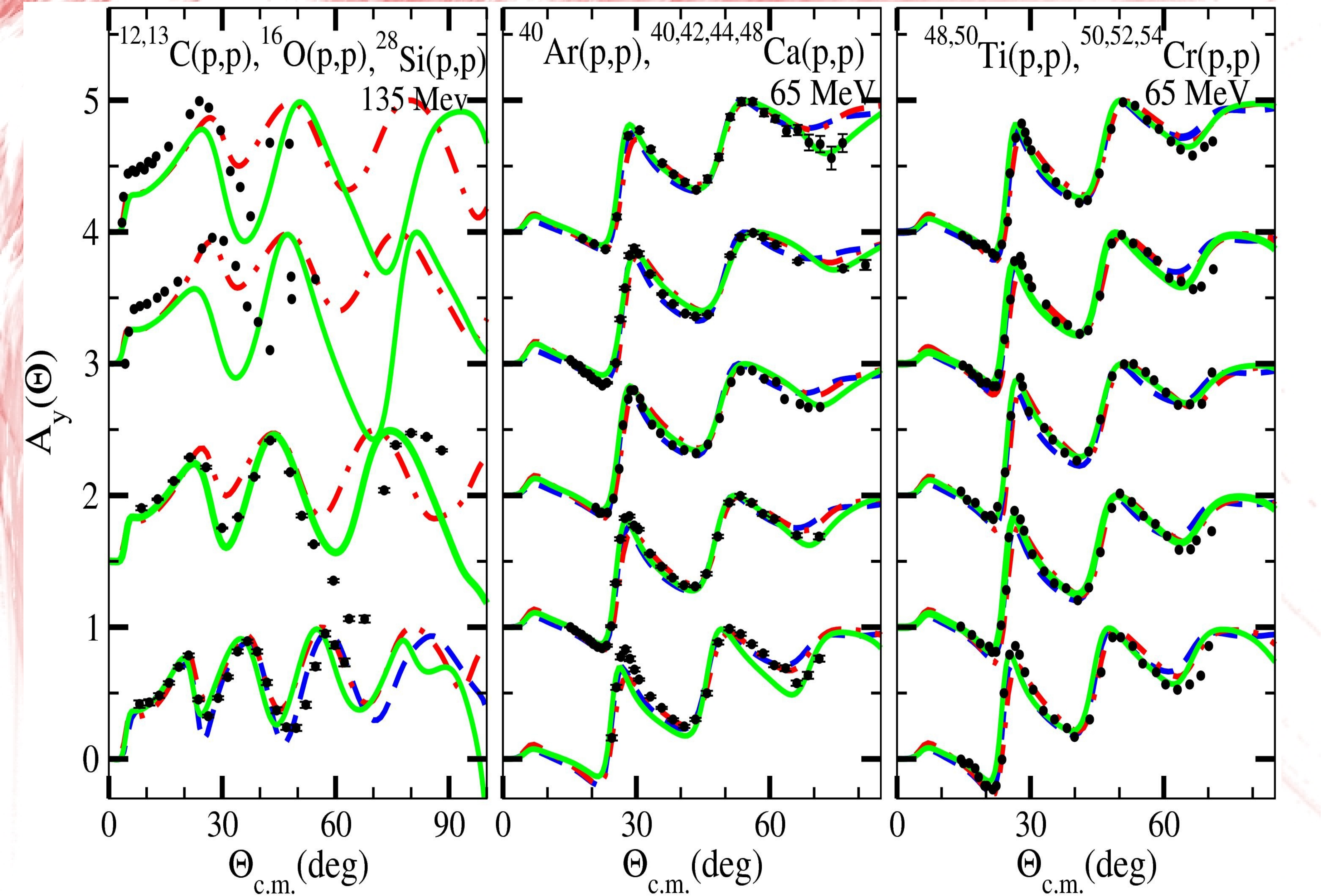
+1000 mb
+600 mb
+0 mb
-600 mb
Experiment
KD OMP
MD OMP
WP OMP
3000
2500
2000
1500
1000
500
σ_T(mb)
30
60
90
120
150
$E_{lab(MeV)}$
^{31}P
^{32}S
^{39}K
^{48}Ti



$\sigma(\Theta)/\sigma_{Ruth}$
^{12}C, ^{13}C, ^{16}O, ^{28}Si(p,p)
135 MeV
^{16,18,20,22}O(p,p)
43 MeV
43 MeV
43 MeV
47 MeV
^{44}Ar(p,p)
33 MeV
^{40}Ca(p,p)
30 MeV
^{40}Ar(p,p)
30 MeV
^{42}Ca(p,p)
30 MeV
^{42}Ar(p,p)
33 MeV
$\Theta_{c.m.}$(deg)
$\Theta_{c.m.}$(deg)
$\Theta_{c.m.}$(deg)

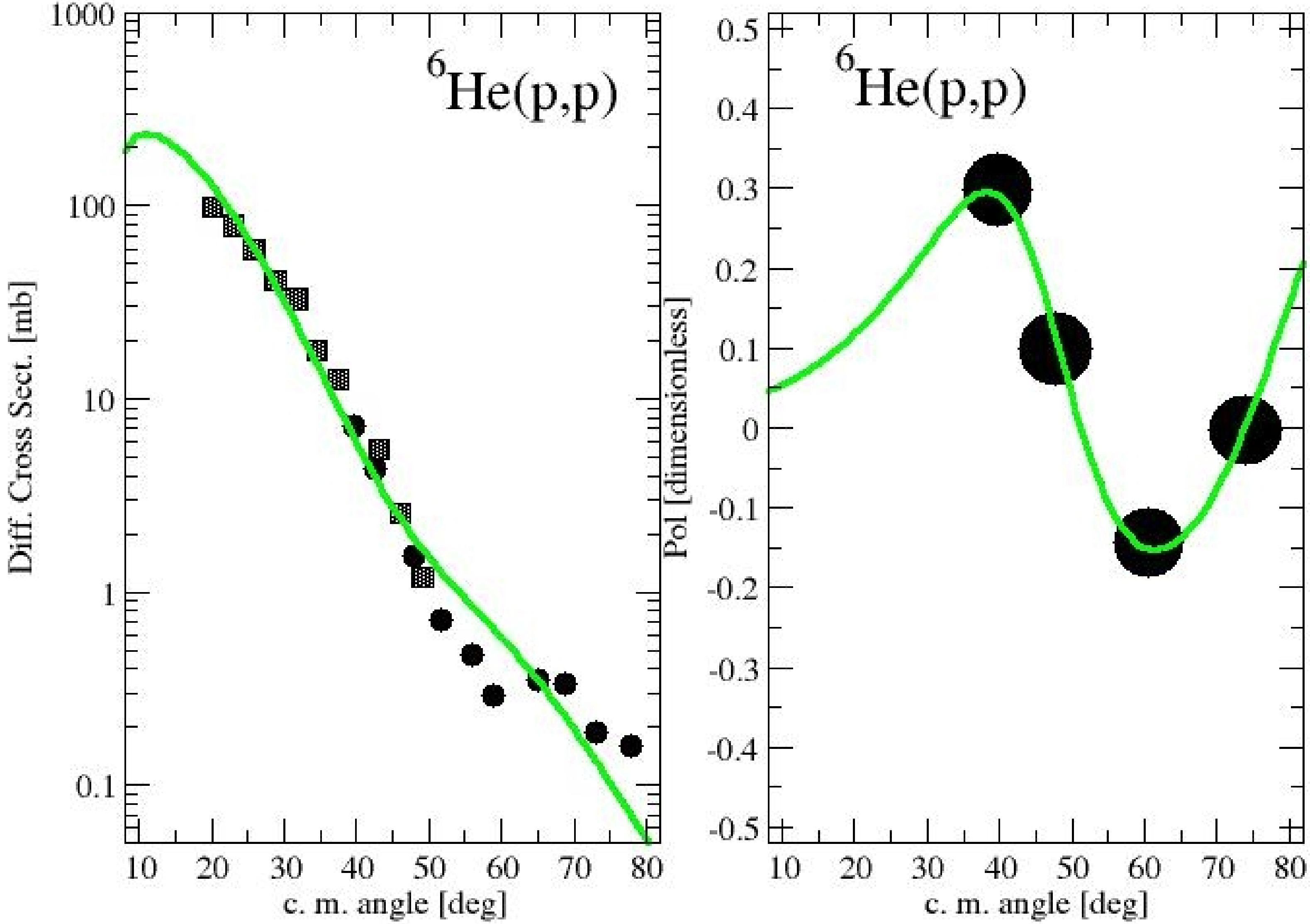


^{34}Ar(p,p)
47 MeV
^{30}S(p,p)
53 MeV
^{40}Ar(p,p)
65 MeV
^{45}Sc(p,p)
50 MeV
^{40,42,44,48}Ca(p,p)
65 MeV
^{50,52,54}Cr(p,p)
65 MeV
$\sigma(\Theta)/\sigma_{Ruth}$
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
30
60
90
$\Theta_{c.m.}$(deg)



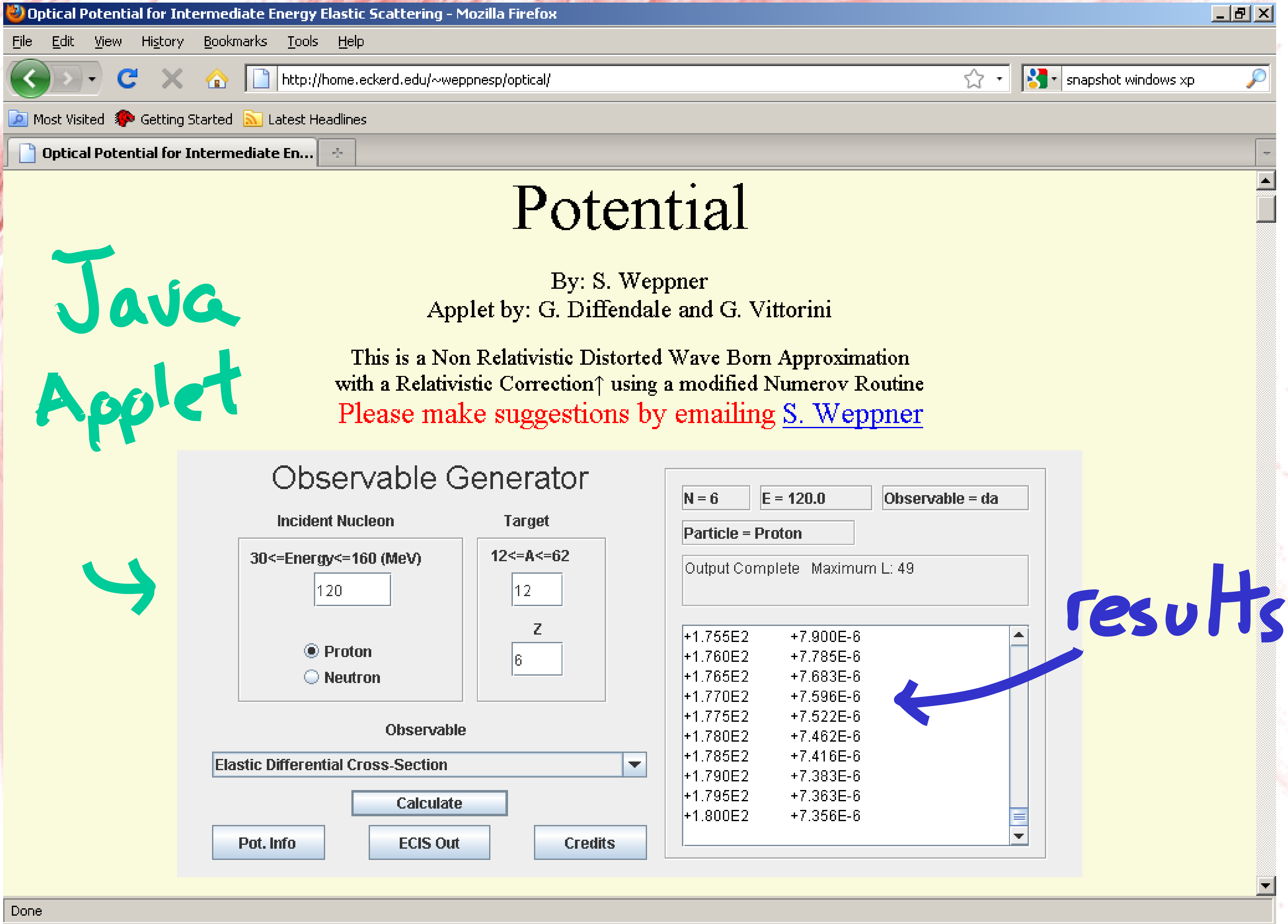
^{12,13}C(p,p), ^{16}O(p,p), ^{28}Si(p,p)
135 Mev
^{40}Ar(p,p), ^{40,42,44,48}Ca(p,p)
65 MeV
^{48,50}Ti(p,p), ^{50,52,54}Cr(p,p)
65 MeV
$A_y(\Theta)$
$\Theta_{c.m.}$(deg)

My try at the polarized proton RIKEN experiment …



Optical Potential Calculator

A Java Applet which calculates the elastic observables and also produces the potential parameters


Optical Potential for Intermediate Energy Elastic Scattering - Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://home.eckerd.edu/~weppnesp/optical/
snapshot windows xp
Most Visited Getting Started Latest Headlines
Optical Potential for Intermediate En...
Potential
By: S. Weppner
Applet by: G. Diffendale and G. Vittorini
This is a Non Relativistic Distorted Wave Born Approximation
with a Relativistic Correction↑ using a modified Numerov Routine
Please make suggestions by emailing S. Weppner
Java Applet
Observable Generator
Incident Nucleon
Target
30<=Energy<=160 (MeV)
120
12<=A<=62
12
Z
6
Proton
Neutron
Observable
Elastic Differential Cross-Section
Calculate
Pot. Info
ECIS Out
Credits
N = 6
E = 120.0
Observable = da
Particle = Proton
Output Complete Maximum L: 49
+1.755E2 +7.900E-6
+1.760E2 +7.785E-6
+1.765E2 +7.683E-6
+1.770E2 +7.596E-6
+1.775E2 +7.522E-6
+1.780E2 +7.462E-6
+1.785E2 +7.416E-6
+1.790E2 +7.383E-6
+1.795E2 +7.363E-6
+1.800E2 +7.356E-6
results
Done

The complete volume term

Real:

$$\mathcal{V}_V = V_{V_0} + V_{V_1}A + V_{V_2}A^2 + V_{V_3}A^3 + V_{V_5}E + V_{V_6}E^2 + V_{V_7}E^3$$
$$+ \mathcal{P}(N-Z)\left(V_{V_{i0}} + V_{V_{i1}}A + V_{V_{i2}}A^2 + V_{V_{i3}}A^3 + V_{V_{i4}}A^4 + V_{V_{i5}}E + V_{V_{i6}}E^2\right)$$
$$+ MN\left(V_{V_{m0}} + V_{V_{m1}}A + V_{V_{m2}}A^2 + V_{V_{m3}}A^3 + V_{V_{m5}}E + V_{V_{m6}}E^2\right),$$

Imaginary:

$$\mathcal{W}_V = W_{V_0} + W_{V_1}A + W_{V_2}A^2 + W_{V_3}A^3 + W_{V_5}E + W_{V_6}E^2 + W_{V_7}E^3$$
$$+ \mathcal{P}(N-Z)\left(W_{V_{i0}} + W_{V_{i1}}A + W_{V_{i2}}A^2 + W_{V_{i3}}A^3 + W_{V_{i4}}A^4 + W_{V_{i5}}E + W_{V_{i6}}E^2\right)$$
$$+ MN\left(W_{V_{m0}} + W_{V_{m1}}A + W_{V_{m2}}A^2 + W_{V_{m3}}A^3 + W_{V_{m5}}E + W_{V_{m6}}E^2\right).$$

Antisymmetric   Magic Number

Asymmetry and Charge Exchange (N-Z) Terms

Fermi and Gamow-Teller Sum rules lead to (N-Z) factors on the isospin dependent terms

Volume integrals of the NN asymmetry piece have a long history as input to calculate charge exchange cross sections

- Spin Dependent - Gamow-Teller Resonances
- Spin Independent - Fermi Resonances

Osterfeld, Reviews of Modern Physics 64, 490 (1992)

Neutron scattering at 135 MeV - total reaction cross section

Target	σ [mb]
^{16}O	269
^{18}O	168
^{20}O	44
^{22}O	unphys
^{24}O	unphys

Target	σ [mb]
^{40}Ca	514
^{42}Ca	432
^{44}Ca	282
^{46}Ca	61
^{48}Ca	unphys

Nucleon-Nucleon

$$V_{asym} \approx \sum_{i}^{A} V_{isoNN}(\tau_{proj} \cdot \tau_i)$$

$$V_{asym} \approx \sum_{i}^{A} V_{isoNN} \frac{1}{2}(\tau_{proj_+}\tau_{i_-} + \tau_{proj_-}\tau_{i_+} + 2\tau_{proj_z}\tau_{i_z})$$

$$V_{asym}(t_{proj} = +\frac{1}{2}) - V_{asym}(t_{proj} = -\frac{1}{2})$$

$$\approx V_{isoNN}(\frac{N}{2}\tau_{proj_-} - \frac{Z}{2}\tau_{proj_+})$$

$$-V_{isoNN}(\frac{N-Z}{4}(\tau_{proj_z}(+\frac{1}{2}) + \tau_{proj_z}(-\frac{1}{2})))$$

Nucleon-Nucleus

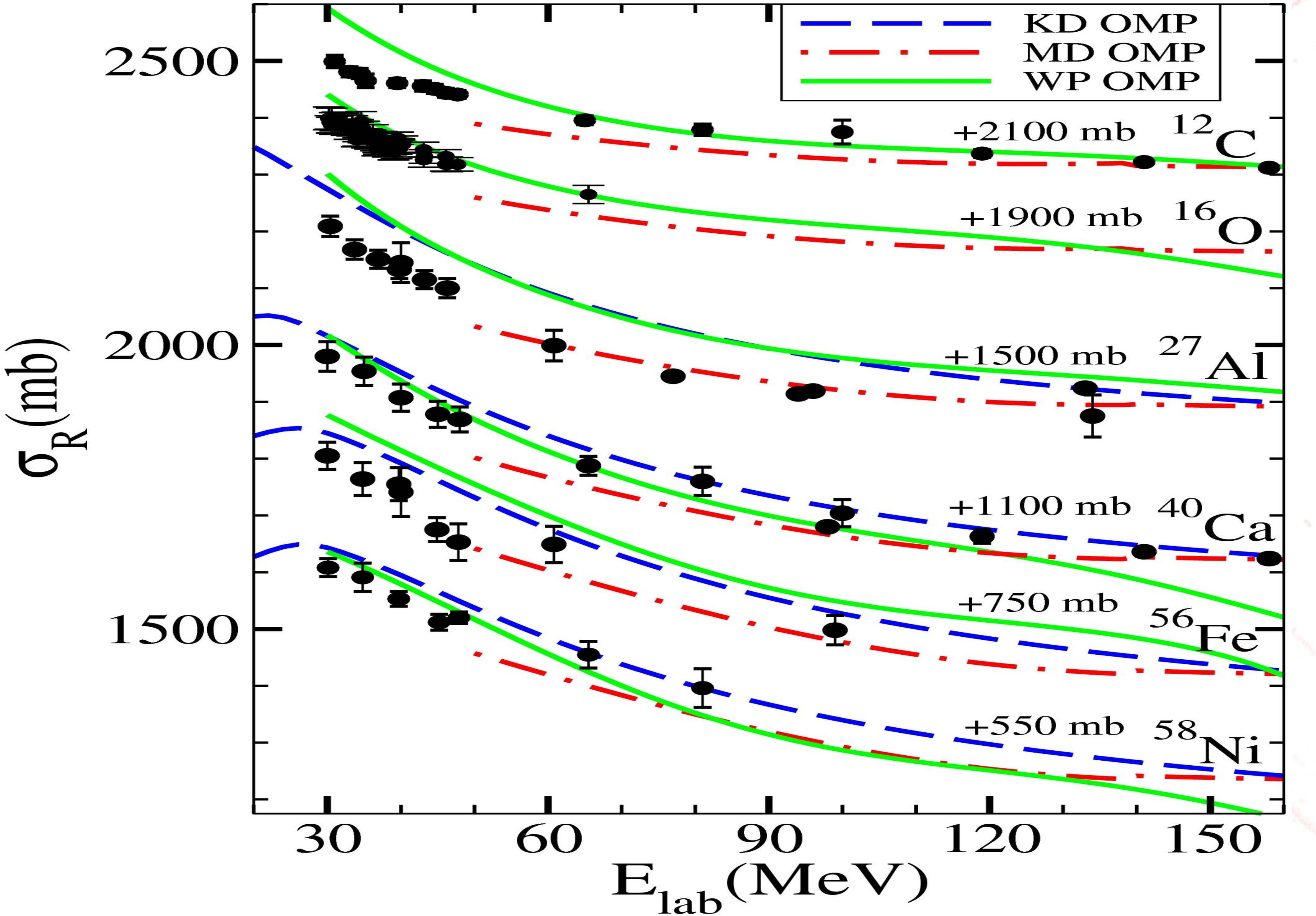
$$V_{asym} = V_{isoNA}(\tau_{proj} \cdot \tau_{targ})$$

$$= V_{isoNA}(\tau_{proj_+}\tau_{targ_-} + \tau_{proj_-}\tau_{targ_+} + \tau_{proj_z}\tau_{targ_z})$$

$$V_{asym}(t_{proj} = +\frac{1}{2}) - V_{asym}(t_{proj} = -\frac{1}{2})$$

$$\approx V_{isoNA}(\tau_{proj_-}\tau_{targ_+} - \tau_{proj_+}\tau_{targ_-} + \tau_{targ_z})$$

$$|\tau_{targ_-}| = |\tau_{targ_+}| \propto \sqrt{N - Z}$$


KD OMP
MD OMP
WP OMP
+2100 mb ^{12}C
+1900 mb ^{16}O
+1500 mb ^{27}Al
+1100 mb ^{40}Ca
+750 mb ^{56}Fe
+550 mb ^{58}Ni
2500
2000
1500
σ_R(mb)
30
60
90
120
150
E_{lab}(MeV)

Neutron scattering difference : 40Ar – 40Ca

Energy	Model	Real Vol. Int. Diff	Imag. Vol. Int. Diff.	Elastic CS	React CS	Total CS
MeV		MeV fm3	MeV fm3	mb	mb	mb
	KD	3.1	0.0	14	-28	-14
50	MD	32.7	0.0	-36	-9	-45
	WP	10.8	8.0	+68	-51	+17
	KD	5.5	0.0	-19	-4	-23
150	MD	39.7	0.0	-106	-5	-111
	WP	-0.8	21.5	+188	-287	-99

Conclusions

- Development of a new global optical potential

- It has the apparatus for systematic analysis of the “traditional” terms

Future steps:

- A better term for anti-symmetry

- Inclusion of charge-exchange constraints

Thanks!