# Instructor's Guide

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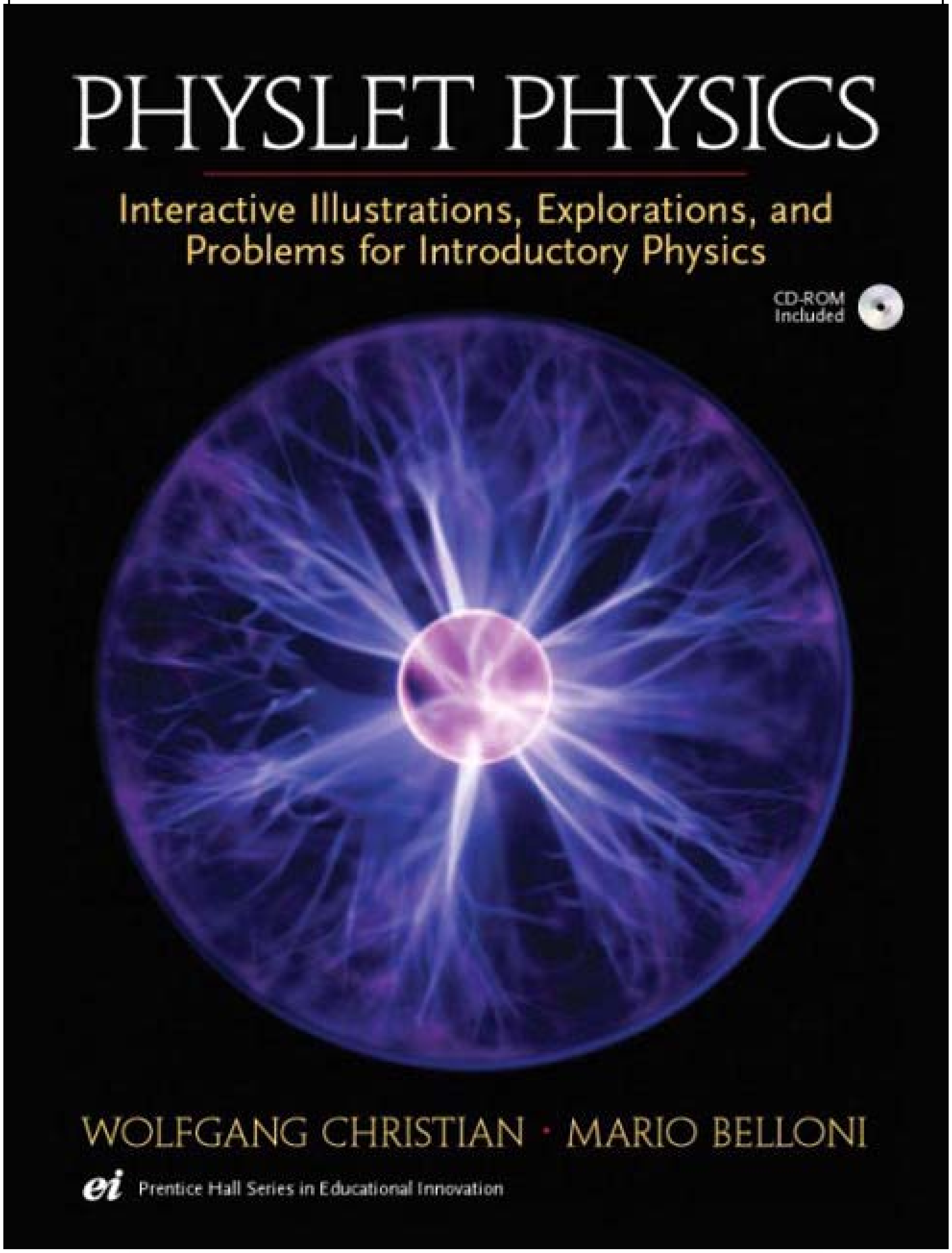

# PHYSLET PHYSICS

## Interactive Illustrations, Explorations, and Problems for Introductory Physics

CD-ROM Included

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# Acknowledgements

Many people have had a hand in this project. We are grateful to them for the insights and motivations they have brought us. A complete acknowledgement of those involved in the Physlet Physics book is found in the preface to that book. Here, we would like to give additional acknowledgement to those who participated in the development of the instructors' guide.

We are thankful to Davidson College students Peter Campbell, Todor Penev, and Laura Cupples for their assistance in working out and checking solutions and to Aaron Titus for contributing the suggested Exploration use for Chapter 13. This manuscript was reviewed by Mario Belloni and Wolfgang Christian. They gave us valuable feedback that has improved this guide as well as much support and encouragement.

Finally, this work was supported by the National Science Foundation under contracts DUE-9752365 and DUE-0126439.

# Part I

## *Discussion of the Pedagogy*

# Chapter I-1: Introduction

By now it is hard to imagine an instructor who has not heard the call to "teach with technology," as it has resounded through educational institutions and government agencies alike over the past several years. However, teaching with technology has often resulted in use of technology for technology's sake and the development of tools that are not pedagogically sound. For example, PowerPoint lectures, a popular response to the teach with technology push, are more colorful, but generally no more interactive than chalkboard lectures. The physics community has, to its credit, worked to use technology in a variety of highly effective ways including wireless classroom response systems that allow for in class quizzing of students and MBLs (mirco computer based laboratories) that free students from the drudgery of data collection so that they can spend more time understanding the underlying physical concepts. Into this, we offer *Physlet Physics*, a collection of ready to run interactive computer simulations designed with a sound use of pedagogy in mind. Our aim is to provide a resource that will use technology in teaching in a way that enhances student learning and interactive engagement with physical concepts which is, at the same time, flexible enough to be adapted to a variety of pedagogical strategies and local environments.

## Illustrations, Explorations and Problems

*Physlet Physics* provides instructors with a collection of exercises spanning the introductory physics sequence. These exercises are computer animations (Java applets) with physics content: Physlets = **Phy**sics content simulated in Java app**lets**. Every chapter of *Physlet Physics* contains three quite different Physlet exercises: **Illustrations**, **Explorations** and **Problems**.

**Illustrations** are designed to demonstrate physical concepts. Students need to interact with the Physlet, but the answers to the questions posed in the Illustration are given or are easily determined from interacting with it. Many Illustrations provide examples of physics applications. Other Illustrations are designed to introduce and illustrate a particular concept or analytical tool. Typical uses of Illustrations would include "reading" assignments prior to class and classroom demonstrations. For example, consider Illustration 13.2 from the chapter on statics as shown in Figure I.1.1. The text of this Illustration asks students to observe the position of the center of mass as they move a block. This Illustration also explains how to calculate the center of mass and discusses the difference between center of mass and center of gravity.

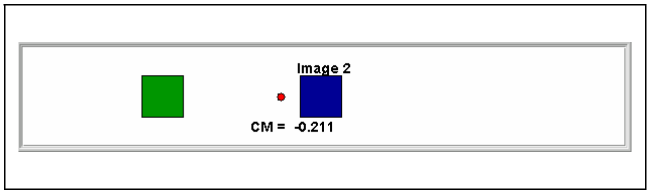


**Figure I.1.1: Demonstrating the position of the center of mass between two blocks of unequal mass in Illustration 13.2.**

**Explorations**, meanwhile, are more tutorial in nature. They provide some hints or suggest problem-solving strategies to students in working problems or understanding concepts. Some Explorations ask students to make a prediction and then check their predictions, explaining any differences between predictions and observations. Other Explorations ask students to change parameters and observe the effect, asking students to develop, for themselves, certain physics relationships (equations). Typical uses of Explorations would be in group problem solving and homework or pre-laboratory assignments. Explorations are also often useful as Just-in-Time Teaching exercises (see chapter I.3). Consider Exploration 13.2 (Figure I.1.2). This Exploration asks students to apply what they learned about center of mass in Illustration 13.2 in order to explain how to build a mobile. In the first part (Figure I.1.2a), students must use the position of the center of mass (must be located somewhere below the support string) to determine the mass of the unknown block (green block). Students will continue to use conditions for static equilibrium to determine the masses of the orange and red blocks (Figure I.1.2b). As they work through this Exploration, they can verify their calculations because the position of the center of mass must remain beneath the support string from the ceiling.

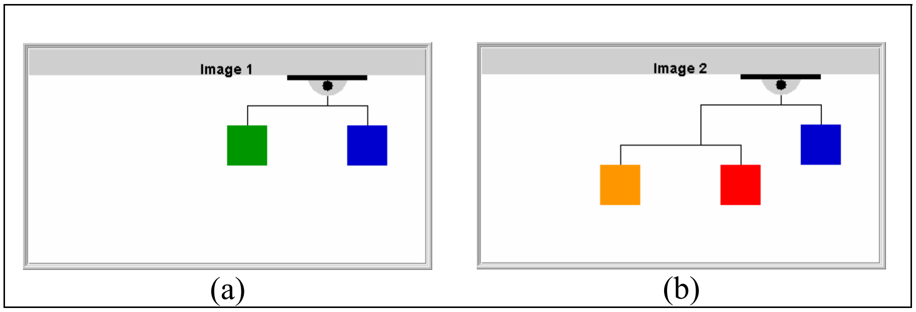
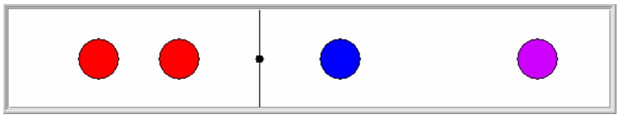


**Figure I.1.2: Applying the concept of the center of mass to understanding how to balance a mobile in Exploration 13.1.**

**Problems** are the kinds of exercises you might assign for homework. They require the students to demonstrate their understanding without as much guidance as is given in the Explorations. They are on many different levels (high school physics to calculus-based university physics). Some Problems ask conceptual questions, while others require detailed calculations. Typical uses for the Problems would be for homework assignments, in-class concept questions, and group problem solving sessions. From the same chapter as the examples above, consider Problem 13.12 (in Figure I.1.3). Here students can use what they have learned by seeing how the position of the center of mass changes as a mass gets moved in Illustration 13.2 and completing Exploration 13.1 which forced them to check their center of mass calculations to solve the problem.



**Problem 13.2:** Four spheres are shown in the image. A blue sphere is half as massive as a red one and a purple sphere is twice as massive as a red one. Where should the purple one be placed in order for the center of gravity to be at the location of the black dot **(position is given in meters)**?

**Figure I.1.3: Center of mass calculation required for Problem 13.12.**

## Resources for *Physlet Physics*

Part I of this instructor guide provides a general overview and the pedagogy behind *Physlet Physics*. Specifically, Chapter I.2 describes pedagogical advantages we have found using Physlet based exercises over the years and then Chapter I.3 describes how *Physlet Physics* exercises can be used with a variety of specific pedagogical strategies that have evolved from physics education research on how students learn. And Chapter I.4 provides some technical information on computer requirements for use.

After this overview, Chapters I-5 through I-11 discuss the specific content of *Physlet Physics*, providing brief highlights and examples from each section in the form of "Section Highlights". These "Section Highlights" should give you a flavor of the variety of exercises included and a sense of how they might be used in the classroom. Since *Physlet Physics* was not written to accompany a specific textbook, each section highlight also describes what material is (and is not) covered in that particular section.

Part II is an answer key for the book. It provides answers for all the Problems and complete solutions for selected Problems. It also gives some specific suggestions for classroom use of the Illustrations. In the answer key, we also indicate the topic(s) covered in each Illustration, Exploration and Problem. Additionally, each Problem has an indication if calculus is required (i.e., not appropriate for algebra-based physics) and a difficulty level indicator (1-3):

Level 1: Straight forward, one-step problems
Level 2: Problems requiring more than one step
Level 3: Multi-step problems that all but the best students would likely need help to answer.

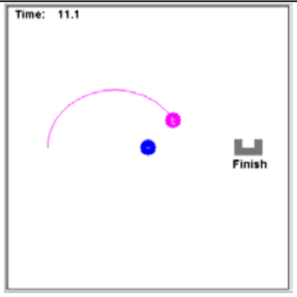
Of course, the level assignment should only be used as a guide as not all Problems clearly fall into one of the designated levels.

Finally, Part III is a collection of worksheets to accompany the Explorations. Since the Explorations are more tutorial in nature, we have developed ready-to-use worksheets for students (both as pdf files and Word documents for local modification). The worksheets are designed to provide structure to the tutorial nature of Explorations. At times, they provide additional guidance to students in solving the questions asked in the Explorations. All worksheets are designed to help students organize and clearly explain their solutions. They can be modified locally and distributed to classes of students as they work independently or in groups on particular problems. A collection of completed worksheets serve as the solutions manual for the Explorations.

## Classroom Environments

"All teaching is local." -- Unknown

Adoption and adaptation of all curricular materials is, of course, dependent on the local environment and many instructors may be hesitant to use *Physlet Physics* in classrooms that do not have a computer for every student. This need not be the case. The exercises in *Physlet Physics* are designed to be flexible enough for a one-computer-one-classroom situation to a computer-rich environment where every student has access to a computer during the entire class. As an example, consider the following Problem (Problem 22.8) shown in Figure I.1.4. In this Problem, the student can change the initial velocity (x and y components) of the pink test charge as well as the charge of the central blue charge. Students are asked to determine initial conditions so that the pink charge will land in the gray "Finish" box.



A positive test charge of 1 x $10^{-5}$ C with a mass of 0.9 kg is shown near a variable charge with a fixed position. You can change the charge of the central charge and the initial velocity of the positive charge. Set the charge of the central charge to -20 x $10^{-5}$ C:

a. What initial velocity must you give the test charge so that the test charge can make it from its starting place to the finish line in a circular path?

b. For an arbitrary negative central charge, Q, what initial velocity must you give the test charge so that the test charge can make it from its starting place to the finish line in a circular path? Your answer should be a formula for v in terms of Q. When you have an answer test it with Q = -10 x $10^{-5}$ C and Q = -30 x $10^{-5}$ C.

**Figure I.1.4: Connecting circular motion with Coulomb's law in Problem 22.8**

In a one-computer (with a data projector) classroom, the instructor can run the animation with some initial conditions that will not solve the problem. Then, the instructor can work with the class to solve the problem, testing suggestions made by the students. With a few computers around the classroom, an instructor can show this problem to students and then ask that they move to a computer to test it when they think they have a correct solution. In the computer-rich environment, students can try to game it (by simply trying different initial conditions), but it is unlikely that they will develop the correct mathematical relationship from such an inefficient approach. In all cases, the same problem can be used a classroom with very different computer resources.

# Chapter I-2: General Pedagogy of Physlets

During the years the vast collection of Physlet Illustrations, Explorations, and Problems were developed, we have used them in our classrooms and have continually sought to understand the value they might have. We have viewed countless student solutions, talked to students informally in office hours, carefully considered comments on end of semester evaluations and surveys, and even conducted problem-solving interviews with students. In the end, it has become clear that Physlet-based materials, by their very nature, offer many advantages over their traditional paper-based counterparts.

In this chapter we list and discuss the most significant aspects of Physlets that we believe makes them a valuable component of any physics course. Throughout, we have included comments from introductory physics students at Davidson College. These comments were made in writing by students on either their end-of-semester evaluation (which did not specifically ask about Physlets) or in response to a survey (specifically addressing Physlets) that was given in the introductory courses over a three-year period.

## Physlet-based materials help students to visualize abstract concepts.

> *" Physlets do help me visualize problems at hand and are quite a bit more interesting than the text."* – Student Comment
>
> *"The (Physlet) problems show one what actually occurs and just doesn't force one to guess what may be happening."* – Student Comment

When we ask students what they think of Physlets, or review end-of-semester evaluations, what is striking is how often they mention that Physlets help them to visualize situations. This appears to be the most obvious benefit to students. If nature is only presented to a student through words (verbal or written) and static pictures, the student is left to construct an understanding through internal visualization. While the ability to internally visualize is important, students are called upon to do it quite often in a typical physics course. Any weakness they have in their ability to internally visualize will seriously hamper their progress toward understanding the concepts of physics.

In introductory physics, motion is often a central concept. In this case, Physlet exercises will obviously help students to better visualize a situation. A verbal or written description of motion can not possibly convey a concept involving motion as effectively as a description that actually contains the motion. We have included a large number of Physlets that specifically ask students to connect their visualization of motion with other representations of motion.

Increased visualization is also going to be a key benefit when a concept involves either abstract ideas (such as electric and magnetic fields) or phenomena that take place on a scale that is not readily observed in our everyday lives. As an example, consider Illustration 18.4 shown in Figure I.2.1.

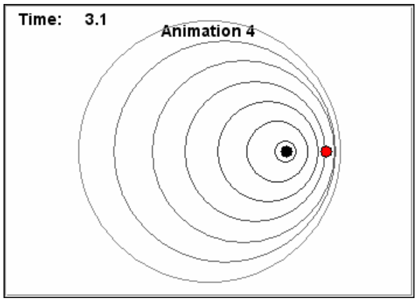


**Figure I.2.1: Illustration 18.4, a demonstration of the Doppler effect.**

The screenshot shown in Figure I.2.1 does even come close to doing this Illustration justice. If you were viewing this Illustration you would see the sound source (black dot) moving toward the red dot. As the sound source moves it emits sound waves (represented by the black circles) that are symmetrical around the location of the sound source at the instant produced. Upon viewing this Illustration it is obvious why the waves appear closer together when the sound source of moving toward the observer and why they would be farther apart if the source were moving away. A student may believe the Doppler effect after hearing the effect of a buzzer being swung on a string in front of the class, and he/she may be able to solve problems using the Doppler effect after working through the mathematics, but neither of these insures that the student understands **why** there is a Doppler effect. The Physlet-based exercise can help the student to understand why by helping him or her to visualize the situation.

Notice that the student viewing the animation of the Doppler effect will also see that the effect varies depending on the location of the observer. This point is usually glossed over in a typical introductory course because the mathematics of observers not in line with the motion is more difficult. Because of their visual nature, it is not unusual for Physlet-based exercises to expose students to concepts that are too difficult or abstract to cover with only words, static pictures and mathematical equations.

## Physlet-based materials discourage novice-problem solving approaches.

> *"[The Physlets] are a lot harder because the data often times does not seem to lend to a possible formula with a solution."* – Student Comment

Introductory level students will often use a plug-n-chug approach of problem solving when presented with a typical end-of-the-chapter textbook problem.[1] The student is likely to start the problem by identifying the known and desired variables and then search through a list of equations, shopping for one that has the letters corresponding to the identified variables. Once they find one, they plug in the knowns, solve for the desired variable, box it in, and then move on to the next problem. Missing from the student's

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[1] The propensity of students to employ a plug-n-chug approach to problem solving has been extensively documented in the research literature. An excellent introduction to this topic is included in Maloney, D. P. (1994). Research on Problem Solving: Physics in Gabel, D. (Ed) Handbook of Research on Science Teaching and Learning, The National Science Teachers Association.

solution process are the most important aspects of true problem solving: Defining the problem by identifying relevant concepts and principles, relating conceptual ideas to the mathematics, and evaluating the reasonableness of the final answer.

It has been documented[2] that students can solve a large number of traditional physics problems successfully without gaining significant understanding of the underlying physics principles and concepts. Since students are not forced to conceptualize the problem when solving a traditional problem, it is unreasonable to expect significant gains in true understanding through this type of problem solving activity.

It is often the case with Physlet-based problems that the novice approach of plug-n-chug is impossible to implement. For example, consider Problem 23.6 and a similar traditional problem shown in Figure I.2.2.

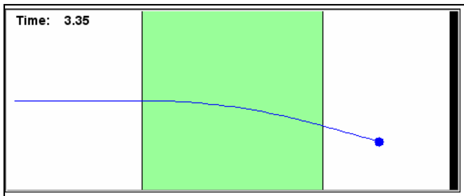
| | |
|---|---|
|  An electron is shot into a region of constant electric field shown in green **(position is given in centimeters and time is given in µsec, $10^{-6}$ s)**. What is the magnitude and direction of the electric field? | An electron is shot into a region of constant electric field. If it takes 1.65 µs to pass through the field and is deflected 14 cm, what is the magnitude of the electric field? |
| **Physlet Problem** | **Traditional Problem** |

**Figure I.2.2: Comparison of *Physlet Physics* Problem 23.6 with a traditional problem.**

The traditional problem identifies the important quantities for the student. In doing so, it gives the student a list of variables that can be plugged into a matching formula. Even though this is a somewhat difficult problem since it requires multiple steps (finding the acceleration, then finding the force, and finally the field), a plug-n-chug approach will still be effective. In the Physlet version of the question, no numbers or variables are explicitly given. In addition, the problem is actually overstated. The amount of "data" that could be collected from the Physlet is almost unlimited. This requires that students do at least some conceptualization before beginning the algebraic manipulation. They must determine what is important and how the important information can be extracted from the Physlet. Only then can they proceed to solve the problem. As one of our students put it when asked about the difficulty of Physlet problems, "Theoretically they should not be more difficult, but they tend to take more thinking and are not as simple as just taking a couple of set numbers."

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[2] For example, see Kim, E. & Pak, S. (2002). Students do not overcome conceptual difficulties after solving 1000 traditional problems. American Journal of Physics, 70(7) pp. 759-765.

## Physlet-based problems are more like problems in the real world.

> *"[The Physlets] help to see a practical use of physics and they do mock real life situations."* – Student Comment
>
> *"The [Physlets] are left up to interpretation. Because of this, the data collected may not be exactly like the actual data therefore making what the student calculates or observes wrong."* – Student Comment

Real problems are very different from textbook problems. Real problems do not have all the necessary information clearly highlighted. The solving of a real problem will entail sorting through lots of irrelevant, and sometimes misleading, information. And finally, real solutions often involve measurement, and measurement means uncertainty. As discussed in the previous section on the reduction of novice problem solving approaches, Physlet-based problems naturally provide too much information while also leaving the student to determine what information is important to answer the question. But Physlet-based problems can also bring an experimental and uncertain aspect to problem solving.

Consider the second student quote given in the introduction to this section. This student appears to view being "left up to interpretation" as a negative and that there is "actual data" which can make "what the student calculates or observes wrong". This is a very interesting outlook and one which is likely shared by many others. After a lifetime of solving problems with prefabricated data leading to a precise answer, it is not surprising that students do not appreciate, or respond positively to, situations in which data must be interpreted. Because of their nature, Physlets offer a wonderful opportunity to expose students, on a routine basis, to ideas of data acquisition and interpretation that are typically only dealt with in the laboratory.

## Physlet-based material can improve assessment of student understanding.

> *"The material in interactive problems tends to be more challenging, however because they can be seen it is slightly easier to do. Being able to actually see the problem is beneficial."*– Student Comment

Sometimes when teachers give students a task to be completed we do so mainly because we believe completing the task is valuable for the student's learning. Such is typically the case with questions and problems assigned for homework. In other circumstances, such as exams, our goal is to assess how well our students have mastered a particular topic. We have found evidence that Physlet-based questions can be superior to traditional paper-based questions when the goal is to gauge true student understanding.

Dancy[3] investigated this issue by comparing student responses to traditional conceptually-based questions with responses to nearly identical Physlet-based questions.

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[3] Dancy, M. (2002). Investigating animations for assessment with an animated version of the Force Concept Inventory. Doctoral Dissertation.

She found that the distribution of responses was statistically different on one third of the questions and that the incidences where the Physlet version increased performance were equal to the number of incidences where performance was higher on the traditional question. In other words, the Physlet version often affected the way students answered a question but the Physlet version did not necessarily improve or decrease performance.

These results, along with an analysis of individual think-aloud interviews with students answering different forms of the questions, led Dancy to assert that in general, the Physlet version of the question was more valid. In other words, when the Physlet version decreased performance it was because it shifted students without a true understanding from the correct answer to an incorrect answer. For example, on one question students tended to have memorized the answer and gave this memorized answer when seeing the paper version of the question but gave the answer they really believed when seeing the Physlet version. Likewise when the Physlet version increased performance it could often be connected to students misunderstanding or misinterpreting what was written. When they saw the Physlet version, they had a better sense of what the question was asking and were therefore better able to express their understanding.

Finally, Dancy found that while performance on the traditional questions was statistically correlated with the students' English ACT score, no such correlation was found with the Physlet version of the questions (correlations were found with math ACT score for both versions of the questions). This finding, that verbal ability appears to play a larger role in the traditional paper-based assessment, is consist with the finding that the Physlet-based question tended to be clearer to the student. While reading ability is important, it is also important to get a clear assessment of a student's physics understanding independent of his/her ability to read. It appears that Physlet-based questions are generally superior at doing this.

## Physlet-based exercises are interactive.

> *"Use of Physlets on the computer is good for experimenting and observing."* – Student Comment

It has been shown that interactive techniques are superior at helping students develop a deep understanding of physical phenomena.[4] A number of successful techniques have been developed to bring interactivity into the physics classroom. Although Physlets by themselves could never substitute for true interactive teaching, they are well suited for most interactive teaching methods. In Chapter I.3, we discuss several of these methods in detail. Here we will only illuminate the reasons why Physlet-based materials are so well suited for interactive teaching.

It is often beneficial for students to predict an outcome before observing a situation. If their prediction is incorrect then they can work to resolve the discrepancy. This model of learning is beneficial for students as it forces them to confront their misunderstandings

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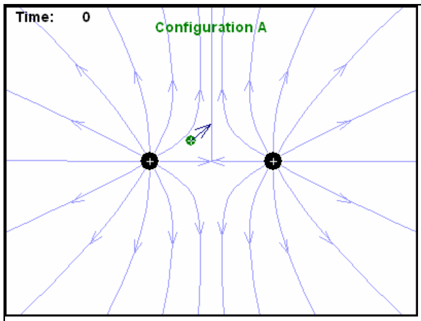
[4] For example, see Hake, R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. American Journal of Physics, 66, pp. 64-74.

rather than just add pieces of knowledge without integrating that knowledge with what they already know and think they understand. Basically, if students are not lead to explicitly confront their misunderstandings, they often don't. In *Physlet Physics*, especially in the Explorations, we often ask students to make a prediction and then interact with the Physlet to test their predictions. For example, consider Exploration 23.2 shown in Figure I.2.3. In this Exploration the student is presented with a situation involving the electric field due to two charges and asked to predict the path a positive test charge will follow if released. Many students will believe that the charge will follow the electric field lines exactly. Once they have made their prediction they can then play the animation and see that the path is not identical to the field lines. Hopefully the students will then confront their ideas and begin to understand why lines of force do not indicate path. There are several more parts to this question (not shown in the figure) that present students with a similar situation and ask them to make new predictions. These additional parts allow the students to try out their revised ideas. This particular Exploration also makes a good Peer Instruction question, which is discussed in Chapter I.3.

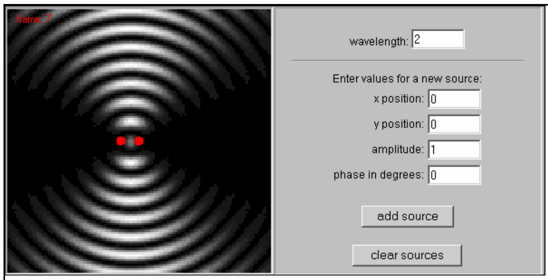


The animation shows two fixed charges and a test charge **(position is given in meters and time is given in seconds)**. The electric field lines due to the fixed charges and the force vector on the test charge are shown. The test charge will move under the action of the electric field when the animation is played.

Using Configuration A, drag the test charge to the approximate position of (-0.8 m, 0 m). Write down a prediction for the path the charge will follow after being released at this point. **After** you have made your prediction, play the animation. Was your prediction correct? If not, what caused your error?

**Figure I.2.3: Excerpt from Exploration 23.2 showing an interactive exercise.**

The previous example was highly structured. However, it can also be beneficial to let the student "play" with a given system to try out his or her ideas and to develop an intuitive understanding. Physlet-based exercises can give real-time feedback to students as they manipulate the parameters of a situation. As an example consider Illustration 37.1 shown in Figure I.2.4.



wavelength: 2

Enter values for a new source:

x position: 0

y position: 0

amplitude: 1

phase in degrees: 0

- What happens if the distance between the sources is increased or decreased?
- What happens if the phase between the sources is changed?
- What happens if the wavelength of the emitted waves is increased or decreased?
- What happens if more sources are added?

**Figure I.2.4: Excerpt from Illustration 37.1 showing a virtual ripple tank.**

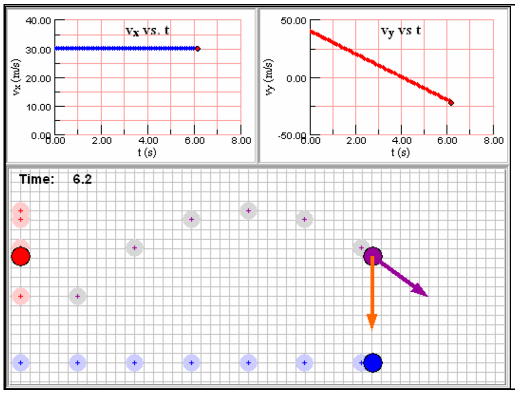
Illustration 37.1 provides students with a virtual ripple tank. The student can add wave sources, controlling the wavelength, position, amplitude and phase of each new source. There are some guiding questions, shown in the figure, included with the Illustration. However, the students are also free to try things beyond the directions given. Because of the flexibility of this particular Illustration there are a number of ideas students could explore.

Two aspects of interactive learning that make it so valuable for the development of genuine understanding are the ability to "probe" a given situation and the feedback that is received. The interactive, dynamic nature of Physlet-based material makes it ideally suited for the development of conceptual understanding.

## Physlet-based materials often use multiple representations.

> *"In addition to class demonstrations, the Physlets help bring to life sterile mathematical equations."* – Student Comment
>
> *"As individuals learn in different ways, little harm is done by presenting a different perspective on a given problem."* – Student Comment

The idea that students learn best when they see the same ideas presented in different ways is not new. But traditional physics instruction often falls well short of achieving this goal with a heavy reliance on the written and spoken word. Perhaps a few static diagrams are introduced, but by and large the students are exposed to the often complex ideas of physics in a few narrow modes. Physlet-based material can not only bring motion to the arena, which greatly helps students visualize ideas, but they can also provide simultaneous representations within a single Physlet. For example, consider Illustration 3.4 shown in Figure I.2.5.



A purple ball undergoes projectile motion as shown in the animation. The blue and red objects illustrate the x and y components of the ball's motion. Ghost images are placed on the screen every second. The velocity vector is shown in purple and the acceleration vector is shown in orange.

**Figure I.2.5: Excerpt from Illustration 3.4 showing multiple representations.**

A large number of representations are built into this Illustration. First of all, students can actually view the motion of the projectile (the purple ball). At the same time, they can watch the motion in just the x or y directions by focusing on the red and blue balls. As the balls move they leave ghost images, which give a lasting visual reference of the ball's path and velocity, since the ghost images are left in set time intervals. In addition the motion is plotted graphically, which can help students to connect their ideas about motion to their ideas about graphs. Finally, yet another representation is given, the velocity and acceleration vectors.

This particular Illustration is particularly rich in its provision of multiple representations but this theme has been integrated through the *Physlet Physics* book. You will find that the features in the above problem, such as showing real-time graphs and vectors, as well as many other forms of representation are massively incorporated into the Illustrations, Explorations and problems of the book.

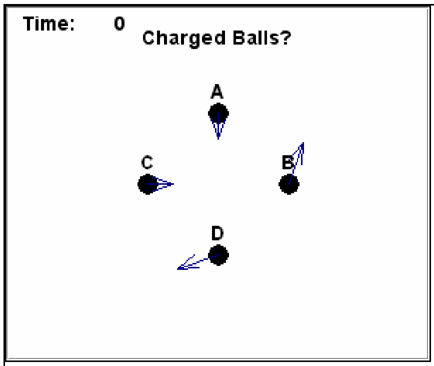
## Physlets are simple, focusing on substance rather than coolness.

Too often educational technologies are developed based on the technology, and pedagogy comes as an afterthought. We did not use this model in developing the curricular material found in *Physlet Physics*. Our model was to come up with an idea, based on what we thought students needed to understand, and then make that idea fit into the Physlet paradigm. The curriculum development drove the technical development as new features were added to the Physlet code to meet pedagogical goals. The result is a curricular package that is not designed to entertain and is more impressive to teachers and their students than to computer programmers.

Physlets are also very simple. Only the important aspects of a situation are included. There are not distracting graphics, animations, or sounds. This allows students to attend to the substance of the Physlet without being distracted by unnecessary or overly flashy additions.

## Physlets can model non-physical situations.

Because the Physlets are not constrained to correct physics, some of the exercises included in *Physlet Physics* ask students to identify non-physical animations and explain what is wrong with them. This forces students to realize that Physlets are simply a simulation, not the “real world,” and physics involves describing real observations. It also provides a different way to probe student conceptual understanding. In order to solve such problems, students must be able to describe what should happen not only mathematically, as is required for most traditional problems, but in terms of a dynamic simulation. For example, Problem 22.2 shown in Figure I.2.6 asks students to determine if anything is wrong with the animation. The student must drag the charges around to determine the force each exerts on the other. This animation does not follow the notion that there are only two kinds of charge and that like charges repel while unlike charges attract. A different kind of thinking is required to answer these questions and Physlets are well suited to probe students in this manner.



Four charged objects are shown on the screen along with vectors representing the forces on each object. You can click-drag on any object to change its position **(position is given in meters)**.

What, if anything, is wrong with the animation?

**Figure I.2.6: Problem 22.2 showing a nonphysical situation.**

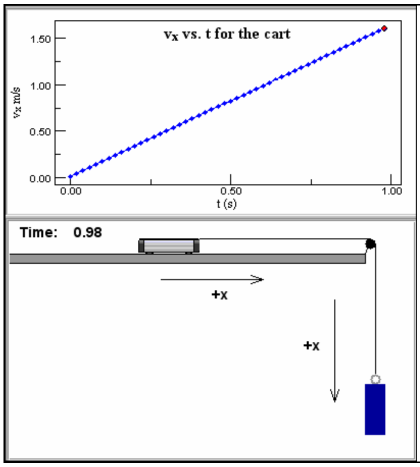
## Physlet-based materials are cheaper and easier to maintain than laboratory equipment and are not restricted to in class use.

> *"The Physlets in general are very helpful because they demonstrate concepts without taking the time of an actual demo."* – Student Comment

Many of the curricular materials found in *Physlet Physics* mimic hands-on activities typically done in the laboratory or in-class demonstrations. We have never advocated that Physlet materials replace real hands-on activities or demonstrations with real equipment. However, because of their similarity, we believe the same benefits associated with such "real" activities can also be realized with Physlet-based activities. The biggest difference is that Physlets are much easier to use.

*Physlet Physics* is much cheaper than laboratory equipment and will not need repairing. This allows for more opportunity for students to be engaged in laboratory type activities where students can quickly change parameters and see the outcome. Even more important, students can work through a Physlet activity without being supervised. Students do no have to be in a particular location or working at a particular time in order to benefit from the Physlet activities. They only need a working computer.

As an example consider Problem 7.6 shown in Figure I.2.7. The problem is very similar to a typical laboratory experiment. We often have students answer these types of problems before coming to lab as a pre-lab exercise to prepare them for the laboratory. We discuss these types of questions further in Chapter I.3.



A 2.5-kg cart on a low-friction track is connected to a string and then to a 0.5-kg hanging mass as shown in the animation. Neglect any effects of the massless pulley on the motion of the system **(position is given in meters and time is given in seconds)**.

During the animation:

What is the total work done by the tension on the two-object system?

**Figure I.2.7: Excerpt from Problem 7.6 showing a typical laboratory based situation.**

## Physlets are scriptable.

Many teachers using *Physlet Physics* will find a problem that they somewhat like but would prefer a slight modification. It is actually easy to modify an existing problem. If you look at the source code for a web page with a Physlet in it, you will see that there is some JavaScript near the top. The JavaScript is what tells the underlying Physlet what objects to create and how to make them interact. By changing the JavaScript you can modify the problem. If you are interested in learning how to script Physlets, the book *Physlets: Teaching Physics with Interactive Curricular Material* is your guide.

We have made a conscious effort to develop the materials in *Physlet Physics* around a sound pedagogical base. As we have outlined in this chapter we have been successful at implementing many of our intentional goals and have been pleasantly surprised to find a number of unexpected pedagogical advantages emerge with classroom use. We believe the curricular material offers many advantages to the physics student and is an effective merging of technology and pedagogy.

# Chapter I-3: Using *Physlet Physics* in the Classroom

The material in *Physlet Physics* is designed to be used with most teaching styles and at most levels of introductory physics. We recognize that the style and preferences of individual instructors, as well as the vastly different student population, call for various teaching approaches. We believe the material in *Physlet Physics* will complement almost any teaching style. If you prefer a traditional lecture-based approach you will find that the Illustrations bring a visual and dynamic nature to the lecture presentation. If you teach in a more interactive classroom, you will find that many of the Explorations and Problems bring out conceptual difficulties that will lead to interesting group discussions. Many of the questions also make excellent Just-in-Time Teaching or Peer Instruction questions as is discussed below.

We have also attempted to include material that would be appropriate for conceptual high school physics to advanced calculus-based university students. You should be able to find ample material that meets the needs of your particular students. Questions that require calculus are marked in the solutions section of this instructor's manual.

Over the years that we have been using Physlet-based material in the classroom, we have discovered that it is essential to **orient students to the technology**. Chapter 1 in the *Physlet Physics* book provides some Physlet-based material for this orientation. If you do not orient students in some fashion it is likely that your students will become exceptionally frustrated. This orientation should include,

- *How to use the mouse to get position measurements from the screen*. Point out to students that mouse coordinates are usually shown in the lower right-hand corner of the applet. It is also helpful to point out that it is wise to pick an easy to locate spot on an extended object to make consistent measurements over time.
- *How to initialize the applet*. Many of the applets do not run automatically when the page is loaded. The student may need to click on a link that will initialize the applet. There many be several of these links in a page. Students need to know to look for them.
- *How to click-drag to move objects around*. Students may not realize that they can move objects on the screen and will just stare at the screen instead of interacting with it.
- *How to get the Physlets to run*. Although Physlets will run on most machines, there are some browser and compatibility issues as was outlined in Chapter I.1. Students need to be aware of these issues and may need help finding a computer they can run the Physlets on.

In addition to these basic issues, be mindful of issues related to particular applets. For example, in the optics bench applet, the focal point of a lens can be changed by clicking on it and then dragging on the hotspot. Students need to be made aware of this. We have tried to include this type of information in the students' materials but students may still have trouble interacting with the applet.

As the students use more of the Physlet-based material they will become comfortable and will understand the basic features. However, in the beginning it is vital that the instructor makes sure they understand how to use the Physlet and that students are able to ask question if they are confused. We typically orient students by having them complete a very simple Physlet-based Problem that requires measurement during their first lab session. Since other students and the laboratory instructor are present, we can quickly deal with frustrations and usage issues. After the laboratory, students are required to access the Physlet from the computer they are most likely to use to complete out of class assignments and report back any technical difficulties they might have encountered.

Finally, it is important that students see Physlet-based material during class time. If an instructor assigns Physlet Problems for homework but never uses them during class and never goes over solutions, then students are unlikely to see Physlet-based materials as valuable and are likely to be frustrated by them. As we discussed in Chapter I.2, answering a Physlet-based question or solving a Physlet-based Problem is very different from anything the students are accustomed to. They must learn to think conceptually and avoid plug-n-chug approaches. This can be very difficult for students and they need to see proper approaches modeled in the classroom. Also, as with any teaching tool, the students need to believe the instructor sees the activity as useful. An instructor that never mentions the Physlet-based Problems in class will convey the attitude that such exercises are unimportant.

In the following sections we will summarize several teaching techniques that we have found to be very beneficial for learning and that complement the material in Physlet Physics. We hope you will find the ideas presented here helpful as you plan your instruction. At the end, we discuss student reactions to Physlets.

## Just in Time Teaching (JiTT)

JiTT[5] is an exemplary example of combining teaching with technology. It also lends itself so well to Physlets that some Physlet adopters have used the terms interchangeably. JiTT is an approach used to prepare and motivate students to learn during class. In this approach students answer a question before class and turn in their response using either e-mail or an html form. The instructor then reads the responses before class and is therefore able to tailor the class discussion around the students' responses. In the following paragraphs we briefly discuss implementation of the JiTT approach and conclude with a discussion of why Physlets fit with this approach so well. We are only able to give a short introduction here, for more details on implementations, as well as a collection of class-tested JiTT questions, see the book referenced in the footnote.

The JiTT question should be carefully chosen, as not just any question is effective. Typically, the question deals with a topic not yet covered in class, thereby forcing students to actually read the text, and perhaps talk to each other, in preparation for class. We have found that, when giving questions based on material not yet covered in class, simple conceptual questions are the most effective at getting students to think about the

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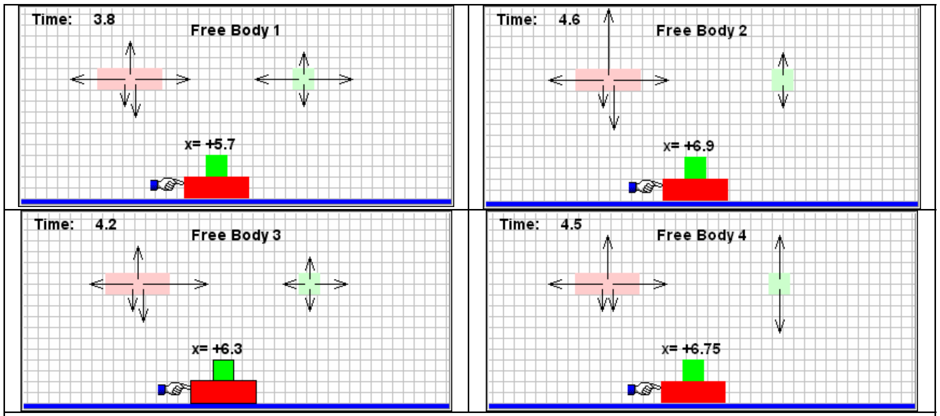
[5] Novak, G. M., Patterson, E. T., Gavrin, A. D., & Christian, W. (1999). *Just-In-Time Teaching: Blending Active Learning with Web Technology*. Prentice Hall: Upper Saddle River, NJ.

material and come to class with questions. Problems are usually not effective as students can often solve a problem by plug-n-chug and do not have to think about the underlying concepts. It is good to ask students questions which are simple but which get at the heart of difficulties the students are likely to have. This helps the students to see where their difficulties lie and will make them more alert during class as they seek to fill in the gaps of their understanding. However, you should occasionally include a question which most students are likely to answer correctly, if they read the text, in order to keep them from getting discouraged. We have found JiTT to be very effective at both encouraging students to review the text before class (which saves class time essentially reading it to them) and at priming their minds to grapple with the more challenging concepts during class (instead of postponing it to when they attempt the homework).

In the JiTT approach students submit their responses electronically before class begins. Most instructors require student answers a few hours before class time and then spend the time before class reviewing student submissions. Review of JiTT responses alerts an instructor to common misunderstandings or difficulties. Based on student responses an instructor may decide to spend more or less time on a particular topic, tailoring class-time more precisely to the needs of the students. Many instructors will also strip out the names from student submissions and then share all submissions with the class. It is helpful for students to see how others responded and provides an easy bounce into class discussion.

We have found Physlets to be a perfect match to the JiTT approach. As discussed in Chapter I.2, Physlet-based materials are excellent at forcing students to think conceptually, even when they are problem solving. Also, because of the interactive nature of Physlet-based questions they open up avenues of thought and discussion that are not typical of a static question. For example, consider Problem 4.1 shown in Figure I.3.1.



A red block is pushed and moves as shown in the animation. In addition, a green block sits on the red block and moves as well. Which free-body diagram is correct? Give reasons why the other three diagrams are incorrect.

**Figure I.3.1: Problem 4.1, an excellent JiTT question.**

This Problem asks students to determine the correct free-body diagram for two blocks. Students must realize that there is no acceleration in order to determine that free-body diagram #2 is correct. The responses of three students who answered this as a JiTT question are given below.

Student 1: I think the free body diagram number **1 is the correct one**. Number 2 does not show horizontal forces acting on the smaller block; Number 3 shows that the net force will be toward right, which means that the objects will be accelerating toward right, but the objects are moving in constant velocity, so no acceleration, no net force toward right; Number 4 again does not show horizontal forces on the smaller block, and the mg on the big block is represented too small.

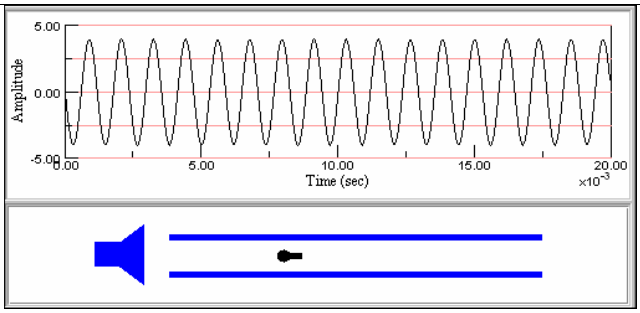
Student 2: **Free-body diagram 2 is correct**. 1 is incorrect because the normal force is less than the combination of the forces of gravity for the red and green boxes, and because the green box has both a friction and an anti-friction vector, which is incorrect. 3 is wrong for the same reasons as 1 and because the force vector that is pushing the red box is bigger than the friction vector. If this were true, then the boxes would be accelerating. 4 is wrong because the gravity vector for the green box on the left is not the same as the gravity vector for the green box on the right.

Student 3: **Free-body diagram #3 is the correct one**. #1 is incorrect because the force being applied by the hand has the same magnitude as the force of friction and if this was the case the object wouldn't be moving. #2 is incorrect for the same reason as #1 and also in #2 the small block doesn't show any force being applied to the small object in the horizontal direction, and both objects have the same horizontal components. Finally #4 is incorrect because the same reason as #1 and also the vertical components of the force do not cancel each other out, and they should since the object is not moving in the vertical direction.

The students' responses are very rich and provide the instructor with a starting point for classroom discussions. Also note that the Physlet version of this question requires that students realize, on their own, that acceleration plays an important role. If the students were simply given the four free-body diagrams and told that velocity remained constant they would have been clued in by the question statement as to what might be important. Although many would have still exhibited the "motion requires a force" model of thinking, the highlighting of the constant velocity in the question statement would have cued some in. The Physlet allows us to ask questions of students without giving them answers they would not otherwise obtain.

## Pre-Laboratory Exercises

Because Physlet-based exercises often mimic laboratory situations, we have found that they make excellent pre-lab exercises. We often require students to answer a Physlet question before coming to laboratory. Using the JiTT philosophy, students submit their answers electronically before coming to class and the instructor can review submissions before the laboratory session begins. We have found the pre-lab exercises to be an excellent way of introducing students to the ideas, and sometimes even the equipment, they will encounter in lab. This allows the students to work through the laboratory faster and with less confusion. For example, consider Problem 18.11 shown in Figure I.3.2.



Using a speaker, a standing sound wave has been set up inside a tube. A movable microphone lies inside the tube (**position is given in centimeters and time is given in seconds**). The graph shows the sound recorded by the microphone as a function of time. Move the microphone back and forth to study the changing amplitude of the sound it receives.

a. For what microphone position(s) does the amplitude of the sound go to zero? What is such a location called?
b. For what microphone position(s) is the amplitude of the sound a maximum? What is such a location called?
c. From the locations of the nodes, determine the wavelength of the sound waves.
d. From the graph, determine the frequency of the sound waves.
e. Using the wavelength and the frequency, find the velocity of the sound waves in the tube.

**Figure I.3.2: Problem 18.11 which mimics a hands on laboratory experiment.**

Problem 18.11 is very similar to a lab that is typically performed during a study of sound. In the lab, students are asked to connect a speaker to a tube and determine the velocity of sound by measuring the locations of the nodes. Problem 18.11 asks students to do the same thing, except in a virtual environment. By assigning this Problem as a pre-lab exercise, students will come to the laboratory with an understanding of the concepts and calculations relevant to the lab and can then focus their attention on data acquisition and analysis. Note that this question would also be an excellent post-lab question on an exam in order to test students on the laboratory procedures, without having to get out the equipment again.

## Peer Instruction

Peer Instruction[6] is an approach that is used to bring interactivity to traditional lectures. In this approach conceptual questions are posed to students during lecture. The students

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[6] Mazur, E. (1997). *Peer Instruction*. Prentice Hall: Upper Saddle River, NJ.

are asked to think about the question briefly and commit to answer as the instructor polls the class. After giving an initial answer, the students then turn to their neighbors to discussion their answers. The instructor can then poll the students again and lead them into a discussion. We find Physlet-based questions to be valuable Peer Instruction questions because they are excellent at bringing out simple, conceptual difficulties. They are also useful because they can quickly help students understand the question asked. One of the keys in Peer Instruction is for all the students to understand the question so that their discussion can be focused on the physics content (and not arguing about what the question means) and an animation clearly shows what happens. Additionally, we have found that it is simply faster to use animations to ask a question than text. Consider Problem 14.10 as shown in Figure I.3.3 in comparison with the associated ConcepTest from Mazur's *Peer Instruction* book.

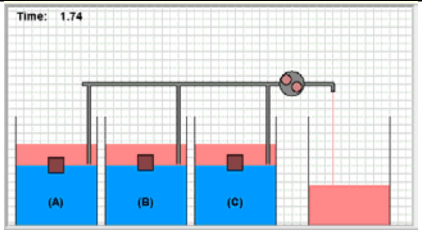
| | |
|---|---|
|  The animation is color coded as follows: blue is water, red is oil, and brown is a wood block initially floating at the interface. A pump, which starts at t = 1 s, removes the oil. Which animation is physical? | ConcepTest 11.11 (from *Peer Instruction*): Consider an object that floats in water but sinks in oil. When the object floats in water, half of it is submerged. If we slowly pour oil on top of the water so it completely covers the object, the object<br>1. moves up.<br>2. stays in the same place.<br>3. moves down. |

**Figure I.3.3: Physlet for use in Peer Instruction.**

Using the Physlet, students can quickly identify which of the animations (A-C) matches their prediction and the discussion that ensues can focus on the reasons that they picked a given animation.

## Cooperative Group Problem Solving

We also believe many Physlet Problems are excellent for cooperative group problem solving. Heller[7] et. al. have made the case that traditional end-of-chapter textbook problems are not adequate for group problem solving. After many years of teaching problem solving using cooperative learning, they have found that good group problems must be challenging enough so there is a real advantage to working in a group. They advocate using what they call context-rich problems with groups. Some of the features they advocate for group problems include: not being solvable in one step by plugging numbers into a formula, leaving the unknown variable unspecified in the problem statement, and giving more information than is required to solve the problem. All of these characteristics are typical of Physlet-based problems. Furthermore, Physlet-based problems are particularly useful for incorporating the last two features. Leaving the unknown variable unspecified, or giving more information than is necessary, in a paper-based problem statement makes the statement a bit contrived. However, both of these

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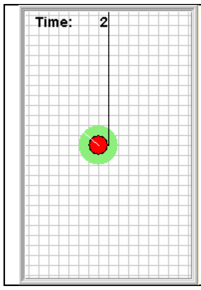
[7] Heller, P., Hollabaugh, M. (1992). Teaching Problem Solving Through Cooperative Grouping. Part 1: Designing Problems and Structuring Groups, American Journal of Physics, 60(7), pp. 637-644.

functions are fulfilled naturally with a Physlet based problem. Consider Problem 11.5 as shown in Figure I.3.4 below.



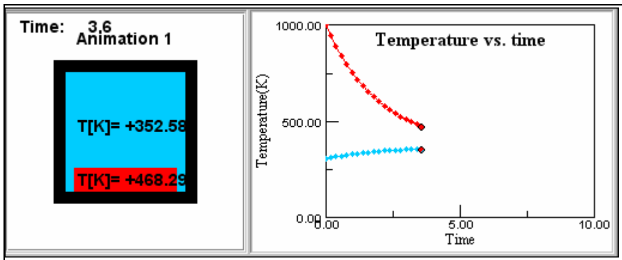
Problem 11.5: A giant 2.5-kg yellow yo-yo, made of two solid green disks and a massless red hub, is shown **(position is given in meters and time is given in seconds)**. Determine the torque that the string exerts on the yo-yo.

**Figure I.3.4: Finding the torque on a yo-yo (Problem 11.5) as a group problem solving exercise.**

Although this is very similar to a typical end of the chapter problem, it serves the purposes of a collaborative group problem solving exercise because the data that students need to take from the Physlet is not apparent (they need to find the acceleration, the radius of the inner red hub as well as the out green hub) and it is not well suited to plug-n-chug because it involves multiple steps (finding the acceleration, moment of inertia of the yo-yo, drawing a force diagram and finally solving for the torque). Student groups using this in a recitation session found it to be a challenging and engaging problem that they felt helped them to understand the material more clearly.

## Ranking Task Exercises

A ranking task[8] is an exercise that gives students a set of circumstances and asks the students to rank the circumstances based on some variable. The authors of the ranking task book referenced in the footnote four reasons for using this type of exercise: they frequently elicit students' natural ideas rather than a memorized response, they provide a way to frame questions in a manner that is novel for almost all students, and they can help students develop a legitimate understanding of some concepts. We agree with their assessment and have subsequently developed many Physlet-based ranking tasks that are included in *Physlet Physics*. You will find ranking task exercises in almost every chapter. As an example, consider Problem 19.7 shown in Figure I.3.5.



Run the animations and rank the materials in order of their specific heat capacity from smallest to largest **(temperature is given in kelvin and time is given in arbitrary units)**. In each animation the same mass of material starts at the same temperature and is put into a thermally insulated container of water. The graph shows the temperature of the water and the material as a function of time.

**Figure I.3.5 – Problem 19.7, a ranking task exercise.**

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[8] O'Kuma, T. L., Maloney, D. P., & Hieggelke, C. J. (2000). *Ranking Task Exercises in Physics*. Prentice Hall: Upper Saddle River, NJ.

Problem 19.7 asks students to rank three materials based on their specific heat. Playing each animation shows students how the temperature varies with time for the unknown substance and the container of water. Students must use this information to determine the relative specific heats of the materials.

## Interactive Lecture Demonstrations

Physlets can also be used to complement these demonstrations to provide ways to explain observations. Exploration 35.1, shown in Figure I.3.6, for example can be used in conjunction with a demonstration that asks students to predict what will happen to a real image formed by a converging lens when half of the lens is blocked. After asking students to make and then discuss predictions and then discussing the observation, students find this exercise useful to help them understand the demonstration. By recognizing that multiple rays leave a given point on the object, they are no longer surprised that a half-blocked lens still has an image.

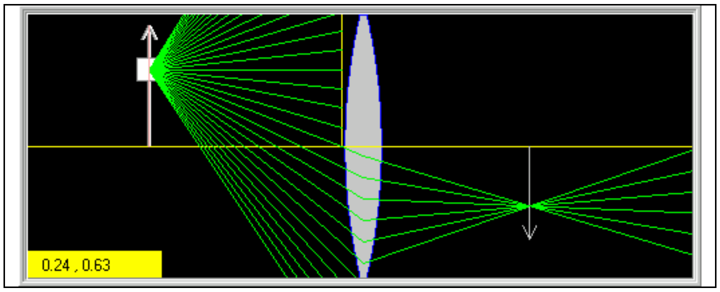


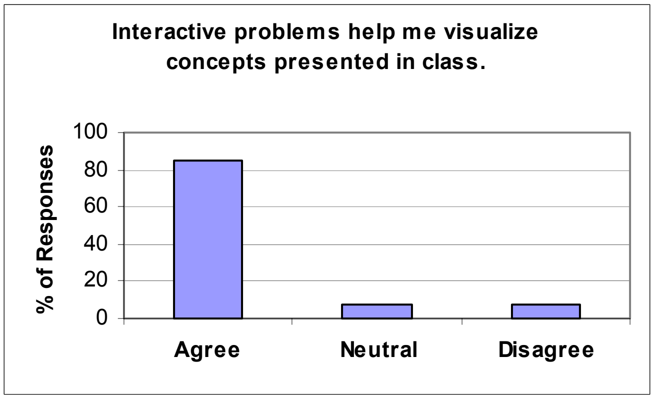
**Figure I.3.6: Image formation in Exploration 35.1 used as part of an ILD.**

## Student Perceptions

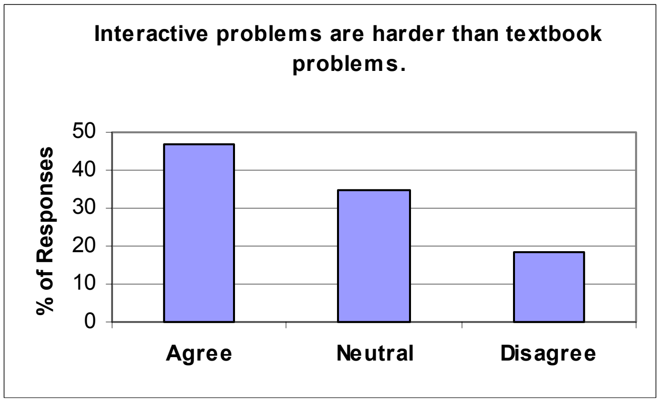
So what do students think of Physlets? We surveyed the students at Davidson College who used Physlets in their course. These surveys were given to approximately 100 students over a period of 3 years. A summary of the survey results follows.

We asked the students to respond agree, neutral, or disagree to several statements regarding Physlet use. Note that to the students, “interactive problems” meant Physlets.

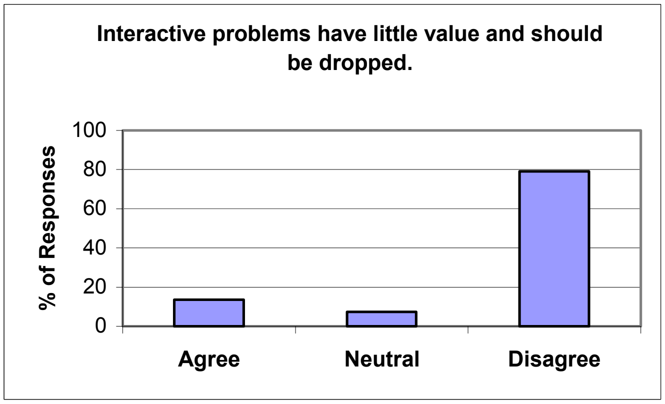
1. Interactive problems help me visualize concepts presented in class.



2. Interactive problems are harder than textbook problems.



3. Interactive problems have little value and should be dropped.



The results of questions one and three indicate that most students find it helpful to have Physlet-based mateial incorporated into their course. Those students who did not fully support the Physlets were vague in their reasons. For example, "I don't like the Physlets. They don't seem to help me focus my studies at all, and are very time-consuming. The Physlets don't help me understand." Since none of the students who were unfavorable toward Physlets were able to articulate any specific reason they did not like them, it seems likely that these students were simply unhappy or frustrated with the course as a whole.

Question two turned out to be very interesting. We believed that students would find the Physlet problems to be more difficult so it was somewhat surprising when less than half of the students agreed. In order to make sense of their opinions we needed to look carefully at the written comments they included with their rankings.

Many students stated that the Physlet problems were harder because they were given as Just-In-Time teaching problems whereas the textbook problems were assigned after the material was discussed in class. Other students noted that they were harder because the student had to think more or work to figure out what information was given. For example,

> "I believe this because for the Physlets, one not only needs to figure out what to do, but what information he/she/ needs to do it with. The textbook most of the time simply gives one the information needed to solve the problem."

Others noted that the level of difficulty depended on the particular problem and could not be generalized based on a Physlet-textbook designation alone.

Almost 20% of students felt the Physlet-based problems were actually easier. In general, it was the increased visualization that helped these students. For example,

> *"The material in interactive problems tends to be more challenging, however because they can be seen it is slightly easier to do. Being able to actually see the problem is beneficial."*
>
> *"Harder than some, but easier than others. It is made easier because I can see what is happening."*

In our experience, most students like the Physlet-based problems as long as they are properly oriented to using Physlets, the use Physlets often enough to become comfortable interacting with them, and the instructor is supportive. Overall, students find the Physlet-based material interesting and beneficial to their learning.

# Chapter I-4: Technical Considerations

Note: The following information is also found in the preface of *Physlet Physics*.

## *Browser Tests and System Requirements*

### Browser Tests

*Physlet Physics* provides physics teachers with a collection of ready-to-run, interactive, computer-based curricular material spanning the entire introductory physics curriculum. All that is required is the *Physlet Physics* CD and a browser that supports Java applets and JavaScript to Java communication. This combination is available for recent versions of Microsoft Windows and most versions of Unix. Although we occasionally check Physlets using other combinations, Microsoft Windows 2000 and Windows XP with both IE and the new Open Source Mozilla browser are our reference platforms.[9]

To check whether your computer already has Java installed, go to the Preface Chapter on the CD and navigate to the Browser Tests and System Requirements page. There you will find two buttons.

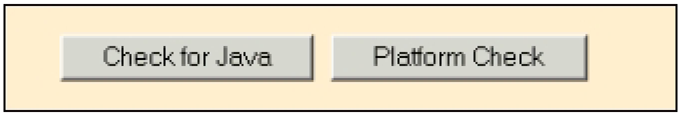


**Figure I.4.1: The "Check for Java" and Platform Check" buttons.**

Click the buttons to check for Java and your computer platform. If your browser fails the Java test, please continue reading for information on getting and installing Java.

### Microsoft Java

Most, but not all, versions of Windows include the Microsoft Java Virtual Machine (JVM). To test if any version of this JVM is installed on your computer, type the `jview` command at a DOS prompt. If a program runs, you have a Microsoft JVM. If you receive an error that no program by that name exists, you don't.

The Microsoft JVM is installed and updated on your computer with Windows Update. Previously it could be downloaded separately, but now Microsoft only uses Windows Update. The main Web page from Microsoft about Java is http://www.microsoft.com/java.

### Sun Microsystems Java

The Sun JVM is downloadable from the Java website: http://java.sun.com. After downloading the file to your hard drive, double-click on its icon to run the installer. Follow the instructions the installer provides.

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[9] Physlets have been tested on Linux and various versions of Unix. The only major operating system vendor that does not support Physlets is Apple Computer since the standard Macintosh and Power PC browsers do not support JavaScript to Java communication.

You can check your computer for a properly installed Sun JVM and change its properties by clicking on the Java plug-in icon in the Windows Control Panel.

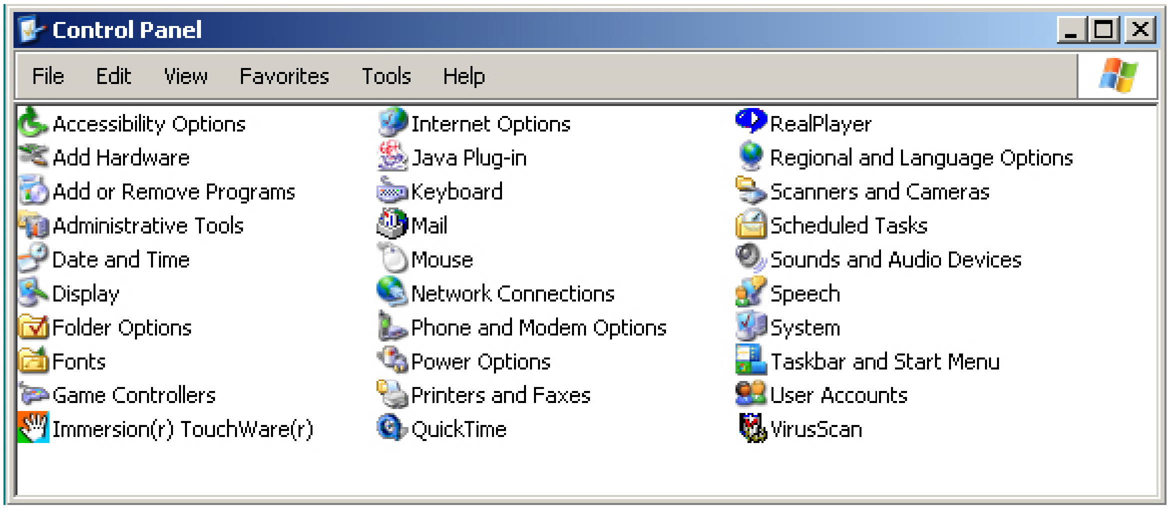
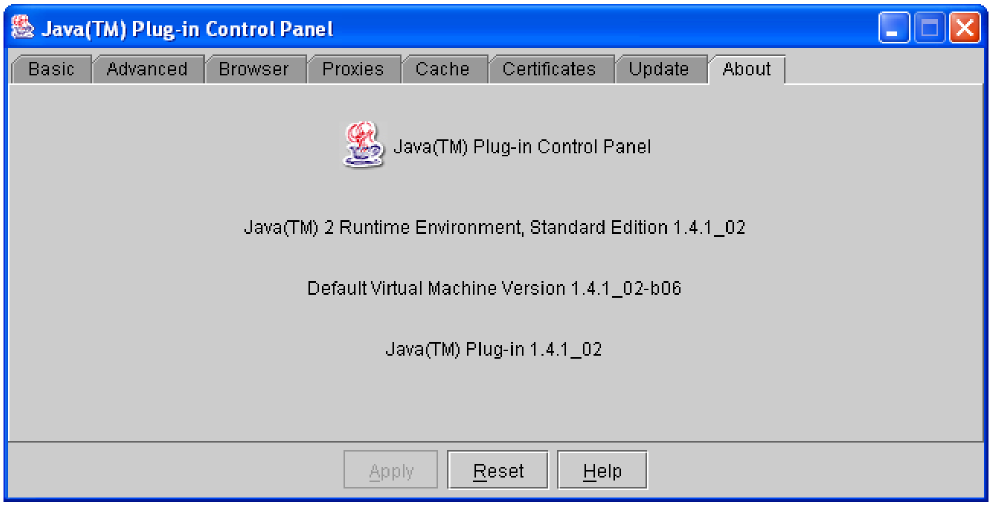


**Figure I.4.2: The Control Panel folder showing the Java plug-in icon.**

The following dialog box will appear:



Java(TM) Plug-in Control Panel

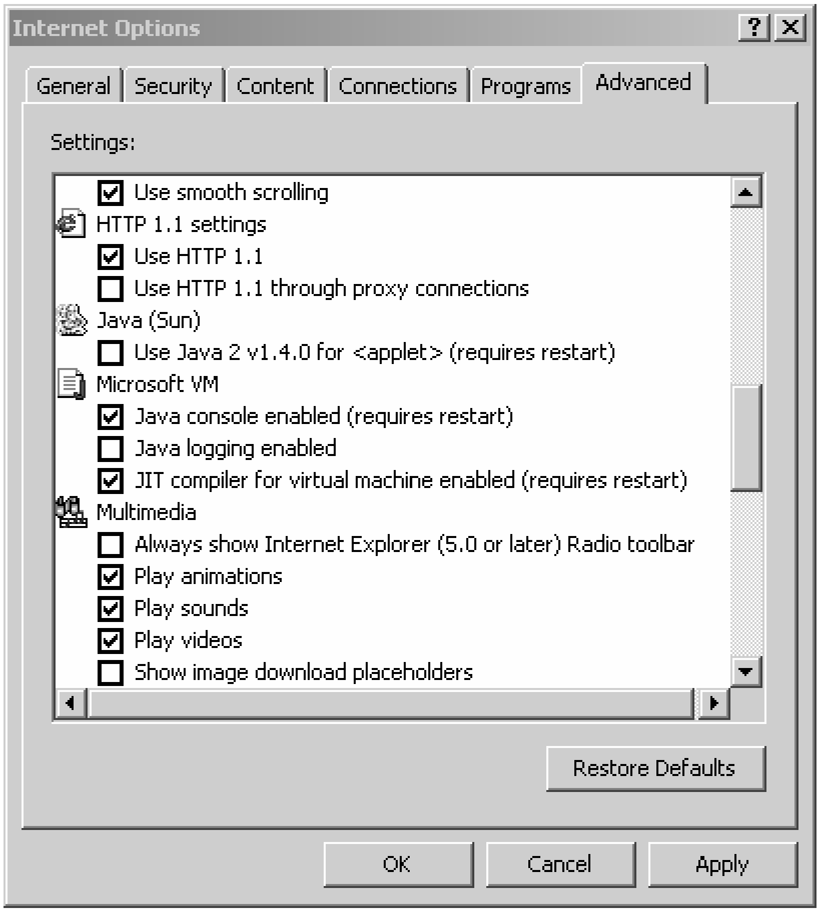
Java(TM) 2 Runtime Environment, Standard Edition 1.4.1_02

Default Virtual Machine Version 1.4.1_02-b06

Java(TM) Plug-in 1.4.1_02

**Figure I.4.3: The Java plug-in dialog box is accessed from the Windows Control Panel.**

Although it is possible to simultaneously install Java VMs from Microsoft and Sun Microsystems on Windows computers, a browser can only run one VM at a time. You can switch between these two JVMs in Internet Explorer. Start IE and click the *Advanced* tab under *Tool|Internet Options* from the IE menu bar. The following dialog box shown in Figure 4 will appear.



Settings:

- ☑ Use smooth scrolling
- HTTP 1.1 settings
  - ☑ Use HTTP 1.1
  - ☐ Use HTTP 1.1 through proxy connections
- Java (Sun)
  - ☐ Use Java 2 v1.4.0 for <applet> (requires restart)
- Microsoft VM
  - ☑ Java console enabled (requires restart)
  - ☐ Java logging enabled
  - ☑ JIT compiler for virtual machine enabled (requires restart)
- Multimedia
  - ☐ Always show Internet Explorer (5.0 or later) Radio toolbar
  - ☑ Play animations
  - ☑ Play sounds
  - ☑ Play videos
  - ☐ Show image download placeholders

**Figure I.4.4: The advanced Internet Options dialog box is accessed from within Internet Explorer.**

Figure 4 shows that this computer that has two Java VMs and that it is currently configured to run the Microsoft VM. The option for the Java (Sun) VM will not appear unless the Sun Java Run-Time Environment has been installed. You will need to close all browser windows if you decide to switch VMs. You do not, however, need to restart the computer.

## Non-Microsoft Browsers

Netscape, Opera, and Mozilla offer alternatives to the Microsoft Internet Explorer on Windows operating systems. You can download this browser from the Mozilla website: http://www.mozilla.org.

After downloading the file to your hard drive, double-click on its icon to run the installer. Follow the instructions the installer provides. The Mozilla browser requires that the Sun JVM be installed on your computer.

# Chapter I-5: Mechanics

The mechanics chapters represent the starting point for a study of physics. The Illustrations in these chapters tend to focus on introducing topics and provide an interactive alternative to the standard textbook treatment. As we discuss below, you will find these chapters to be particularly rich in multiple representations of concepts as motion graphs, energy-bar graphs, and mathematical modeling are integrated throughout.

### What Is Covered?

- All mechanics topics typically covered in an introductory course are found in the mechanics chapters of *Physlet Physics*.
- Chapter 1 provides an introduction to Physlets and Physlet-based exercises and is designed to get students through some of the difficulties they may encounter as they embark into unfamiliar territory.
- Reference frames and relative motion are in a separate chapter, Chapter 9.
- The first Newton's laws chapter (Chapter 4) deals with the basic concept of force, free-body diagrams and the forces of weight, tension, and normal force. The Newton's Laws II chapter (Chapter 5) goes beyond Chapter Four to include friction (including air friction), circular motion, and springs.
- The Rotations about a Fixed Axis chapter (Chapter 10) introduces the concepts associated with rotation: angular variables, torqure, angular momentum and the angular momentum of particles. General Rotations (Chapter 11) extends this to objects that rotate and translate thereby including rolling and collisions between an object in translational motion with a rotating object (e.g. mass dropped on a rotating merry-go-round).

## *Highlights of Mechanics Chapters*

### Focus on Graphical Representations of Motion

We have made extensive use of motion graphs throughout the mechanics chapters. As students view the actual motion, they also see graph being created of the motion, thereby reinforcing the connection between actual motion and a graphical representation. Although the ability to construct and interpret graphs is an important scientific skill, there is evidence that students often have great difficulty interpreting kinematics graphs.[10] Because Physlets can easily create a motion graph in real time with the motion, they provide an avenue for helping students to develop their understanding of motion graphs that can not be replicated through traditional means. As an example, consider Exploration 2.4 shown in Figure II.5.1.

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[10] Beichner, R. J. (1994). Testing Student Interpretation of Kinematics Graphs. *Am. J. of Phys*, **62**(8), pp. 750-762.

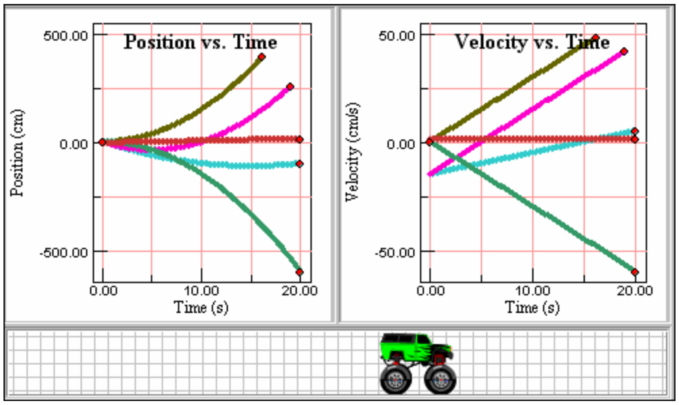
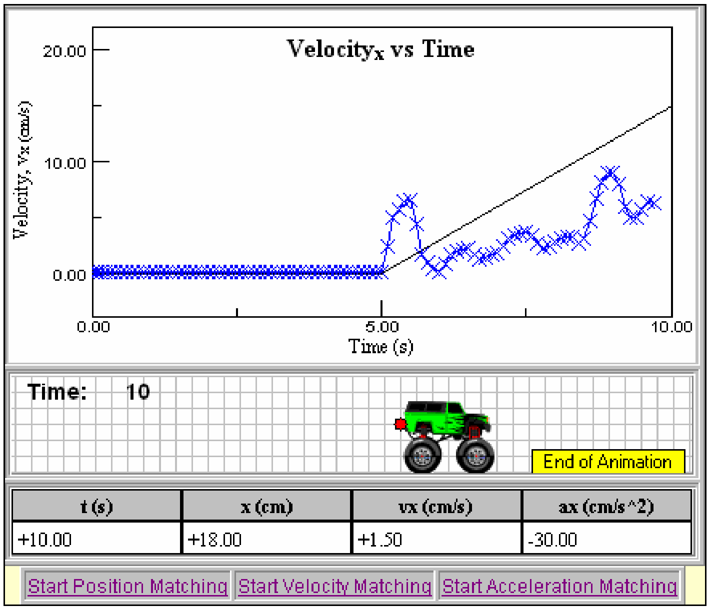


**Figure II.5.1: Exploration 2.4 which allows students to input initial conditions and then see both the motion and motion graphs.**

In this Exploration, students can input initial values for the monster truck and then see the position and velocity graphed as the truck follows the motion below. The questions in the Exploration guide students into experimenting with the parameters to see the effect on the graphs. This is one of many Illustrations, Explorations, and Problems that explicitly connect the graphical representation of motion to a visual representation.

## Kinesthetic Learning

A tremendous advantage of Physlet-based exercises is that they are interactive. This interactive nature has been exploited numerous times in Physlet exercises that require students to use their mouse to control the Physlet. For example, consider Problem 1.3 shown in Figure I.5.2.



| t (s) | x (cm) | vx (cm/s) | ax (cm/s^2) |
|---|---|---|---|
| +10.00 | +18.00 | +1.50 | -30.00 |

**Figure I.5.2: Problem 1.3 which has students attempt to match a position, velocity or acceleration graph by dragging a monster truck.**

Students are shown a position, velocity or acceleration graph and asked to move the monster truck, by dragging, to replicate the graph. Students will find that it is easiest to match the position graph and will have the most difficulty with the acceleration graph. This activity is fun for the students and can help them to connect physical motion with graphical representations. Other Physlet-based exercises that require physical inputs are Exploration 3.2, Illustration 4.3, Exploration 4.3, and Exploration 7.1.

## Energy Bar Graphs

When energy considerations are an integral part of a Physlet, we have often included an energy bar graph. This type of representation has been shown[11] to be a useful tool to help students understand energy concepts and to solve problems related to work and energy. An example Physlet that utilizes energy bar graphs is shown in Figure I.5.3.

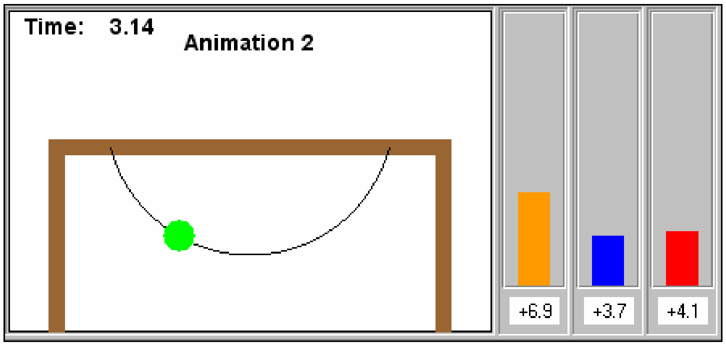


**Figure I.5.3: Illustration 7.1, a ball sliding on a wire with bar graphs representing kinetic energy, potential energy and energy lost to friction.**

In the Illustration shown in the figure, a ball is released on a wire and slides back and forth. The first bar (orange) represents the amount of kinetic energy of the ball, the second bar (blue) shows the amount of potential energy, and the third bar (red) shows how much energy has been lost to friction. As the ball slides back and forth, the heights of these bars change in real time to reflect the various energies. The energy bar graphs provide students with an alternative, qualitative, way of thinking about energy and changes in energy.

## Visualization of Abstract Concepts: Reference Frames

Along with providing visualization of multiple representations such as plots, charts, and bar graphs associated with animations, students find the visualization of certain specific concepts in mechanics to be particularly useful. Exploration 9.3, shown in Figure I.5.4, allows students to see motion from a variety of reference frames. Comparing the motion seen from different reference frames helps students develop a stronger conceptual understanding of reference frames. This visualization helps students understand how and why reference frames are used in ways that a static or mathematical construction can not begin to provide.

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[11] Van Heuvelen, A. & Zou, X. (2001). Multiple Representations of Work-Energy Processes. *Am. J. of Phys*. **69**(2), pp. 184-194.

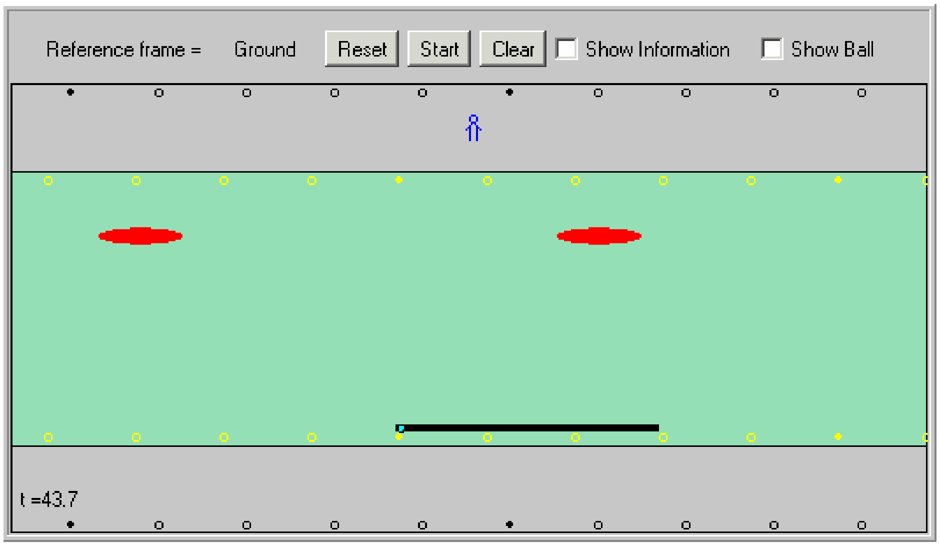


**Figure I.5.4: Exploration 9.3 compares the motion of objects viewed from three different reference frames (person on shore, boat in river, and river itself). Applet authored by Fu-Kwun Hwang, National Taiwan Normal University**

## Visualization of Abstract Concepts: Kepler's Laws

*Physlet Physics* examples that allow students to explore Kepler's laws, give students a laboratory that is unavailable in any other way. It is a bit difficult to design and construct a solar system, after all. As students change the velocity and initial positions of planets in Exploration 12.1 as shown in Figure I.5.5, for example, they can quickly see the change in the orbits and how Kepler's laws are applied. As students see motion described (and required) by Kepler's laws, they are in a better position to make the connection between celestial gravity and terrestrial gravity and to understand Newton's law of universal gravitation.

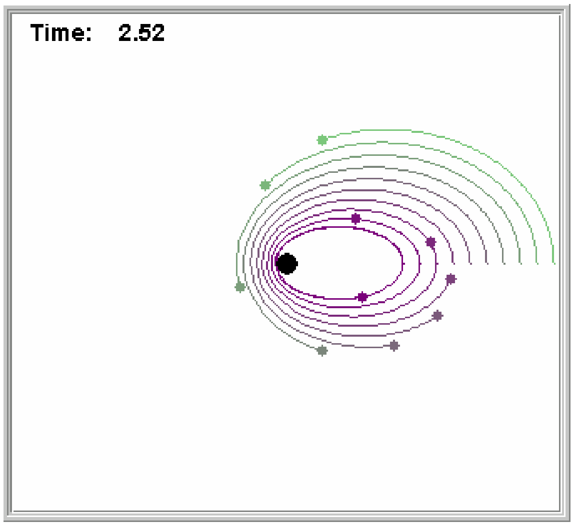


**Figure I.5.5: Building a solar system in Exploration 12.1.**

## Visualization of Abstract Concepts: Rotational Motion

Even though the extension from linear systems to rotating systems seems logical, because of the mathematics, many students find it difficult to understand the corresponding concepts of rotational motion: angular acceleration and velocity, torque, moment of inertia, and angular momentum. The visualization provided by the exercises in *Physlet Physics* allows students to see the motion and make connections between the motion they observe and the parameters that they change. Consider for example, Exploration 10.3 as shown in Figure I.5.6. Students change the mass of the pulley, the hanging block and the size of the pulley and as they see the effects of the changes (in the motion of the falling block), they can recognize the need for a description (equation) that incorporates all of the parameters, leading naturally to the use of torque and moment of inertia. Once students recognize the need for such descriptions, these (necessarily) mathematical descriptions become less intimidating because students have a picture of motion (a movie, if you like) in mind to attach to the equation.

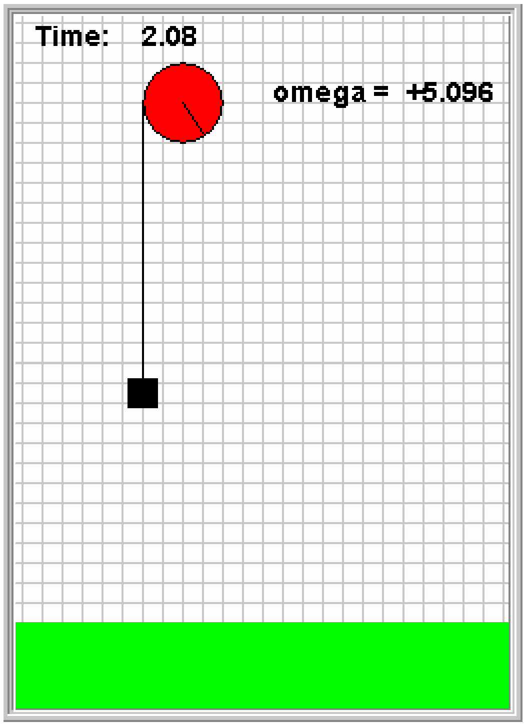
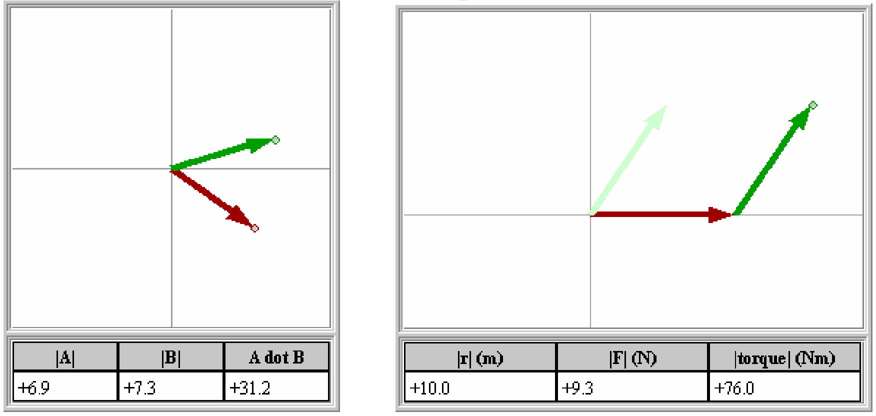


**Figure I.5.6: Changing parameters to see the effect on the motion in Exploration 10.3.**

## Understanding Abstract Concepts: Vectors

Crucial to understanding physics is a solid understanding of vectors. The interactive nature of the vectors in *Physlet Physics* helps students understand vector components, vector addition and dot products. For example, in Illustration 6.1 and Exploration 11.1 as shown in Figure I.5.7 (a) and (b), students change the length and orientation of the vectors and they can immediately see the dot-product and magnitude of the cross product (torque) of these vectors. This helps students develop a sense of what a dot or cross product is and why it is used in physics. Additional specific exercises designed to help students understand vectors include Illustration 3.1, Exploration 3.1 and associated Problems 3.1 and 3.2, Illustration 10.1, and Exploration 11.1.



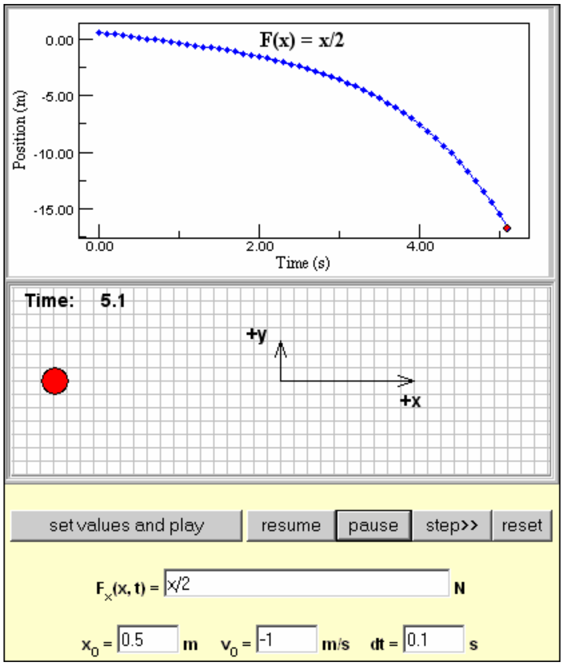
| \|A\| | \|B\| | A dot B |
|---|---|---|
| +6.9 | +7.3 | +31.2 |

| \|r\| (m) | \|F\| (N) | \|torque\| (Nm) |
|---|---|---|
| +10.0 | +9.3 | +76.0 |

**Figure I.5.7: (a) Exploration 6.1 and (b) Illustration 11.1 have students move the vectors and observe how the dot product and cross product (torque) change.**

## Unique Problems – Connections with Mathematics

There are a number of Physlet-based exercises that require students to enter values in a formula to match to an outcome. We believe this type of exercise can be very beneficial for students because it directly connects mathematical representations with more intuitive representations which can strengthen their overall conceptual understanding. Also, these types of exercises have a "game" aspect to them which can be fun and challenging. As an example, consider Exploration 4.8 shown in Figure I.5.8.



Time: 5.1

$F_x(x, t) =$ x/2 N

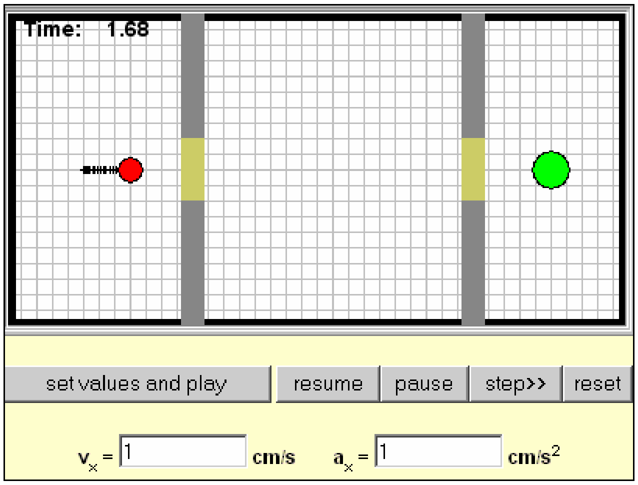
$x_0 =$ 0.5 m $v_0 =$ -1 m/s dt = 0.1 s

**Figure I.5.8: Exploration 4.8 which encourages students to connect mathematics with motion and a graphical representation.**

In this Exploration students are encouraged to input values for the force function as well as values for the initial position and velocity. They can then see the results of their inputs as both motion and a position vs. time graph. This type of exercise is truly interactive as students make predictions and then test them. The Physlet-based exercise provides them direct feedback using multiple representations. Other Physlets which use a similar pedagogy include Explorations 2.4, 2.5, 4.4, 5.5-5.7, 6.4, 8.2, 10.1-10.2, and 12.1-12.2.

## Unique Problems – Kinematic Putt-Putt Golf

There are several "putt-putt" problems contained in the mechanics chapters. These problems contain an element of fun since students can calculate the correct initial conditions to "score" and then try their predictions. They also provide a very unique type of problem for the student to work on that requires students to think a bit outside the proverbial box. An example of this type of problem is shown in Figure I.5.9.



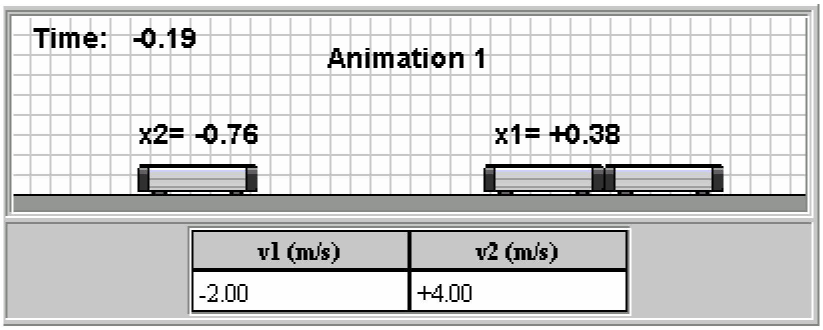
$v_x$ = 1 cm/s $a_x$ = 1 cm/s$^2$

**Figure I.5.9: Problem 2.15, Kinematic Putt-Putt Golf**

Students must determine the initial velocity and acceleration that will project the ball through the two gates to the green hole. The gates open at set times and are closed otherwise so getting the ball into the hole requires careful calculations. Other putt-putt type problems are 2.6 and 2.16.

## Unique Problems- Identifying Non-Physical Animations

Since the physics is not built into Physlets, many Physlet-based exercises can model non-physical events. *Physlet Physics* takes advantage of this by asking students to identify the non-physical animations. For example, in Problem 8.7 as shown in Figure I.5.10, students need to identify which collisions could actually occur. Students often try to solve these types of problems by assuming conservation of energy when they should check for conservation of momentum. Addressing this issue explicitly by providing animations that obey conservation of energy, but do not conserve momentum in a collision provides a way for instructors to gauge how well students understand conservation laws. Other problems, such as Problem 8.6 and Problem 12.5, that ask students to identify non-physical animations address similar fundamental physical descriptions.



| v1 (m/s) | v2 (m/s) |
|---|---|
| -2.00 | +4.00 |

**Figure I.5.10: Problem 8.7, Identifying non-physical animations.**

# Chapter I-6: Fluids

The two chapters in the fluids section of *Physlet Physics* provide nice examples of applications of previous material. The Physlets exercises in this set provide a visualization of fluid problems: specifically fluid flow and a virtual measurement of pressure as a function of depth. They are designed to help students make connections whether it is between the buoyant force and pressure as a function of depth or Bernoulli's principle and conservation of energy.

## What Is Covered?

- The two chapters in this section cover static and dynamic fluid flow.
- There is no discussion of turbulent flow and a limited discussion of viscous flow.

## *Highlights of Fluids Chapters*

### Multiple Approaches to Abstract Concept: Buoyant Force

Static Fluids (Chapter 14) provides multiple ways to understand buoyancy. Although students calculate the weight of the water displaced to find the buoyant force, many question (and rightly so) how the displaced water causes a force. Illustrations 14.3 as well as Explorations 14.1 and 14.2 (shown in Figure I.6.1) address this by showing the water displaced (the water is displaced into a nearby container to explicitly show this) and either showing or asking students to calculate the pressures at the top of the liquid and at the bottom of the floating object.

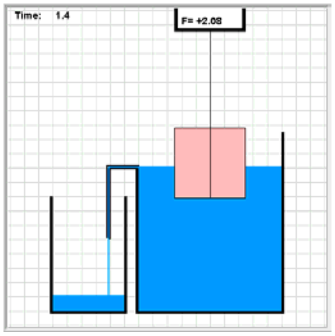


**Figure I.6.1: Buoyant force in Exploration 14.2**

Forcing students to do this calculation as well as seeing how the force varies as you change parameters, helps students to understand the origin of the buoyant force (which is not due to the liquid that has fallen out of the container; that simply provides a convenient way to calculate the value of the force). The associated problems are designed to reinforce this idea (particularly Problem 14.10 in association with Exploration 14.3).

### Visualizing Dynamics: Continuity Equation and Bernoulli's principle

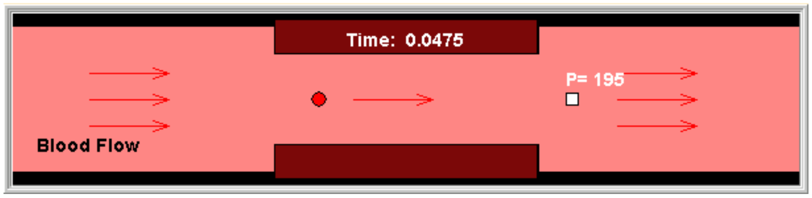


**Figure I.6.2: Exploration 15.1 and the continuity equation**

In dynamic fluids (Chapter 15), the Physlet-based exercises emphasize the application of the continuity equation and Bernoulli's equation for ideal fluids while providing a few examples of non-ideal fluids. For ease of visualization, the Physlets animate static textbook diagrams to help explain the continuity equation. For most students, seeing a volume of fluid going from a wider to in a narrower pipe where it moves faster, makes the associated equation more concrete (and obvious to many students). See, for example, Illustration 15.1 or Exploration 15.1 (shown in Figure I.6.2) and Problem 15.1.

## Connections to Mechanics

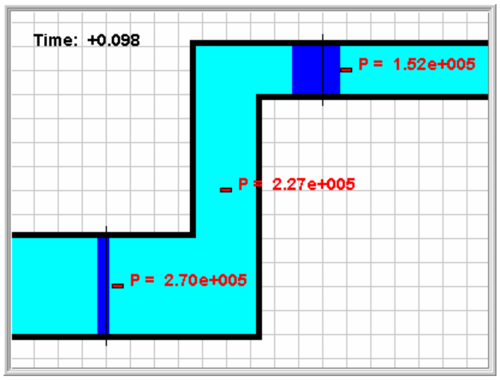


**Figure I.6.3: Using Exploration 15.2 to develop Bernoulli's equation**

In Exploration 15.2, shown in Figure I.6.3, students use their knowledge of mechanics to develop Bernoulli's equation while Exploration 15.3 is an animation of a standard textbook application of Bernoulli's principle: water leaking from a large barrel. Both provide some hints on applying Bernoulli's equation that will be useful for both textbook problems as well as the associated Physlet-based problems (Problems 15.3-15.9, Problem 15.4 is shown in Figure I.6.4). Before the student knows what measurements to make (how far did the stream of water travel, height of water in the container), he/she must connect Bernoulli's principle with projectile motion and then use equations.

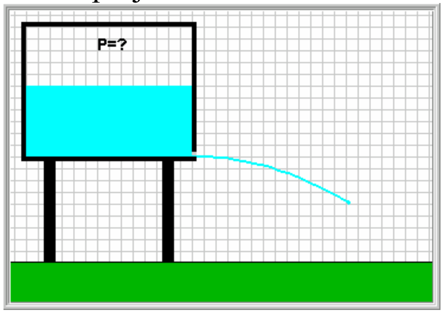


**Figure I.6.4: Application of Bernoulli's principle in Problem 15.4 (and Exploration 15.3).**

# Chapter I-7: Oscillations and Waves

An analysis of waves and wave motion is typically very mathematical and very difficult to demonstrate with traditional means. However, the mathematics leads to a visible representation that is less abstract than many topics in physics. The Physlet-based exercises in these chapters utilize the power of visual and dynamic representations to make the connection between the mathematics and the concepts of waves and oscillations.

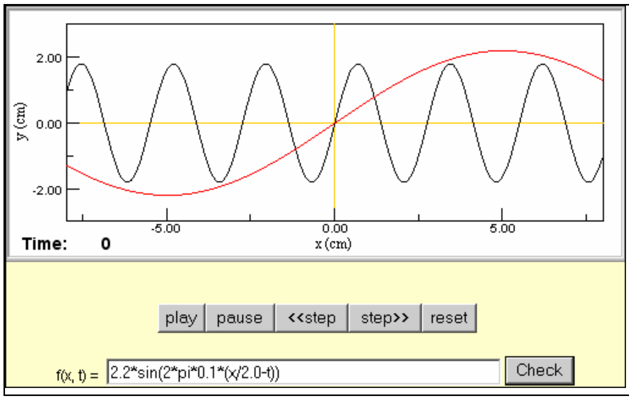
## What Is Covered?

- Chapter 16 develops the mathematical ideas underlying wave motion focusing on simple harmonic motion associated with springs and pendulum. Advanced topics of damping/driving forces, Fourier series, and phase space representation are also covered in this chapter.
- Chapter 17 deals with wave motion in general covering wave types, superposition, resonance, reflection at a barrier, standing waves, and wave velocity.
- Chapter 18 is really an application of the ideas developed in Chapters 16 and 17 to sound waves.
- Superposition and interference of waves is also covered in the optics chapters 37 (Interference) and 38 (Diffraction).

## *Highlights of Waves Chapters*

### Connecting Mathematical and Visible Representations

Students often have difficulty connecting the mathematical representation of a wave to the visual representation of a wave. Many of the Physlet-based exercises push students to make this connection by requiring them to input a mathematical function. For example, consider Exploration 17.2, shown in Figure I.7.1.



f(x, t) = 2.2*sin(2*pi*0.1*(x/2.0-t))

Shown in black is a traveling wave (**position is given in centimeters and time is given in seconds**). Measure the relevant properties of this wave and determine the wave function of the wave. Once you are finished, check your answer by importing a f(x, t) and look at the red wave to see if it matches.

**Figure I.7.1 – Exploration 17.2 which asks students to input a function to match a waveform.**

This Exploration presents students with a wave form. Students must measure properties of the wave form in order to determine the proper mathematical function that describes the wave form. Since this is a traveling wave, the wave must match not only in form, but also in velocity. We have found this type of exercise to be very difficult for students because they must solidly understand the connections between the visible representation of the wave and the mathematical formalism. Having students work through these types of exercises forces them to make these connections. Physlet-based exercises that require students to input a wave function are found throughout these chapters, especially in Chapter 17.

## Learning Through Visualization

As with all other chapters, there are many Physlets in the waves chapters that are excellent for showing concepts in a way that could not be replicated with a static paper-based diagram. The following highlights some of our favorites.

Consider Illustration 16.1, shown in Figure I.7.2.

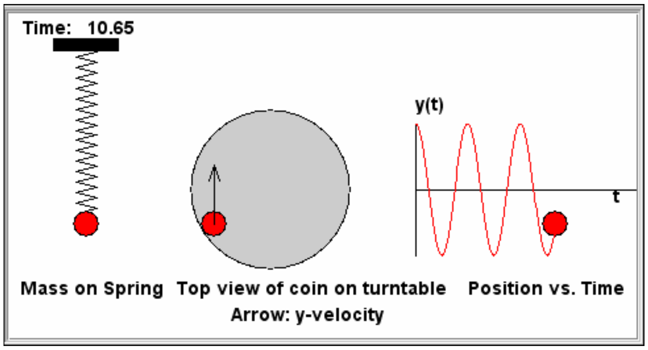


**Figure I.7.2 – Illustration 16.1, an animated version of a static diagram.**

This Illustration is merely an animation of a diagram that is often found in physics textbooks. However, the animation really brings this Illustration to life as it demonstrates the connection between an obvious simple harmonic oscillator (mass on a spring), and a system that can be represented as a simple harmonic oscillator (mass on a turntable), and the graphical representation of the SHM. For the student having difficulty connecting the motion to the graph, seeing the two occur simultaneously can be of tremendous benefit.

Another Illustration we have found to be particularly helpful for visualization reasons is Illustration 17.5, shown in Figure I.7.3, which demonstrates resonance.

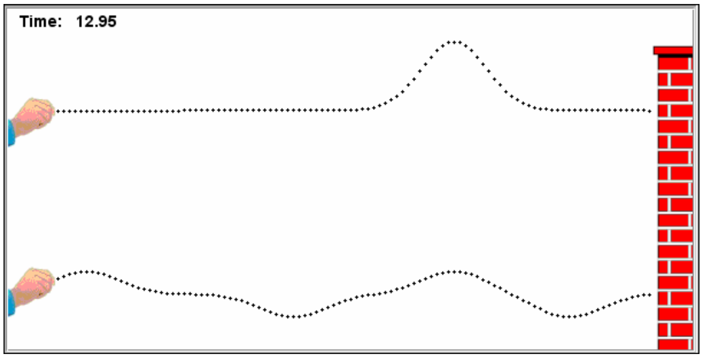


**Figure I.7.3 – Illustration 17.5 demonstrating resonance.**

In this Physlet-based exercise, a hand shakes a rope creating a pulse. In the top rope, the hand sends a pulse synchronized with the existing pulse, in the bottom rope the hand acts out of sync with the existing pulse(s). Students can clearly see that energy is transferred constructively in the top case and nonconstructively in the bottom.

Another excellent example is Illustration 17.4 shown in Figure I.7.4.

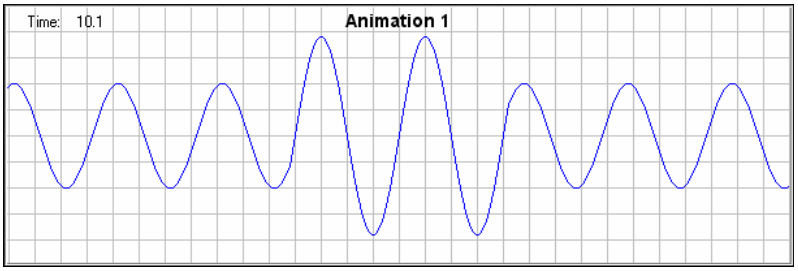


**Figure I.7.4 – Illustration 17.4 demonstrating that a standing wave is the superposition of two traveling waves.**

Illustration 17.4 begins with a right- and left-traveling wave incident from their respective sides. As the waves begin to overlap each other a standing wave is clearly seen. The effect is very striking, even for physics instructors who already have a firm grasp of the underlying concepts. Students can slowly step through the process as the two waves superimpose to understand how two traveling waves can add to a standing wave.

Another Illustration, of which we are particularly fond of for visualization reasons, is Illustration 18.2 shown in Figure I.7.5.

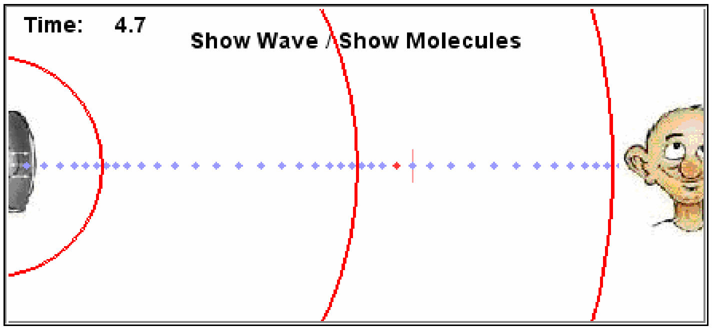


**Figure I.7.5 – Illustration 18.2 demonstrating the molecular view of sound waves.**

This Illustration shows students how sound is a wave, and that it is a longitudinal wave. Students see the molecules of air being compressed by the pressure wave of sound. Many students do not understand the nature of sound and too easily accept that it is a wave without understanding the underlying process. Illustration 18.2 makes this process explicit and in doing so, also shows clearly that the molecules are vibrating in the direction of wave motion, making it a longitudinal wave.

Finally, we would like to point out the Illustration and Exploration dealing with the Doppler effect. Illustration 18.4 is shown in Figure I.7.6. Exploration 18.5 is not shown but is also a wonderful tool for demonstrating the effect of motion on perceived sound waves.

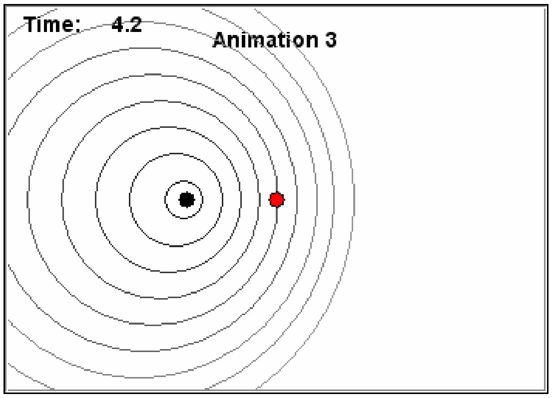
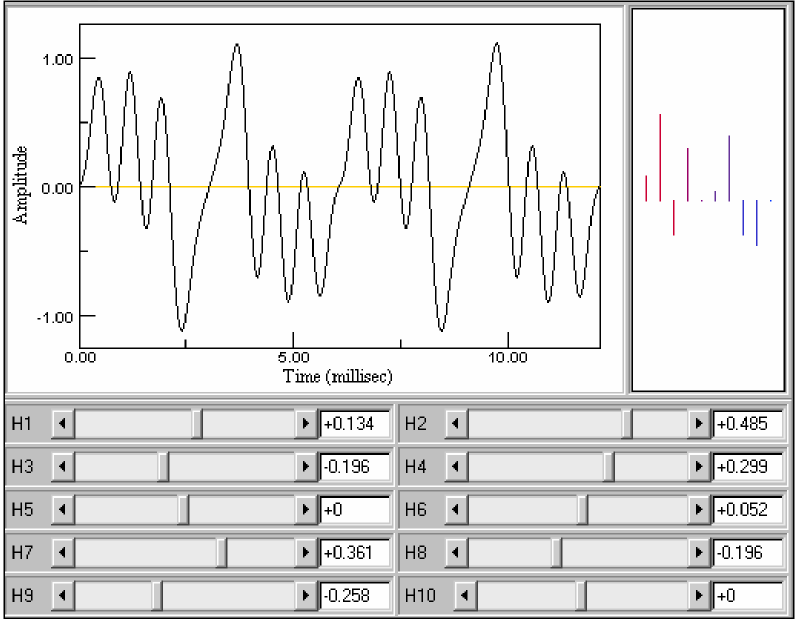


**Figure I.7.6 – Illustration 18.4 demonstrating the Doppler effect.**

The underlying causes of the Doppler effect are amazingly clear after viewing the animations included with Illustration 18.4. These animations show various relative motions between a source of sound and a detector. For example, in Animation 3, depicted in the figure, the detector remains stationary while the source moves toward it. As the animation plays, it is clear that sound emits in all directions symmetrically about the source but the waves arrive bunched up, or far apart, due to the motion of the source. This visualization of the Doppler effect can help students to understand the underlying ideas.

## Wave Superposition

The addition of multiple waves is an important concept to consider in any study of waves. We believe Physlet-based exercises are ideal for showing students the effects of wave superposition. Above, we discussed Illustration 17.4, which demonstrates the addition of waves to produce a standing wave. There are many other superposition-based Physlet exercises throughout these three chapters. For example, Exploration 17.4 allows students to change the parameters of a wave and then view the superposition of their wave and another wave. Exploration 17.1 presents students with several pulses and shows them how these pulses interfere. Explorations 18.1 and 18.2 (shown in Figure I.7.7) allow students to add multiple waves as harmonics.



H1 +0.134
H2 +0.485
H3 -0.196
H4 +0.299
H5 +0
H6 +0.052
H7 +0.361
H8 -0.196
H9 -0.258
H10 +0

**Figure I.7.7 – Explorations 18.1 and 18.2 which allow students to add sine waves and hear the superposition.**

As the amplitude of the different harmonics is varied from zero, the resulting sound is played through the computers speakers for the student to hear. These Explorations provide a wonderful way to introduce students to both the superposition of waves and also of electronically produced music.

# Chapter I-8: Thermodynamics

As is true of earlier chapters, using Physlet-based exercises to aid teaching topics covered in thermodynamics provides several advantages. Specifically for thermodynamics, a model of ideal gas particles in a box provides a concrete visual for abstract concepts as students make connections between microscopic and macroscopic quantities. Students can also change parameters in Physlet-based exercises included here to develop or verify a series of useful equations (e.g. linear expansion, specific heat of solid, work done by a gas, specific heat of a gas at constant pressure and volume, efficiency of engines). The Physlets can help students use the analytic tools including dynamic connections between graphs and thermodynamic processes. Physlets also simplify modeling "real-world" applications specifically showing engine processes and their associated heat and work diagrams. Finally, these thermodynamic exercises challenge students to move away from "equation shopping", and to consider the type of problem (What type of thermodynamic process does the problem animate?) and required information (What data do I need to collect from the animation?) before simply dropping numbers into an equation.

## What Is Covered?

- The three chapters in this section cover temperature and heat, kinetic theory and engines and entropy.
- There is only limited discussion of heat transfer via convection or radiation (and no associated problems).
- Essentially no discussion of non-ideal gases or non-ideal engines.

## *Highlights of Thermodynamics Chapters*

### Effect of Changing Parameters on a System

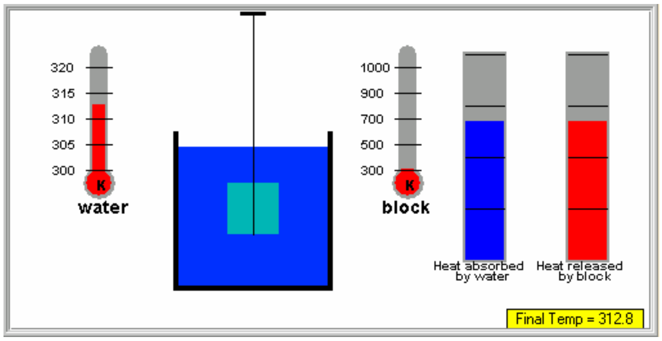


**Figure I.8.1: Calorimetry and Exploration 19.3**

Starting with the first chapter in the thermodynamics section (Chapter 19: Heat and Temperature), the Physlet-based exercises cover the standard range of topics: expansion, calorimetry and heat balance problems. Students can try several of the Explorations that lead them to develop the relationships codified in equations useful for these topics. These exercises also allow them to get data for a variety of idealized scenarios that would take a long time to record in the laboratory. Exploration 19.3, shown in Figure I.8.1, (and Illustration 19.1, Problems 19.6-19.8), for example, provides students with idealized calorimetry data. Students can change the temperatures and mass of the materials and are then asked to find the heat exchanged (Exploration 19.3) or the specific heat (Illustration 19.1). These can be used, for example, to prepare students for a calorimetry laboratory: helping them look at the problem before they get immersed in the thick of data collection.

Similarly, Exploration 19.2 allows students to change different parameters to discover a (albeit idealized) relationship between expansion of a solid, temperature change, and size of the solid while Exploration 19.1 (and Problems 19.1 and 19.2) forces students to make the connection between mechanical energy and heat.

## Microscopic Models

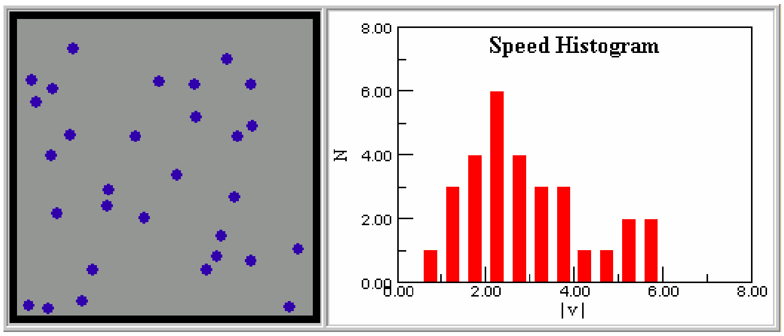


**Figure I.8.2: Gas particles in a box in Exploration 20.1**

For ideal gases (Chapter 20), the visualization aid available from Physlets is particularly useful in developing a microscopic model of gas particles in a box. Here we introduce a set of new Physlet-based exercises that contain hard spherical particles which collide elastically with the walls and each other, modeling an ideal gas. The volume, temperature, and number of particles can be changed and several Illustrations, Explorations and Problems center on these Physlet-based exercises. Exploration 20.3 allows students to explore ideal gas laws while Exploration 20.1, shown in Figure I.8.2, (and Illustration 20.2) makes explicit the connection between the properties of individual particles (momentum and kinetic energy) and the macroscopic quantities of thermodynamic systems: pressure and temperature. As students work through the steps of Exploration 20.1, starting with one particle, calculating the change in momentum, average force and then pressure, changing the mass, temperature and number of particles, they develop (derive for themselves) that the average internal energy of a particle in a gas per degree of freedom is ½kT. They can use Exploration 20.4 (and Problem 20.4) to get a true sense of equipartition of energy as these exercises separate out the contributions to total kinetic energy from atoms and rigid rotor, diatomic molecules.

Since Physlets are dynamic, students can see the moving particles (representing gas atoms) and their different speeds which are ever changing due to collisions. Illustration 20.1 makes the constantly changing speeds explicit as students see a dynamically changing Maxwell-Boltzmann distribution. This can quickly dispel the student belief that all the atoms have the same speed. It also gives students the clearer picture of why the particle speeds are constantly changing and the need for a defined characteristic speed (whether it be rms, average or mean speed).

## Multiple Representations of Thermodynamic Processes

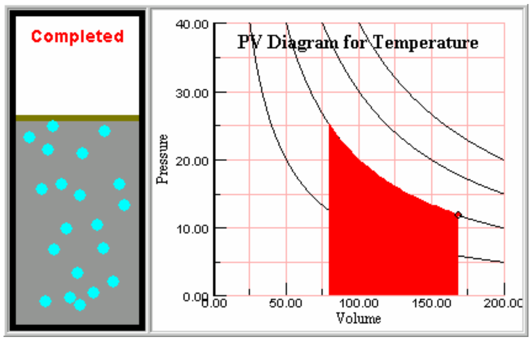
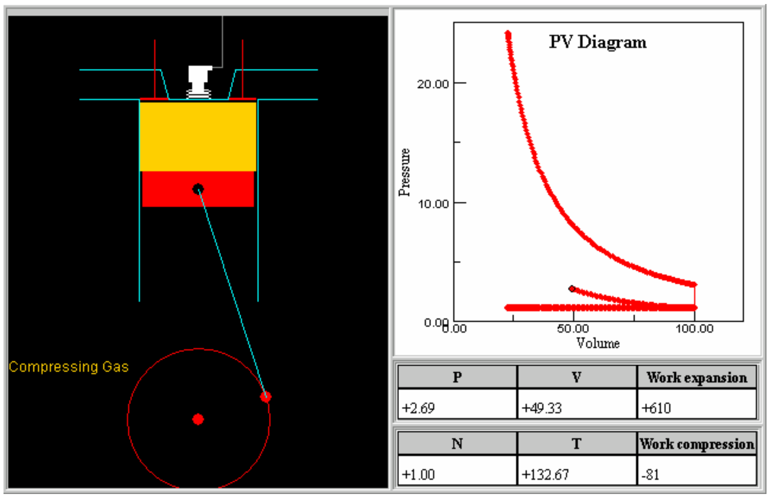


**Figure I.8.3: PV diagram for isothermal expansion in Exploration 20.6**

With Physlets, the tools students use to analyze problems, namely PV diagrams, need not be separated from the fundamental picture of box teeming with moving, colliding particles. Students can see a dynamic expansion on both the PV diagram and the particle container. Furthermore, as in Exploration 20.5 or Exploration 20.6, shown in Figure I.8.3, the work done (area under the curve) is clearly shown. These types of exercises help students understand why we use PV diagrams especially when they can see the dynamic connections between the diagram, the particles in a box, and the work done by the gas. Additionally, students get a much better sense of the difference between the different named thermodynamic processes (isobaric, isothermal, isochoric, adiabatic), if they can directly compare them as in Illustration 20.3 (and as they are asked to calculate the work done or heat added in Problems 20.8-20.11).

Understanding how to read PV diagrams is crucial in analyzing engine cycles as well, so once students are familiar with them, we use these again to illustrate engine cycles: explicitly showing when the gas is in contact with a hot and cold reservoir. Again, the advantage is the ability of the Physlet to easily connect the dynamics of the expansion and compression with the path on the PV diagram and net work associated with the complete engine cycle.

## The Otto Engine



| P | V | Work expansion |
|---|---|---|
| +2.69 | +49.33 | +610 |

| N | T | Work compression |
|---|---|---|
| +1.00 | +132.67 | -81 |

**Figure I.8.4: Otto Engine of Exploration 21.2**

Probably the most complex, but least abstract, engine process in this set is the Otto Engine (Exploration 21.2, shown in Figure I.8.4, and Problem 21.4). This animation shows a model of a combustion engine and the associated PV diagram. As the piston moves up and down, the point on the PV diagram is shown until the complete path is drawn. Students will need to work through this example, stopping at a number of points on the path, to fully understand what is happening. However, they can, quite quickly, determine the efficiency of this engine and it helps them connect the abstract PV diagrams of physics to a simplified version of real-world combustion engines in their cars.

## Multiple Approaches to Abstract Concept: Entropy

Finally, when it comes to entropy, there are several Illustrations and Explorations to allow different approaches to entropy: Exploration 21.3 guides students through the connection between entropy and microstates (statistics) while Illustration 21.3 connects entropy to the Second law of thermodynamics and Illustration 21.4 connects entropy to engine efficiency. Problems 21.6 and 21.7 require entropy calculations for different thermodynamic processes (including engine processes).

# Chapter I-9: Electromagnetism

In these chapters, we exploit the power of Physlets to help students visualize abstractions: electric and magnetic fields (vector and field line representations), equipotential surfaces, force vectors, induced currents and emfs, and microscopic models of electrodynamics. These exercises help students to understand and use the multiple representations employed with these topics. We encourage and even, at times, require that students take advantage of the ability of the Physlet to quickly model what happens when different system parameters are changed. These Physlet-based exercises are also designed to help students make connections between the individual chapters and connections back to mechanics.

## What Is Covered?

- The chapters in this section start with electrostatics and end with Faraday's law, covering introductory physics material on Maxwell's equation.
- Circuits, both DC and AC, are included in their own part of *Physlet Physics*.
- Electromagnetic waves make up the first chapter of the Optics part of *Physlet Physics*.

## *Highlights of Electromagnetism Chapters*

### Visualizing Abstract Concepts: Electric and Magnetic Fields

Since electromagnetism depends so heavily on the understanding of fields, which are abstractions removed very far from students' experiences, the extra visualization provided by Physlets is particularly valuable.

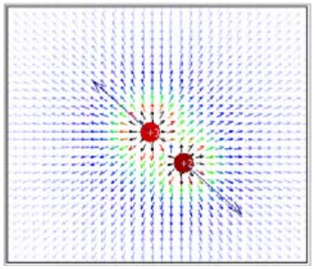


**Figure I.9.1: Forces and electric fields in Illustration 23.2.**

Consider a simple example in electrostatics from Illustration 23.2 shown in Figure I.9.1: the electric field from a two point charges. The Physlet quickly simulates the vector field and students can see the effect of changing the charge and the distance between the charges. Not only can students see the field, this also addresses a common misunderstanding that students have. When the charges are not equal (often not illustrated explicitly in books), students tend to think that the larger charge experiences the larger force since **F** = q **E** and q is bigger. Or, students may think that the smaller charge experiences the larger force because it is experiencing a force due to a larger charge. Using this exercise, though, students can quickly realize that the force is equal and opposite and as they move the charges around, they can see that these forces are a linked pair and constitute a Newton's third law pair.

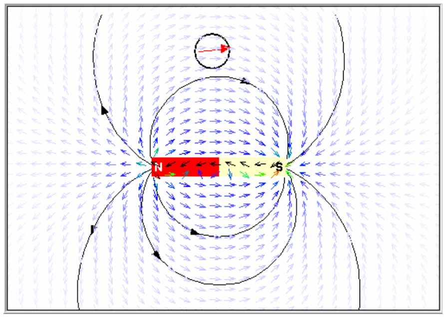
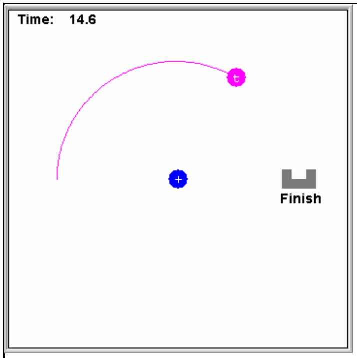


**Figure I.9.2: Representations of the magnetic field from Illustration 27.1.**

Students can also create charge distributions and see both vector field representations as well as field line representations common in textbooks. Similarly, they can see magnetic vector field representations as well as field line plots as is demonstrated in Figure I.9.2. This helps students connect charges (currents/magnets) with vectors and the field line representations in textbooks. Being able to make these connections is crucial to the development of the theory that follows. Also, when students have a chance to create their own electric or magnetic distributions and see the resulting fields, it helps them get a better handle on these abstractions.

## Encouraging Students to Conceptualize

Even though these Physlets do more computational work than a student (or instructor) could do in a finite time frame, they do not do so much work that there is nothing left for the student to do. Quite the contrary! Students must think deeply about the problems and truly understand the concepts to work many of the problems. Consider, for example, Problem 22.8 (Figure I.9.3).



A positive test charge of 1 x $10^{-5}$ C with a mass of 0.9 kg is shown near a variable charge with a fixed position (at the origin). You may change the charge of the central charge and the initial velocity of the test charge **(position is given in meters and time is given in seconds)**.

What initial velocity must you give the test charge so that the test charge can make it from its starting place to the finish line in a circular path?

**Figure I.9.3: Problem 22.8 requiring students determine initial conditions to get the pink charge into the finish box.**

Although the Physlet calculates the field, the student must figure out for herself or himself the appropriate initial velocity so that the pink test charge (of $10^{-5}$ C) will make it into the "Finish" box for different values of the center blue positive charge. As in other

Physlet-based Problems, the student must decide what to measure (the radius of the desired motion). To solve this Problem, the student must use tools from mechanics (circular motion) in combination with Coulomb's law. Students can quickly test answers, but gaming alone (by testing a variety of answers), although fun, probably will not allow them to develop a general relationship.

## Connecting Mathematical and Physical Ideas: Important for Electromagnetism

Because of the mathematical nature of much of electromagnetism, there are a number of Physlet-based exercises that help students to connect these somewhat abstract and advanced mathematical ideas with the physical concepts they represent. For example, the power of the Physlet to calculate a variety of charge configurations is used to help students understand the need for symmetry to use Gauss's law. Figure I.9.4 shows one view of a filament from Illustration 24.2 with a flux "detector" (the red square with its output reading on the bar graph to the left).

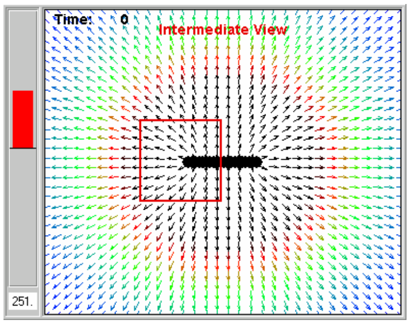


**Figure I.9.4: View of a charged filament in Illustration 24.2**

This same Illustration can show a close-up view of the filament and a view from far away along with flux "detectors" of the same geometry as the charge distribution. In the other two cases, the symmetry is such that you can use Gauss's law to calculate the electric field. Students can quickly compare the direction of the electric field with the geometry of the flux "detector", or Gaussian surface. The same holds true for magnetism and the symmetry needed for Ampere's law as demonstrated in Exploration 28.2 (Figure I.9.5). For Ampere's law, Physlets provide an additional advantage with a dynamic calculation of the path integral as a student moves a cursor around on a screen. In this Exploration, value of the path integral is shown in the bar graph to the right (as well as in a table).

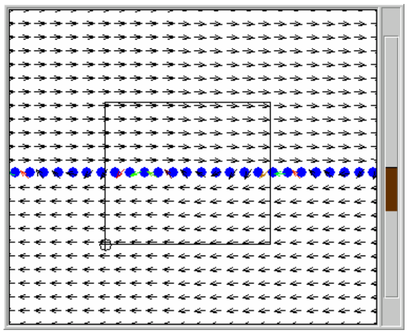


**Figure I.9.5: Black Amperian "loop" (square) surrounding a plane of current carrying wires in Exploration 28.2.**

## Microscopic Models

Students can also use Physlet-based exercises to help visualize what happens at a microscopic level in the presence of electromagnetic fields. Consider Illustration 26.4, shown in Figure I.9.6, which shows a model of a capacitor at the microscopic level. Here, the electrons (represented in blue) separate from the underlying bound charge and build up on the plates of a capacitor. Having a microscopic model can help students understand how current can flow when a capacitor is in a circuit even though charge does not pass between the two plates of a capacitor.

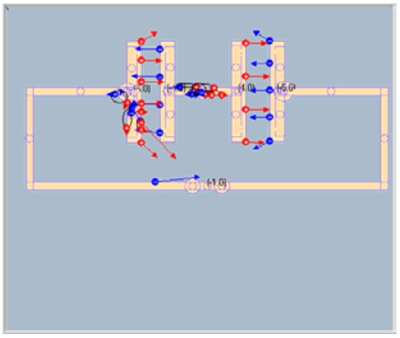
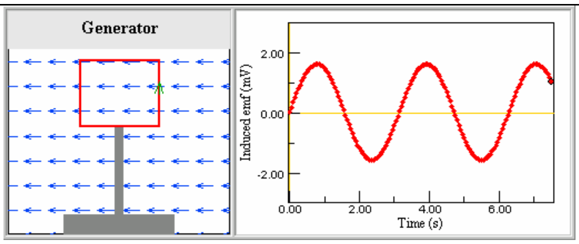


**Figure I.9.6: Microscopic model of charges on capacitors connected in series in Illustration 26.4.**

## Simplification

Physlets can also provide a simplified model of a real-world system, keeping the dynamics of what occurs in the actual application. This can be particularly important in electromagnetism where many concepts are abstract for students. Consider Problem 29.10, shown in Figure I.9.7, which deals with an electric generator (explained in Illustration 29.3) consisting of a wire loop rotating in an external magnetic field.



The graph on the right shows the induced emf through the loop as a function of time **(position is given in centimeters, time is given in seconds, and emf is given in millivolts)**. The green arrow shows the direction and magnitude of the induced current.

a. What is the magnitude of the magnetic field?
b. Looking down on this loop from above, in what direction is it rotating (clockwise or counterclockwise)? Explain your answer.

**Figure I.9.7: Problem 29.10 a simple model of an electric generator.**

The Physlet Problem shows the motion and the associated induced emf, but strips away the rest of the details of a generator to get to Faraday's laws which is the basis for understanding the generator. This Problem demonstrates another feature of Physlet Problems: new types of problems unique to animations. In this Problem, students must decide which way the loop is rotating to create the induced current shown by the green arrow, which is only possible in an animation.

## Connections to Mechanics

Throughout this section, various exercises recall connections to mechanics, particularly in Chapter 25: Electric Potential. For example, in Illustration 25.2, shown in Figure I.9.8, the work required to move the red test charge from one equipotential contour (green and brown contours) to another appears in the bar graph to the right. At the same time, the animation depicts the force on the particle showing that the force changes with position (requiring an integral calculation for the work). Associated Problems require that students measure the change in kinetic energy of a moving charge with the change in potential.

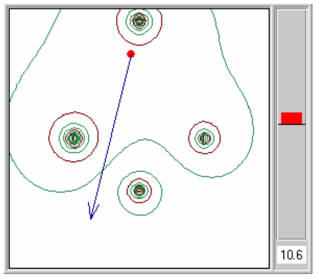


**Figure I.9.8: Equipotential contours and work to move the red test charge in Illustration 25.2.**

## Real-World Applications of Electromagnetism

There are several applications of electromagnetism that are introduced in *Physlet Physics*. For example, Illustration 23.4 shows students how cathode ray tubes operate, Illustration 27.3 demonstrates the mass spectrometer, Illustration 22.4 discusses static cling, and Illustration 29.3 demonstrates the electric generator. In addition to these Illustrations, you will also find Explorations and Problems dealing with these applications.

# Chapter I-11: Optics

Optics is, by definition, very visual. Therefore, the topic lends itself quite well to integration with Physlets. In our experience, students usually find the Physlet-based exercises in this section to be fun and interesting, perhaps because of the nature of "play" that follows so naturally from them.

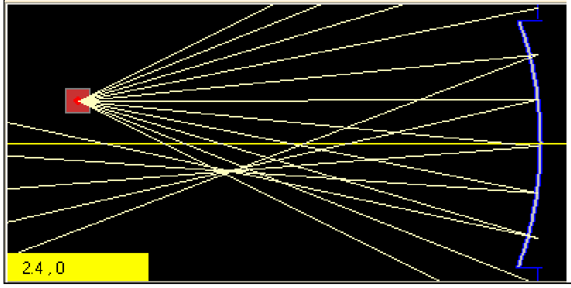
## What Is Covered?

- The chapters in this section cover electromagnetic waves, ray optics (mirrors, lenses and refraction), and wave optics (interference, diffraction, and polarization).
- The optics appendix, "What's Behind the Curtain", presents students with a hidden optical element that could be a flat mirror, concave mirror, convex mirror, converging lens or diverging lens. The student must determine what kind of element is hidden. These questions were placed in an appendix because they encompass ideas from several chapters.
- Chapter 36 combines ideas from previous chapters with a focus on real applications of optics concepts.

## *Highlights of Optics Chapters*

### Multiple Solution Methods in Optics Bench-Problems

Many of the Problems in these chapters ask student to find the focal length of an optical element. As an instructor, you will find that students will approach these problems using a variety of solving methods. For example, consider Problem 33.1 shown in Figure I.11.1.



A point source is located to the left of a mirror. You can drag this point source to any position **(position is given in meters and angle is given in degrees)**. Find the focal length of the mirror.

**Figure I.11.1 – Problem 33.1 which can be answered several ways.**

Notice that students are asked to find the focal length but are not given any variables. They must determine what information is relevant to answer the question (Where does an incoming parallel ray cross the principal axis?) and then interact with the Physlet to get that information (in this case, at x = 2.4 cm). They can then use this information to solve the problem. The student must also find the location of the mirror to determine the focal length. A "plug-n-chug" approach will not be effective in this problem as there is nothing to "plug" until some conceptualization of the problem has been done. It is also important to note that while the problem contains no overt "givens", it provides the means to access a plethora of information, most of which is unnecessary for the solution. Just as with real-world problems, the students must decide how to focus their efforts and what information to ignore.

Now, you may argue that the Physlet is unnecessary. After all, the problem can be asked and solved with just a picture and a ruler. But such an argument misses an important point. Students can not interact with a static picture. We have seen the benefit of interactivity in three very specific ways: the ability of the students to be creative, the learning opportunities provided, and the likelihood of students demonstrating misconceptions.

In the above example a solution was suggested based on finding where a parallel ray crossed the principal axis. If a static picture was all students had to work with, then the above solution would be the only method available. However, when working with the Physlet, students have the freedom to look at the problem from alternative angles. If a class is presented with this problem it is likely that a portion of the class will obtain a correct answer by moving the source along the principal axis to the point where all rays leaving the source are reflected parallel, thereby identifying the focal point. However, there are four methods that a student could correctly use to answer this question. They are

1. Place the source where the rays are reflected parallel, as discussed above.
2. Place the source above the principal axis and follow an incoming parallel ray to see where it crosses the principal axis, that will be the focal point.
3. Place the source on the principle such that the rays are reflected and refocus at the same point of the source, that will be the radius of curvature, which is twice the focal length.
4. Measure the distance from the mirror to the source (the object distance), measure the distance from the mirror to where the rays converge (the image distance), and use the mirror equation to find the focal length.

Physlet Problems allow for creativity on the part of the student that is difficult to replicate in a paper-based problem.

## Unique Problems: What is Behind the Curtain?

The Optics chapters include a number of Problems where students are given a source (point, infinite beam, or object) and a region hidden from view. They are asked to determine what optical element is behind the curtain. In the example shown in Figure I.11.2 (Problem 35.2), students are shown four such regions, asked to identify what is behind each curtain, and then asked to rank the objects in terms of their focal lengths. Students can move and resize the sources.

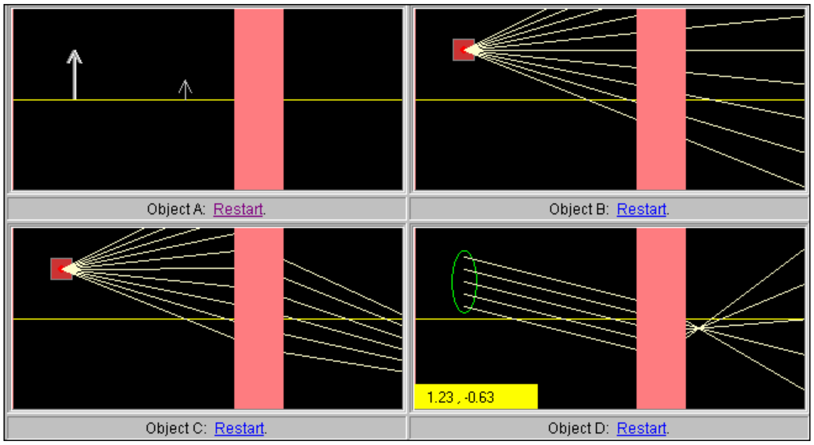


**Figure I.11.2 – Problem 35.2, a behind the curtain problem.**

In this particular problem, different types of sources are used so that students must utilize their understanding of each. For Curtain A, students must change the location of the object and note that the image is always on the same side of the mystery element as the object. At this point it is reasonable for the students to guess that a diverging lens is behind the screen. Further manipulation of the location and height of the object confirms this assertion. For Curtain B, students need to note that the element has no effect on the light. There is nothing behind Curtain B. Curtains C and D both contain converging lenses which students could deduce by observing the direction incoming rays are bent.

In order to rank the focal lengths of the mystery objects, students must first recognize that the focal length of the diverging lens is negative and that it is positive for the two converging lenses. This makes it unnecessary to calculate the actual focal length of the diverging lens, though it could easily be done by noting the locations of the object and image and using the lens equation. The focal lengths of the two converging lenses can be compared by noting where parallel rays converge.

## Simplification: A Model of the Eye and Camera

Students are usually motivated by problems they see as relevant to their everyday lives. Our students found the Physlet that model's the eye to be exciting and relevant, and claimed it helped them to visualize the functioning of the eye. Students are asked to consider the eye in Illustration 36.2 (shown in Figure I.11.3) and in Problems 36.1-36.3. The Physlet allows student to simulate a healthy eye, a nearsighted eye, or a farsighted eye. They can view how either a far away source or near by source is focused by the eye and can use the slider at the bottom to simulate accommodation of the eye. Finally, they can add an eyeglass and change the focal length of this lens to "prescribe" glasses. Note that, the eye is not shown to scale and units are arbitrary because the actual scale is too small to allow the functioning of the eye to be noticeable.

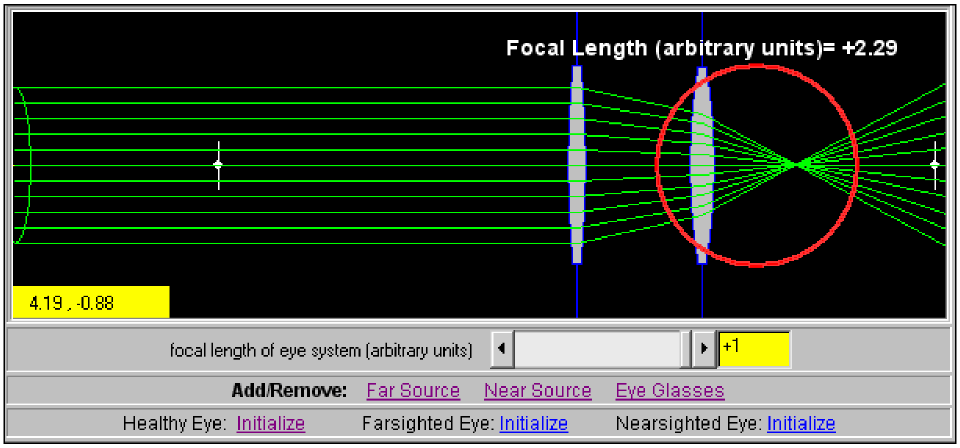


**Figure I.11.3 – A model of the eye from Illustration 36.1.**

In order to answer the Problems associated with the model of the eye correctly, students must have a strong conceptual understanding of the operation of lenses, especially as that operation relates to the optical system of the eye. Students who have this understanding can easily answer this question. Students who have misunderstandings will find it difficult to answer the question. We used this question on an exam and found it to be beneficial for assessing student understanding.

Similar to the model of the eye, Chapter 36 also contains an Illustration (Illustration 36.2) and an Exploration (Exploration 36.1) that models a camera. This model includes normal, telephoto, and wide-angle lenses. The Exploration guides students through the operation of the various lenses.

## Connections to Index of Refraction

You will find a number of Physlets that deal with the index of refraction. Because Physlets can easily show the path of light through various mediums, they are ideally suited for this topic. In the example, shown in Figure I.11.4, students are presented with an applet that displays a point source and a lens.

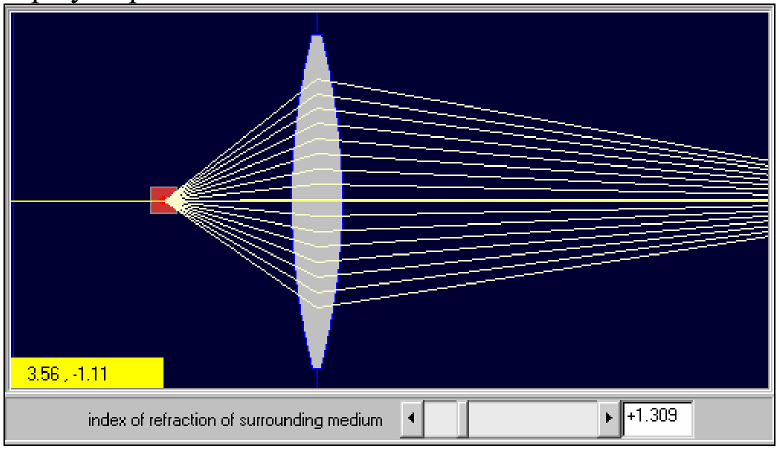
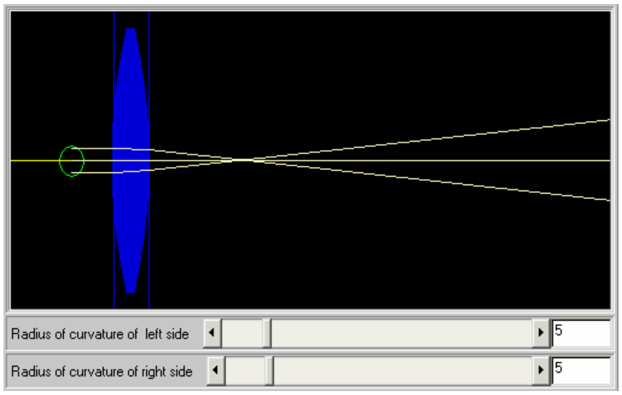


**Figure I.11.4 – Exploration 34.1 showing students that a lens is nothing more than a region of differing index of refraction.**

The index of refraction of the medium surrounding the lens can be changed. As the index of refraction is changed, the color of the background varies as a visual clue to the index change. Students are then asked to find the index of refraction of the lens. In order to determine the index of refraction of the lens, students can change the index of refraction of the medium surrounding the lens until the light passing through the lens is unaffected. At that point, the index of refraction of the lens must be the same as the index of refraction of the surrounding medium.

Although this is a fairly easy problem for students, the process of solving the problem can help students to develop their understandings of refraction and lens operation. The problem reinforces the idea that lenses work by means of a different index of refraction from the surrounding medium. It can also be used to help students see that a "converging" lens is only converging if placed in a medium where the index of refraction of the medium is less than that of the lens. A glass lens in air will be converging, but if the same glass lens is placed in a medium of higher index of refraction it will act as a diverging lens. These issues are important for developing a conceptual understanding of lenses but are often glossed over in traditional instruction. Students are then ready to use Explorations 34.3 and Explorations 35.5 to "build" lenses by changing the index of refraction and radius of curvature of dielectric materials (see Figure I.11.5 below).

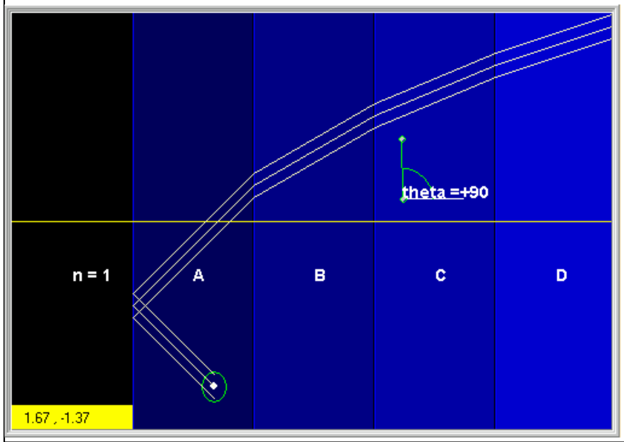


Radius of curvature of left side 5

Radius of curvature of right side 5

**Figure I.11.5: Using the lens maker's equation in Exploration 35.5.**

Physlets are also well-suited to help students understand the connection between total internal reflection and refraction. As students change the angle of the light entering a medium (they simply need to click drag on a light ray to change its angle), they can see the refraction and how that refraction leads to total internal reflection. This allows them to solve more complicated problems such as Problem 34.7 as shown in Figure I.11.6.



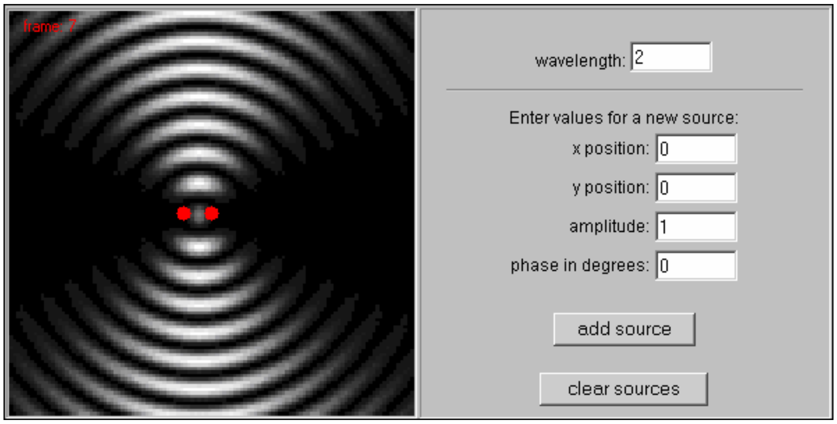
Four materials are next to each other and the change in the index of refraction from one to the next is the same. You can use the moveable green protractor and click-drag to measure angles.

What is the index of refraction of region D? Explain your observations as you drag the source through each of the different materials (why is there total internal reflection between some interfaces, but not others?).

**Figure I.11.6-Total internal reflection in Problem 34.7.**

## Ripple Tank Simulation

Another particularly nice use of Physlets is to simulate a ripple tank. Unlike a physical ripple tank, the applet creates no mess and can be made accessible to students outside of class. There are a number of Problems and Explorations that utilize the ripple tank, especially in the interference and diffraction chapters. The most basic form of the ripple tank in presented in Illustration 37.1, shown in Figure I.11.7.



wavelength: 2

Enter values for a new source:

x position: 0

y position: 0

amplitude: 1

phase in degrees: 0

**Figure I.11.7 – Illustration 37.1, a no mess ripple tank.**

The Illustration can be used in class or as an out of class tool for students to play with as they construct their understandings. The applet allows the user to create any number of wave sources, put them at any location, and specify their amplitude and phase. The wavelengths of all the sources must be the same, but can be set to a desired value. When the applet plays, waves can be seen moving away from the sources, just as with a ripple tank.

## Visualizing Abstract Concepts: Polarization

Making the connection between electromagnetic waves and physical optics is often difficult for students and this difficulty often appears when students try to understand polarization (especially circularly polarized light). A combination of two different representations of polarization can help students understand the connection between waves, light, and polarization. Exploration 39.1, shown in Figure I-11.8, allows students to input values for the amplitude of the electric field in the x and y directions as well as the phase shift between the two waves.

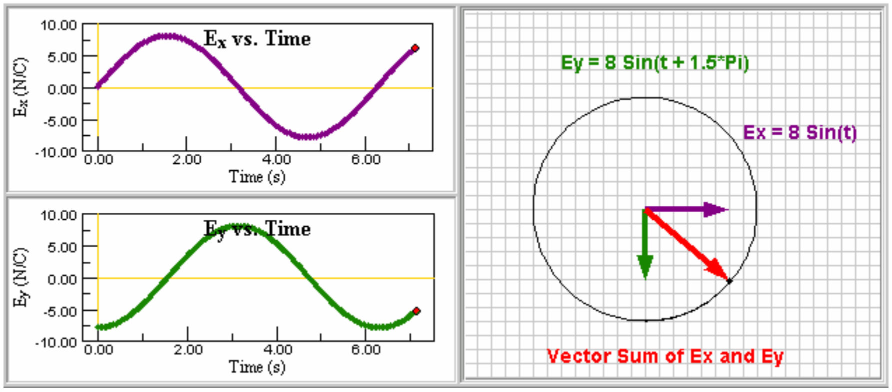


**Figure I.11.8-Circularly polarized light in Exploration 39.1.**

As the text of the Exploration directs students to try different inputs for the amplitudes of the fields and the phase angle, they can learn how to create linearly, circularly and elliptically polarized light and the connection between the equations and the polarization. Another representation of polarization shows a traveling wave. In this representation, used in Illustration 39.2 and shown in Figure I.11.9, students can not only see a polarized traveling wave, they can also see the effect of a polarizing film.

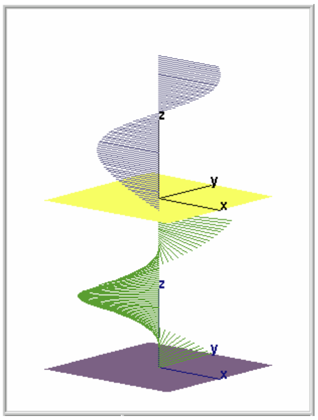


**Figure I.11.9- Circularly polarized light passing through a linear polarizer.**

# Part II

## Discussion of Illustrations, and Solutions to Problems

# Chapter II.1: Introduction

## *Illustrations*

### Illustration 1.1: Static Text Images Versus Physlet Animations

*Topic(s):* Control Buttons, Links
*Purpose:* To introduce the features of Physlet animations.
*Suggested Use:* Have students read through illustration as a way of orientating them to the *Physlet Physics* workbook. Advanced students will probably be able to figure out most of the features without the help of this illustration but younger students or the less technologically inclined will find it a gentle introduction.

### Illustration 1.2: Animations, Units, and Measurement

*Topic(s):* Making Measurements
*Purpose:* To alert students to issues regarding how to take measurements from a Physlet.
*Suggested Use:* Have students read through illustration as a way of orientating them to the *Physlet Physics* workbook. Advanced students will probably be able to figure out most of the features without the help of this illustration but younger students or the less technologically inclined will find it a gentle introduction.

### Illustration 1.3: Getting Data Out

*Topic(s):* Representations of Quantities
*Purpose:* To further orient students to aspects of using *Physlet Physics*.
*Suggested Use:* Have students read through illustration as a way of orientating them to the *Physlet Physics* workbook. Advanced students will probably be able to figure out most of the features without the help of this illustration but younger students or the less technologically inclined will find it a gentle introduction.

## *Explorations*

### Exploration 1.1 - Click-Drag to Get Position

*Topic(s):* Making Measurements

### Exploration 1.2 - Animations, Units, and Measurement

*Topic(s):* Inputting Data

### Exploration 1.3 - Getting Data Out

*Topic(s):* Inputting Formulas

## *Problems*

### P.1.1 (Level 1)

*Topic(s):* Vernier Caliper
*Answer:*

**Animation 1 – 0.32 cm**
**Animation 2 – 2.46 cm**
**Animation 3 – 0.68 cm**
**Animation 4 – 1.29 cm**

### P.1.2 (Level 1)

*Topic(s):* Taking Data
*Answer:*

a) **slope = 1.28 cm/s, intercept = -10.29 cm**
b) **slope = velocity of car at t=0.**

### P.1.3 (Level 1)

*Topic(s):* Interacting with Physlet, Displacement, Velocity, Acceleration
*Answer:*

a) **Position is the easiest, acceleration is the most difficult.**
b) **Since velocity is the change in position relative to a change in time, any errors made in positioning the truck will be compounded in the velocity measurement. Likewise, since acceleration is the change in velocity relative to some change in time, the errors with velocity will be magnified for acceleration.**

# Chapter II.2: One-dimensional Kinematics

## *Illustrations*

### Illustration 2.1: Position and Displacement

*Topic(s):* Displacement, Graphical Representation of Motion
*Purpose:*. To introduce the concepts of position, displacement, and position graphs.
*Suggested Use:* Describe the motion of the trucks to students and have them predict the graph for each animation before it is played. Differences in the graphs and incorrect predictions can then be discussed.

### Illustration 2.2: Average Velocity

*Topic(s):* Velocity, Graphical Representation of Motion
*Purpose:* To introduce the concept of average velocity.
*Suggested Use:* Physlet can be used in class as a visual representation when introducing the concept of velocity. The dynamic-visual nature can help students make the connection between the mathematics and the concepts.

### Illustration 2.3: Average and Instantaneous Velocity

*Topic(s):* Velocity, Graphical Representation of Motion
*Purpose:* To introduce the concept of velocity.
*Suggested Use:* Just as with Illustration 2.2, this illustration can be used to help students make connections between ideas and mathematics. Use as an in-class when introducing concepts.

### Illustration 2.4: Constant Acceleration and Measurement

*Topic(s):* Acceleration
*Purpose:*. To guide students through the process of determining acceleration from a Physlet.
*Suggested Use:* Reading assignment for students who may have trouble figuring out how to determine acceleration on their own. It is important that students know how to make measurements from the Physlet to determine acceleration as they will be required to do so in many problems throughout *Physlet Physics*.

### Illustration 2.5: Motion on a Hill or Ramp

*Topic(s):* Displacement, Velocity, Acceleration, Graphical Representation of Motion
*Purpose:* To further develop ideas about position, velocity and acceleration.
*Suggested Use:* Reading assignment for students who might have difficulty with these concepts. Could also be used in class as a predict and explain exercise. Describe (or show) the motion and have students predict the graph. Differences in the graphs and incorrect predictions can then be discussed.

### Illustration 2.6: Free Fall

*Topic(s):* Free Fall, Graphical Representation of Motion
*Purpose:* To introduce the concepts of free fall.
*Suggested Use:* As with previous Illustrations, this Illustration can be used in class by having students predict the graphs that will be produced from the situations modeled in the three animations. Differences in the graphs and incorrect predictions can then be discussed.

## *Explorations*

### Exploration 2.1 – Compare Position vs. Time and Velocity vs. Time Graphs

*Topic(s):* Displacement, Graphical Representation of Motion

### Exploration 2.2 – Determine the Correct Graph

*Topic(s):* Displacement, Velocity, Graphical Representation of Motion
*Answers:* b) Graph A is correct, c) Graph D is correct.

### Exploration 2.3 – A Curtain Blocks Your View of a Golf Ball

*Topic(s):* Velocity, Graphical Representation of Motion

### Exploration 2.4 – Set the x(t) of a Monster Truck

*Topic(s):* Kinematics, Graphical Representation of Motion

### Exploration 2.5 – Determine x(t) and v(t) of the Lamborghini

*Topic(s):* Kinematics, Graphical Representation of Motion

### Exploration 2.6 – Toss the Ball to Barely Touch the Ceiling

*Topic(s):* Kinematics, Free-Fall

### Exploration 2.7 – Drop Two Balls; One with a Delayed Drop

*Topic(s):* Free-Fall

### Exploration 2.8 – Determine the Area Under a(t) and v(t)

*Topic(s):* Position, Velocity, Acceleration, Graphical Representation of Motion

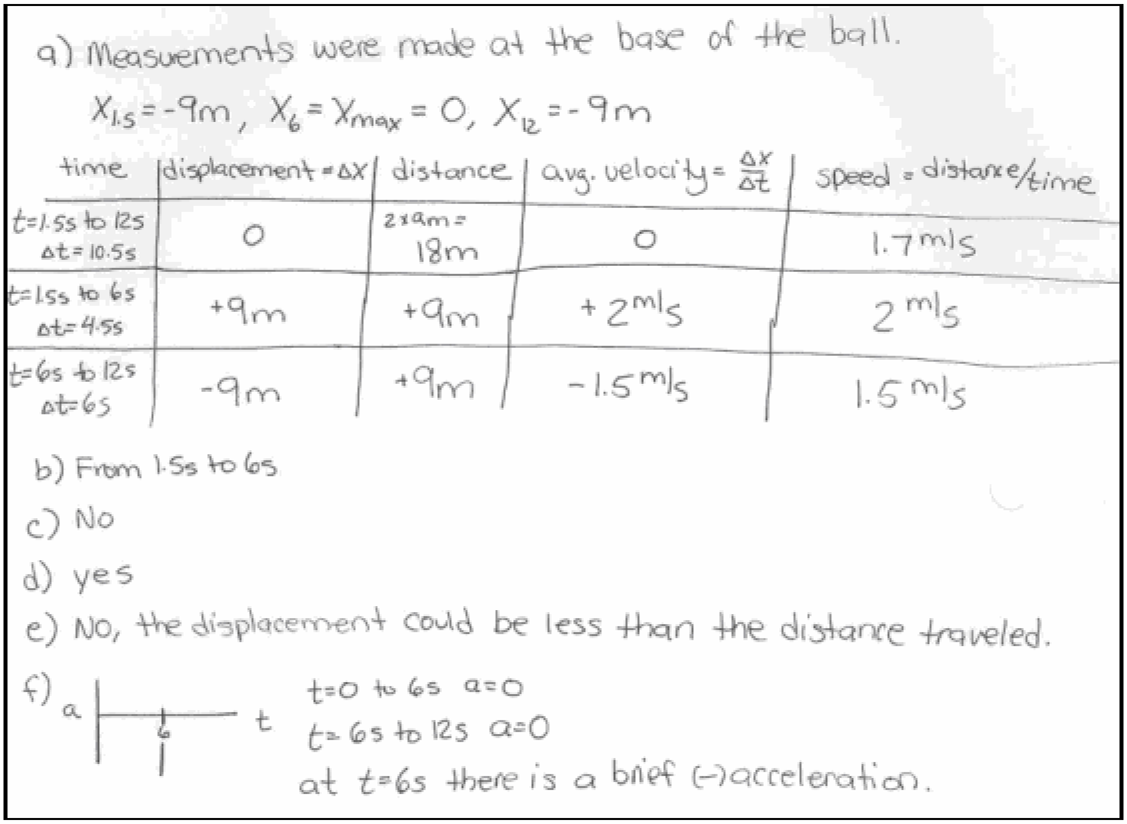
## *Problems*

### P.2.1 (Level 1)

*Topic(s):* Displacement, Graphical Representation of Motion
*Answer:* **Animation 3**

## P.2.2 (Level 1)

*Topic(s):* Displacement, Velocity, Acceleration
*Answer:*



a) Measuements were made at the base of the ball.

$X_{1.5} = -9m$, $X_6 = X_{max} = 0$, $X_{12} = -9m$

| time | displacement = Δx | distance | avg. velocity = $\frac{\Delta x}{\Delta t}$ | speed = distance/time |
|---|---|---|---|---|
| t=1.5s to 12s<br>Δt=10.5s | 0 | 2x9m =<br>18m | 0 | 1.7 m/s |
| t=1.5s to 6s<br>Δt=4.5s | +9m | +9m | + 2 m/s | 2 m/s |
| t=6s to 12s<br>Δt=6s | -9m | +9m | - 1.5 m/s | 1.5 m/s |

b) From 1.5s to 6s

c) No

d) yes

e) No, the displacement could be less than the distance traveled.

f)

t=0 to 6s a=0
t=6s to 12s a=0
at t=6s there is a brief (−)acceleration.

## P.2.3 (Level 1)

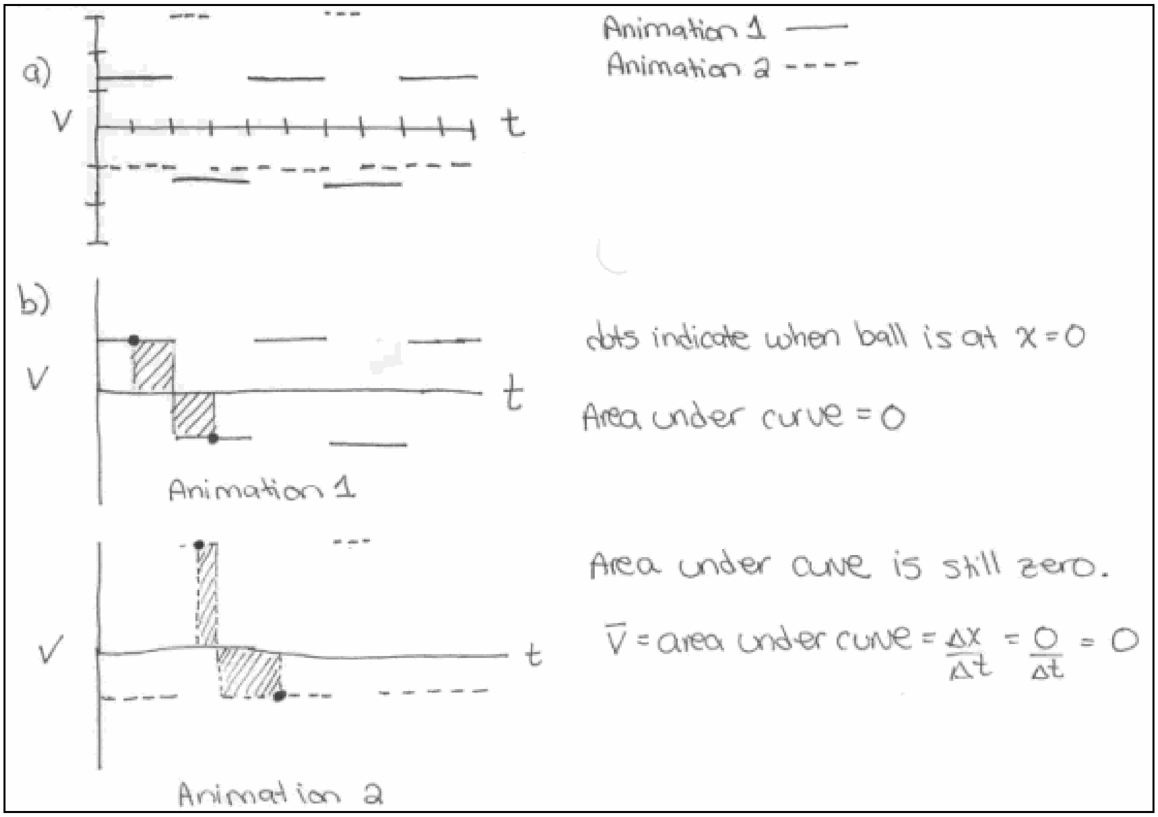
*Topic(s):* Velocity, Graphical Representation of Motion
*Answer:* **Animation 1**

## P.2.4 (Level 1)

*Topic(s):* Velocity, Graphical Representation of Motion
*Answer:* **a) Blue ball wins, b) The blue ball has the greater average velocity as can be seen from the graph.**

## P.2.5 (Level 1)

*Topic(s):* Velocity, Graphical Representation of Motion
*Answer:*



dots indicate when ball is at $x = 0$

Area under curve $= 0$

Area under curve is still zero.

$$\bar{V} = \text{area under curve} = \frac{\Delta x}{\Delta t} = \frac{0}{\Delta t} = 0$$

## P.2.6 (Level 2)

*Topic(s):* Velocity
*Answer:* **7.5 cm/s**

## P.2.7 (Level 2)

*Topic(s):* Kinematics
*Answer:*

Animation 1

$V_0=0,\ t_0=0,\ V_f=2m/s,\ t_f=1s$

$V=V_0+at$

$2m/s=0+a(1s)$

$\boxed{a_1=2m/s^2}$

Animation 2

$X_0=0,\ t_0=0,\ X_f=1.65m,\ t_f=1.2s$

$V_0$ appears to be zero

$X=X_0+V_0t+\frac{1}{2}at^2$

$1.65m=0+0+\frac{1}{2}a(1.2s)^2$

$\boxed{a=2.3m/s^2}$

Animation 3

$X_0=0,\ t_0=0,\ X_f=-2.2m,\ t_f=1.1s$

V appears constant

$\boxed{a=0}$

Animation 4

$X_0=0,\ V_0=0,\ X_f=1m,\ V_f=2.2m/s$

$V^2=V_0^2+2a\Delta X$

$(2.2m/s)^2=0+2a(1m)$

$\boxed{a=2.4m/s^2}$

Animation 5

$X_0=0,\ V_0=-1m,\ X_f=-2m,\ V_f=-3.3m/s$

$V^2=V_0^2+2a\Delta X$

$(-3.3m/s)^2=(-1m)^2+2a(-2)$

$\boxed{a=-2.5m/s^2}$

Animation 6

$X_0=0,\ V_0=3m/s,\ X_f=1.8m,\ V_f=0$

$V^2=V_0^2+2a\Delta X$

$0=(3m/s)^2+2a(1.8m)$

$\boxed{a=-2.5m/s^2}$

## P.2.8 (Level 2)

*Topic(s):* Kinematics, Graphical Representation of Motion

*Answer:*

a) Need to find $a_{purple}$

at $t=0,\ V_{purple}=-5m/s,\ X_0=8m$

$t=1s,\ V_{purple}=-5.5m/s,\ X_1=2.75m$

$v=V_0+at$

$-5.5m/s=-5m/s+a(1s-0)$

$a_{purple}=-0.5m/s^2$

when cars pass

$X_{purple}=X_{yellow}$

$X_{op}+V_{op}t+\frac{1}{2}a_pt^2=X_{oy}+V_{oy}t+\frac{1}{2}a_yt^2$

$8m+(-5m/s)t+\frac{1}{2}(-0.5m/s^2)t^2=5m+(-5m/s)t+0$

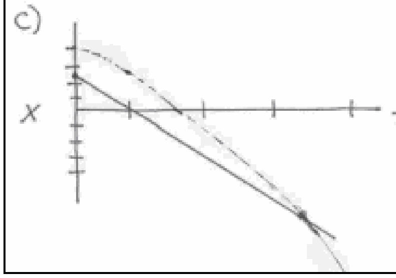
$3-0.25t^2=0$

$\boxed{t=3.5s}$

b) $X_{yellow}=X_{oy}+V_{yellow}t$

$=5m-5m/s(3.5s)=\boxed{-12.5m}$

c)



—— yellow = straight line since $a_{yellow}=0$

----- purple = curved

## P.2.9 (Level 2)

*Topic(s):* Velocity, Acceleration, Graphical Representation of Motion
*Answer:* **a) Increasing, b) constant, c) 0.05 m/s^2**

## P.2.10 (Level 2)

*Topic(s):* Kinematics
*Answer:* **a) 2 m/s, b) 10.2 s, c) 20.7 m, d) 20 m/s**

## P.2.11 (Level 1)

*Topic(s):* Velocity, Acceleration, Graphical Representation of Motion
*Answer:* **a) increasing, b) decreasing, c) displacement, d) 0.53 m/s^2, e) decreases**

## P.2.12 (Level 1)

*Topic(s):* Acceleration, Graphical Representation of Motion
*Answer:* **a) around t=0.12s, t=0.38s, t=0.62s, … b) first max at t=0.26 s, first min at t=0 s, c) zero**

## P.2.13 (Level 2)

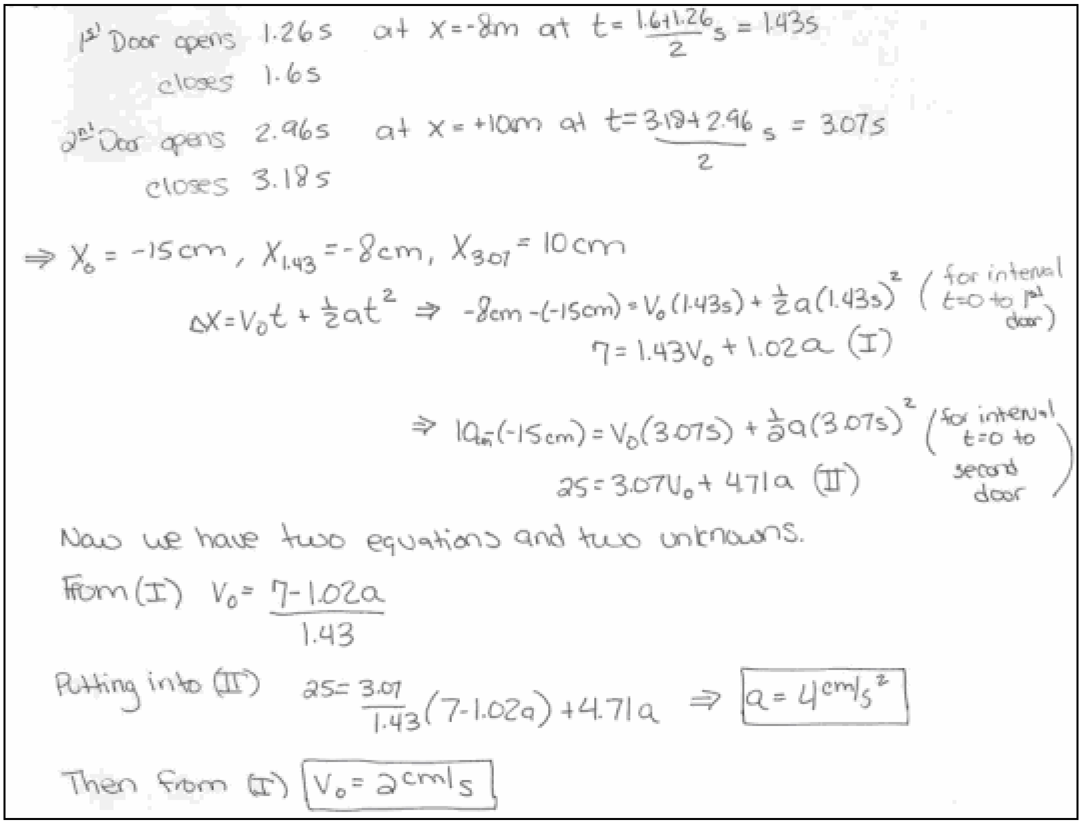
*Topic(s):* Kinematics, Graphical Representation of Motion
*Answer:* **-5.4 m/s**
a = 1.7 m/s^2

## P.2.14 (Level 2)

*Topic(s):* Kinematics, Graphical Representation of Motion
*Answer:* **a) alow = 0, ahill = 2.5 m/s^2, ahigh = 0, b) velocity is increasing, speed is decreasing, c) 4.6 m/s**

## P.2.15 (Level 3)

*Topic(s):* Kinematics
*Answer:* **a=4cm/s^2, v=2cm/s**



1st Door opens 1.26s at x=-8m at $t=\frac{1.6+1.26}{2}s = 1.43s$

closes 1.6s

2nd Door opens 2.96s at x=+10m at $t=\frac{3.18+2.96}{2}s = 3.07s$

closes 3.18s

$\Rightarrow X_0=-15cm,\ X_{1.43}=-8cm,\ X_{3.07}=10cm$

$\Delta X=V_0t+\frac{1}{2}at^2 \Rightarrow -8cm-(-15cm)=V_0(1.43s)+\frac{1}{2}a(1.43s)^2$ (for interval t=0 to 1st door)

$7=1.43V_0+1.02a$ (I)

$\Rightarrow 10cm-(-15cm)=V_0(3.07s)+\frac{1}{2}a(3.07s)^2$ (for interval t=0 to second door)

$25=3.07V_0+4.71a$ (II)

Now we have two equations and two unknowns.

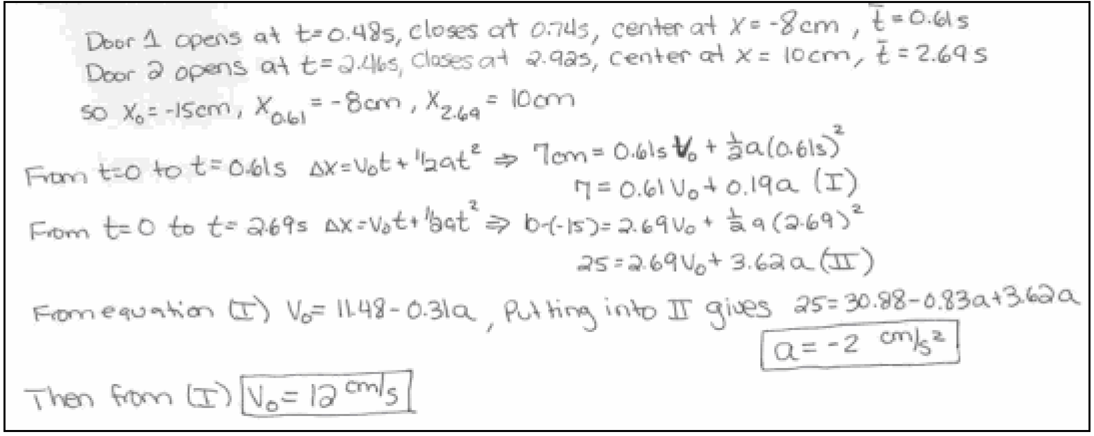
From (I) $V_0=\frac{7-1.02a}{1.43}$

Putting into (II) $25=\frac{3.07}{1.43}(7-1.02a)+4.71a \Rightarrow \boxed{a=4\,cm/s^2}$

Then from (I) $\boxed{V_0=2\,cm/s}$

## P.2.16 (Level 3)

*Topic(s):* Kinematics
*Answer:* **a=-2cm/s^2, v=12 cm/s**



Door 1 opens at t=0.48s, closes at 0.74s, center at x=-8cm, $\bar{t}=0.61s$

Door 2 opens at t=2.46s, closes at 2.92s, center at x=10cm, $\bar{t}=2.69s$

so $X_0=-15cm,\ X_{0.61}=-8cm,\ X_{2.69}=10cm$

From t=0 to t=0.61s $\Delta x=V_0t+\frac{1}{2}at^2 \Rightarrow 7cm=0.61s\,V_0+\frac{1}{2}a(0.61s)^2$

$7=0.61V_0+0.19a$ (I)

From t=0 to t=2.69s $\Delta x=V_0t+\frac{1}{2}at^2 \Rightarrow 10-(-15)=2.69V_0+\frac{1}{2}a(2.69)^2$

$25=2.69V_0+3.62a$ (II)

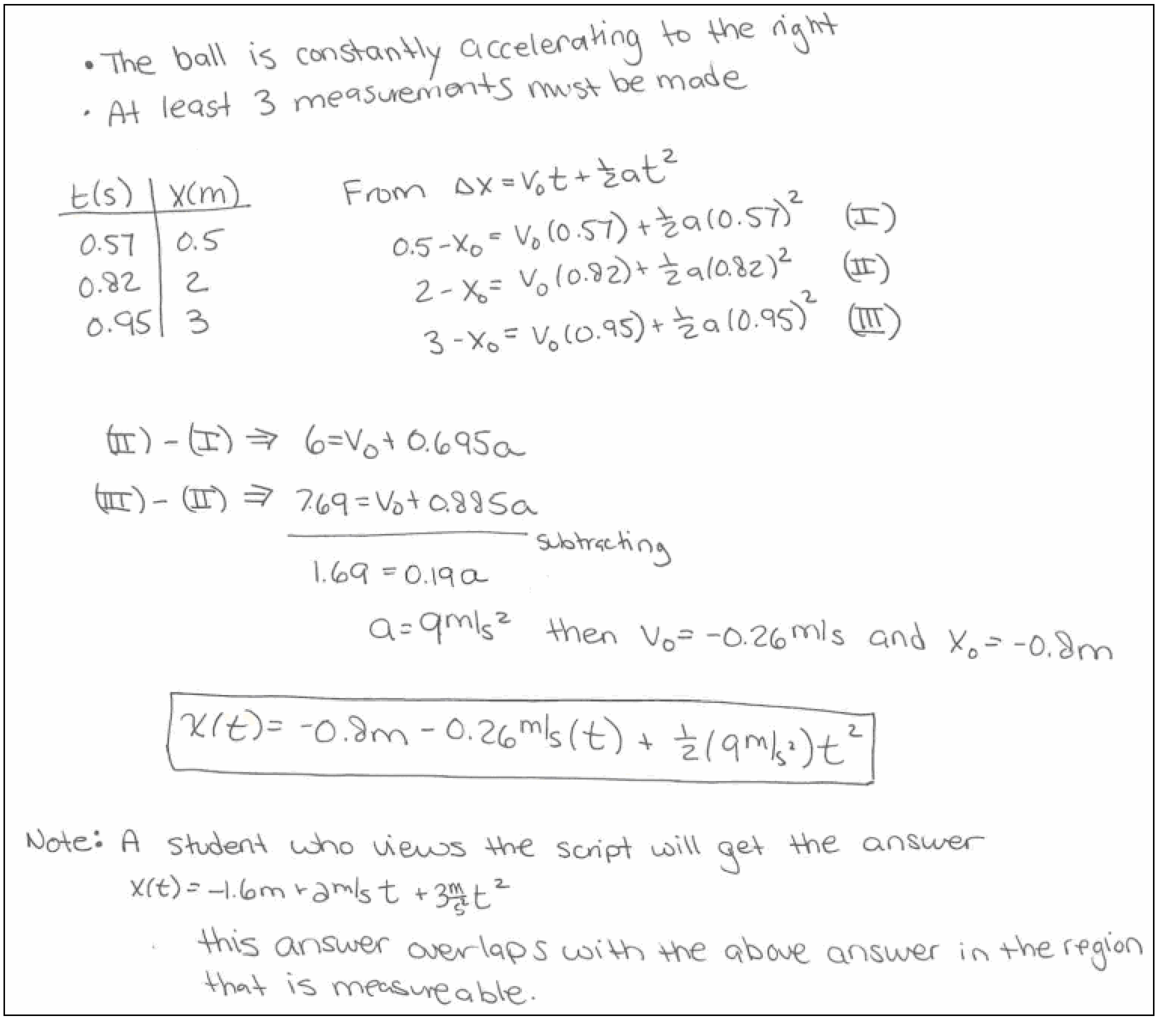
From equation (I) $V_0=11.48-0.31a$, Putting into II gives $25=30.88-0.83a+3.62a$

$\boxed{a=-2\,cm/s^2}$

Then from (I) $\boxed{V_0=12\,cm/s}$

## P.2.17 (Level 3)

*Topic(s):* Kinematics
*Answer:*



- The ball is constantly accelerating to the right
- At least 3 measurements must be made

| t(s) | x(m) |
|---|---|
| 0.57 | 0.5 |
| 0.82 | 2 |
| 0.95 | 3 |

From $\Delta x = V_0 t + \frac{1}{2} a t^2$

$$0.5 - x_0 = V_0 (0.57) + \tfrac{1}{2} a (0.57)^2 \quad (\mathrm{I})$$

$$2 - x_0 = V_0 (0.82) + \tfrac{1}{2} a (0.82)^2 \quad (\mathrm{II})$$

$$3 - x_0 = V_0 (0.95) + \tfrac{1}{2} a (0.95)^2 \quad (\mathrm{III})$$

$$(\mathrm{II}) - (\mathrm{I}) \Rightarrow 6 = V_0 + 0.695a$$

$$(\mathrm{III}) - (\mathrm{II}) \Rightarrow 7.69 = V_0 + 0.885a$$

subtracting

$$1.69 = 0.19a$$

$a = 9 \mathrm{m/s^2}$ then $V_0 = -0.26 \mathrm{m/s}$ and $x_0 = -0.8\mathrm{m}$

$$\boxed{x(t) = -0.8\mathrm{m} - 0.26 \mathrm{m/s}(t) + \tfrac{1}{2}(9 \mathrm{m/s^2}) t^2}$$

Note: A student who views the script will get the answer
$x(t) = -1.6\mathrm{m} + 2\mathrm{m/s}\, t + 3\frac{\mathrm{m}}{\mathrm{s}^2} t^2$

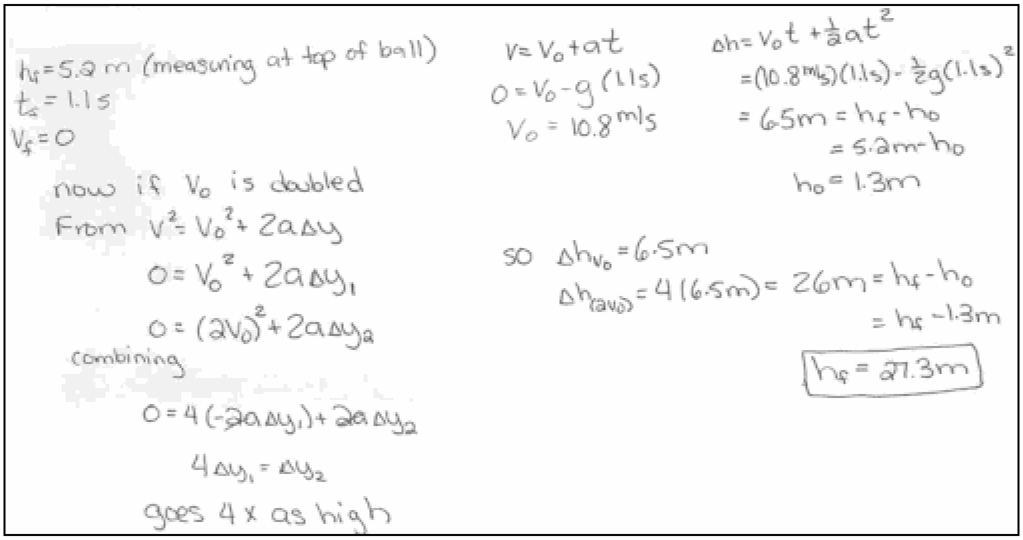
- this answer overlaps with the above answer in the region that is measureable.

## P.2.18 (Level 2)

*Topic(s):* Kinematics, Free-Fall
*Answer:* **a) 5 m/s, b) 2.2m**

## P.2.19 (Level 3)

*Topic(s):* Kinematics, Free-Fall
*Answer:* **27.3 m**



$h_f = 5.2\text{ m}$ (measuring at top of ball)

$t_s = 1.1\text{ s}$

$V_f = 0$

now if $V_o$ is doubled

From $V^2 = V_o^2 + 2a\Delta y$

$0 = V_o^2 + 2a\Delta y_1$

$0 = (2V_0)^2 + 2a\Delta y_2$

combining

$0 = 4(-2a\Delta y_1) + 2a\Delta y_2$

$4\Delta y_1 = \Delta y_2$

goes 4 x as high

$V = V_o + at$

$0 = V_o - g(1.1\text{s})$

$V_o = 10.8\text{ m/s}$

$\Delta h = V_o t + \frac{1}{2}at^2$

$= (10.8\text{ m/s})(1.1\text{s}) - \frac{1}{2}g(1.1\text{s})^2$

$= 6.5\text{m} = h_f - h_o$

$= 5.2\text{m} - h_o$

$h_o = 1.3\text{m}$

so $\Delta h_{V_o} = 6.5\text{m}$

$\Delta h_{(2V_0)} = 4(6.5\text{m}) = 26\text{m} = h_f - h_o$

$= h_f - 1.3\text{m}$

$\boxed{h_f = 27.3\text{m}}$

# Chapter II.3: Two-Dimensional Kinematics

## *Illustrations*

### Illustration 3.1: Vector Decomposition

*Topic(s):* Vectors
*Purpose:*. Introduction to vectors.
*Suggested Use:* Reading assignment or for in class use – have student predict how the various values will change when the vector is drug in a particular way. Then discuss discrepancies in students' thinking.

### Illustration 3.2: Motion on an Incline

*Topic(s):* Constant Acceleration 2-D, Graphical Representations of Motion
*Purpose:* To introduce motion on an inclined plane
*Suggested Use:* Reading assignment or use in class as concepts are discussed.

### Illustration 3.3: The Direction of Velocity and Acceleration Vectors

*Topic(s):* Constant Acceleration 2-D
*Purpose:* To introduce 2-D kinematics.
*Suggested Use:* Reading assignment or for in-class use – have students predict and explain the vector before showing them. Then discuss discrepancies in students' thinking.

### Illustration 3.4: Projectile Motion

*Topic(s):* Projectile Motion, Graphical Representations of Motion
*Purpose:*. To introduce projectile motion.
*Suggested Use:* This would make a great in-class demonstration to help students connect the x and y components of projectile motion. Could just show the Physlet in class and discuss or have students predict what the graphs and/or footprints will look like before viewing.

### Illustration 3.5: Uniform Circular Motion and Acceleration

*Topic(s):* Uniform Circular Motion
*Purpose:* To introduce uniform circular motion
*Suggested Use:* Reading assignment or use in class as concepts are discussed.

### Illustration 3.6: Circular and Non-Circular Motion

*Topic(s):* Uniform Circular Motion, Non-Uniform Circular Motion
*Purpose:* To show examples of non-uniform circular motion.
*Suggested Use:* Reading assignment or use in class as concepts are discussed.

## *Explorations*

### Exploration 3.1 – Addition of Displacement Vectors

*Topic(s):* Vectors

### Exploration 3.2 – Run the Gauntlet, Controlling x, v, and a

*Topic(s):* 2-D motion

### Exploration 3.3 – Acceleration of a Golf Ball That Rims the Hole

*Topic(s):* Constant Acceleration 2D

### Exploration 3.4 – Space Probe with Constant Acceleration

*Topic(s):* Constant Acceleration 2D

### Exploration 3.5 – Uphill and Downhill Projectile Motion

*Topic(s):* Projectile Motion

### Exploration 3.6 – Uniform Circular Motion

*Topic(s):* Uniform Circular Motion

## *Problems*

### P.3.1 (Level 1)

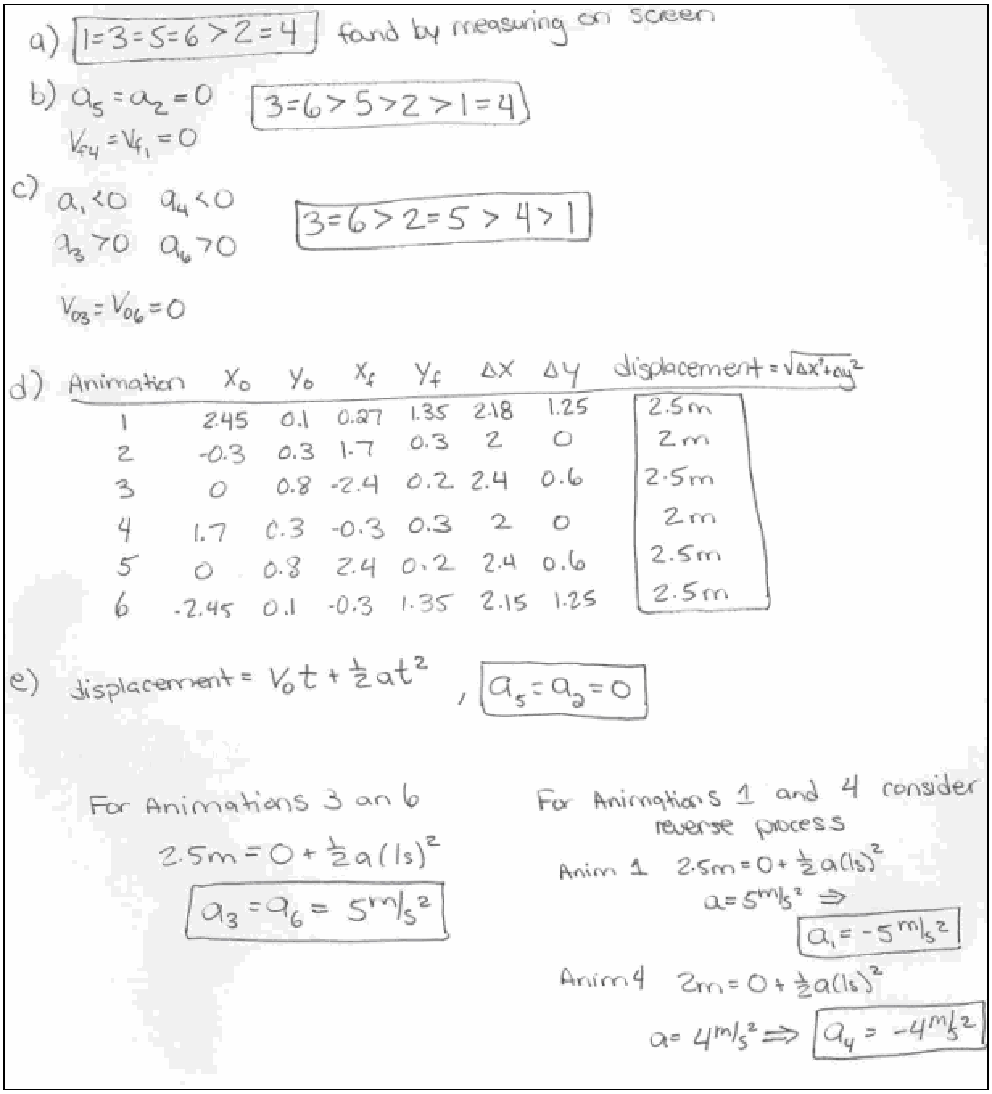
*Topic(s):* Vectors
*Answer:* **a) A=B=C>E=D, b) A=B=C=D>E, c) 0 units in x, 16 units in y**

### P.3.2 (Level 1)

*Topic(s):* Vectors
*Answer:* **a) 15m in x, 10m in y, b) –10m in x, 12m in y, c) 5min x, 22m in y, d) it is the addition of the components**

### P.3.3 (Level 3)

*Topic(s):* 2D Acceleration, Kinematics
*Answer:* **a) 1=3=5=6>2=4, b) 3=6>5>2>1=4, c) 3=6>2=5>4>1, d) 2.5 m for animations 1, 3, 5, and 6, 2m for animations 2, 4, e) a1 = -5 m/s^2, a2 = a5 = 0, a3 = a6 = 5m/s^2, a4 = -4 m/s^2**



a) $\boxed{1=3=5=6>2=4}$ found by measuring on screen

b) $a_5=a_2=0$ $\boxed{3=6>5>2>1=4}$

$V_{f4}=V_{f1}=0$

c) $a_1<0$ $a_4<0$ $\boxed{3=6>2=5>4>1}$

$a_3>0$ $a_6>0$

$V_{03}=V_{06}=0$

d)

| Animation | $X_0$ | $Y_0$ | $X_f$ | $Y_f$ | $\Delta X$ | $\Delta Y$ | displacement $=\sqrt{\Delta x^2+\Delta y^2}$ |
|---|---|---|---|---|---|---|---|
| 1 | 2.45 | 0.1 | 0.27 | 1.35 | 2.18 | 1.25 | 2.5m |
| 2 | -0.3 | 0.3 | 1.7 | 0.3 | 2 | 0 | 2m |
| 3 | 0 | 0.8 | -2.4 | 0.2 | 2.4 | 0.6 | 2.5m |
| 4 | 1.7 | 0.3 | -0.3 | 0.3 | 2 | 0 | 2m |
| 5 | 0 | 0.8 | 2.4 | 0.2 | 2.4 | 0.6 | 2.5m |
| 6 | -2.45 | 0.1 | -0.3 | 1.35 | 2.15 | 1.25 | 2.5m |

e) displacement $= V_0 t + \frac{1}{2}at^2$ , $\boxed{a_5=a_2=0}$

For Animations 3 an 6

$2.5\text{m} = 0 + \frac{1}{2}a(1\text{s})^2$

$\boxed{a_3=a_6=5\,\text{m/s}^2}$

For Animations 1 and 4 consider reverse process

Anim 1 $2.5\text{m} = 0 + \frac{1}{2}a(1\text{s})^2$

$a = 5\,\text{m/s}^2 \Rightarrow$

$\boxed{a_1=-5\,\text{m/s}^2}$

Anim 4 $2\text{m} = 0 + \frac{1}{2}a(1\text{s})^2$

$a = 4\,\text{m/s}^2 \Rightarrow \boxed{a_4=-4\,\text{m/s}^2}$

## P.3.4 (Level 1)

*Topic(s):* Displacement, Velocity
*Answer:* **Displacement, 1=2=3=4, Avg. Velocity 3>2>1=4**

## P.3.5 (Level 1)

*Topic(s):* Constant Acceleration 2D, Graphical Representations of Motion
*Answer:* **c) –0.5 m/s, d) 0.3 m/s, e) 0.58 m/s**

## P.3.6 (Level 1)

*Topic(s):* Constant Acceleration 2D, Graphical Representations of Motion
*Answer:* **a) 0 to 45 s, b) 45 s to 85s, c) 45s, d) 0, e) 0, f) –0.3 m/s, g) 0, h) –51 m, I) 10.05 m**

## P.3.7 (Level 2)

*Topic(s):* Projectile Motion

*Answer:* **at x = 43m, y=28.2m, v = 17.9 m/s**

***min speed is at highest point***

## P.3.8 (Level 1)

*Topic(s):* Projectile Motion
*Answer:* **Animation 2**

## P.3.9 (Level 1)

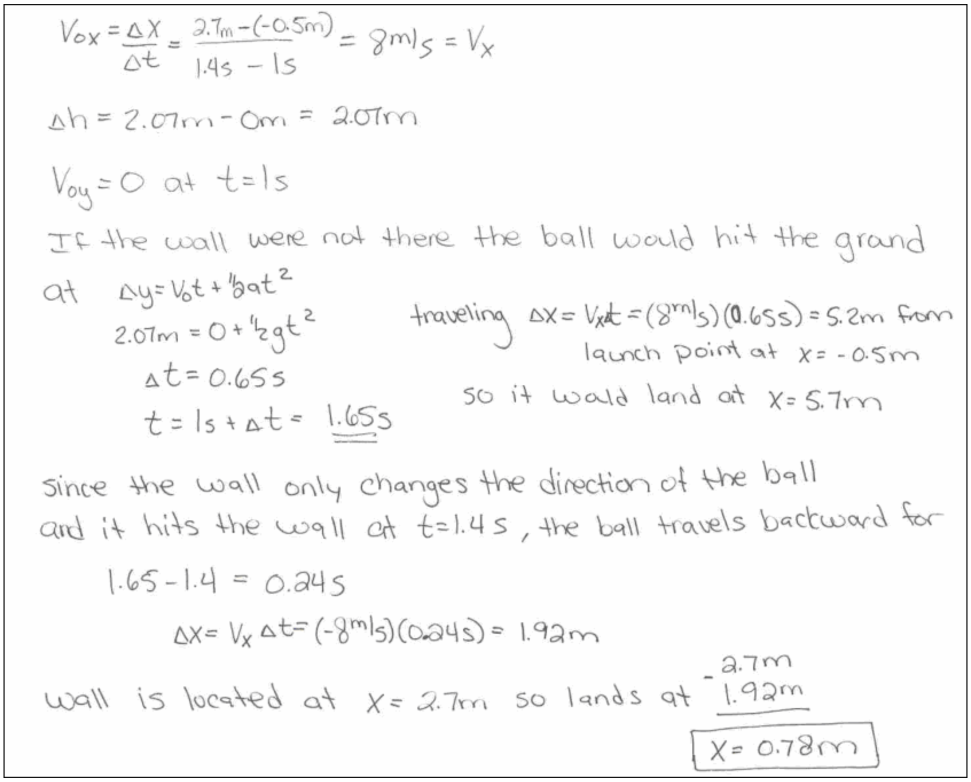
*Topic(s):* Projectile Motion
*Answer:* **a) 59 degrees, b) vx = 10.3 m/s, vy = 17.1 m/s, c) 14.9 m, d) 1.75 s, e) 3.5 s**

## P.3.10 (Level 1)

*Topic(s):* Projectile Motion
*Answer:* **VoCosθ = 12 m/s**

## P.3.11 (Level 2)

*Topic(s):* Projectile Motion
Answer: x = 0.78m



$V_{ox} = \frac{\Delta X}{\Delta t} = \frac{2.7m-(-0.5m)}{1.4s - 1s} = 8m/s = V_x$

$\Delta h = 2.07m - 0m = 2.07m$

$V_{oy} = 0$ at $t = 1s$

If the wall were not there the ball would hit the grand at

$\Delta y = V_0 t + \frac{1}{2} a t^2$

$2.07m = 0 + \frac{1}{2} g t^2$

$\Delta t = 0.65s$

$t = 1s + \Delta t = \underline{\underline{1.65s}}$

traveling $\Delta X = V_x \Delta t = (8m/s)(0.65s) = 5.2m$ from launch point at $X = -0.5m$

so it wald land at $X = 5.7m$

Since the wall only changes the direction of the ball and it hits the wall at $t = 1.4s$, the ball travels backward for

$1.65 - 1.4 = 0.24s$

$\Delta X = V_x \Delta t = (-8m/s)(0.24s) = 1.92m$

wall is located at $X = 2.7m$ so lands at $2.7m - 1.92m$

$\boxed{X = 0.78m}$

## P.3.12 (Level 2)

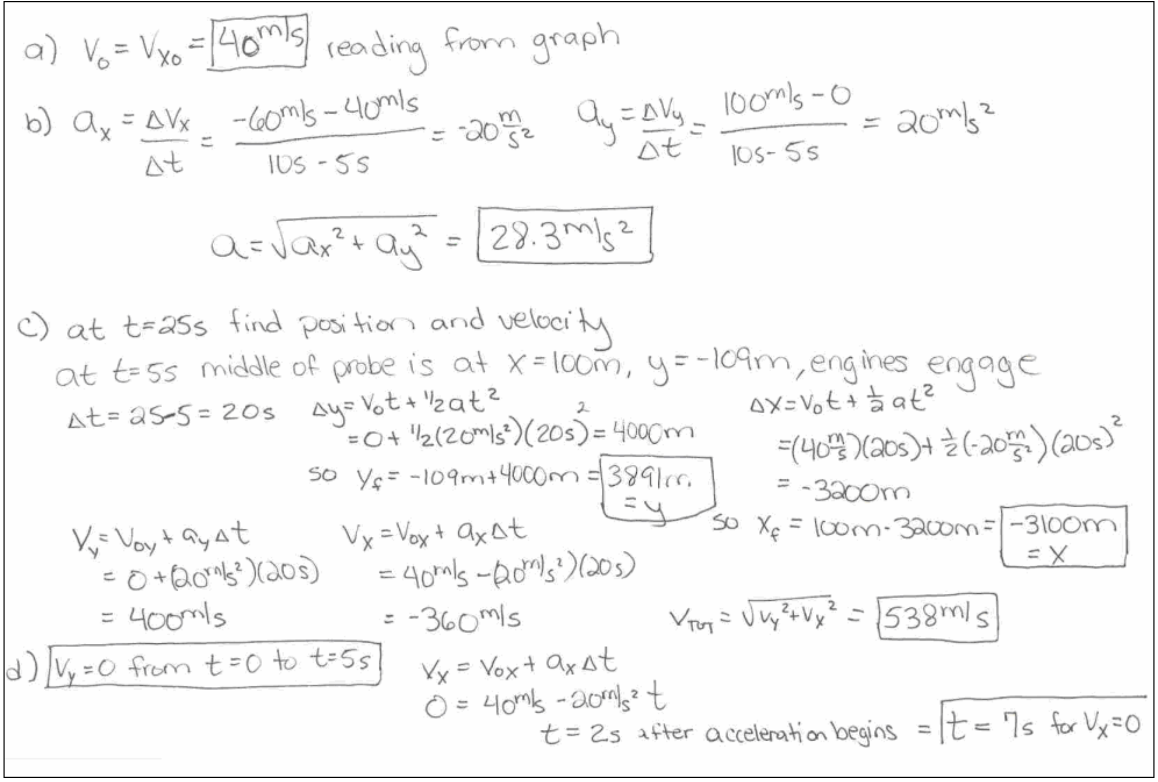
*Topic(s):* 2D Motion, Graphical Representations of Motion
*Answer:* **a) 138 m/s^2, b) 1.5 m/s^2, c) 138 m/s^2, d) v is constant, a is zero, e) ay=0, ax = 0.09 m/s^2.**

## P.3.13 (Level 2)

*Topic(s):* Constant 2D Acceleration
*Answer:* **a) constant, b) 0.88 m/s^2, c) x = -6.7 m, y = -3.9 m, e) 7.8 m**

## P.3.14 (Level 2)

*Topic(s):* Constant 2D Acceleration
*Answer:* **a) 40 m/s, b) 28.3 m/s^2, d) x = -3100m, y = 3890 m, v = 538 m/s, d) vy = 0 from t = 0 to t = 5s, vx = 0 at t = 7s**



a) $V_0 = V_{x0} = \boxed{40\,m/s}$ reading from graph

b) $a_x = \frac{\Delta V_x}{\Delta t} = \frac{-60\,m/s - 40\,m/s}{10s - 5s} = -20\,\frac{m}{s^2}$ $a_y = \frac{\Delta V_y}{\Delta t} = \frac{100\,m/s - 0}{10s - 5s} = 20\,m/s^2$

$a = \sqrt{a_x^2 + a_y^2} = \boxed{28.3\,m/s^2}$

c) at t=25s find position and velocity

at t=5s middle of probe is at x=100m, y=-109m, engines engage

$\Delta t = 25 - 5 = 20s$

$\Delta y = V_0 t + \frac{1}{2} a t^2$
$= 0 + \frac{1}{2}(20\,m/s^2)(20s)^2 = 4000m$
so $y_f = -109m + 4000m = \boxed{3891m = y}$

$\Delta x = V_0 t + \frac{1}{2} a t^2$
$= (40\frac{m}{s})(20s) + \frac{1}{2}(-20\frac{m}{s^2})(20s)^2$
$= -3200m$
so $x_f = 100m - 3200m = \boxed{-3100m = x}$

$V_y = V_{0y} + a_y \Delta t$
$= 0 + (20\,m/s^2)(20s)$
$= 400\,m/s$

$V_x = V_{0x} + a_x \Delta t$
$= 40\,m/s - (20\,m/s^2)(20s)$
$= -360\,m/s$

$V_{TOT} = \sqrt{V_y^2 + V_x^2} = \boxed{538\,m/s}$

d) $\boxed{V_y = 0 \text{ from } t = 0 \text{ to } t = 5s}$

$V_x = V_{0x} + a_x \Delta t$
$0 = 40\,m/s - 20\,m/s^2\, t$
$t = 2s$ after acceleration begins $= \boxed{t = 7s \text{ for } V_x = 0}$

## P.3.15 (Level 1)

*Topic(s):* Uniform Circular Motion
*Answer:* **a) right, b) up-left, c) 0 d) down, e) left, f) up-left**

## P.3.16 (Level 2)

*Topic(s):* Uniform Circular Motion
*Answer:* **a) 0.7 m in x and –0.7 m in y, b) 1.6m, c) 0.5 m/s, d) 0.8 m/s**

## P.3.17 (Level 1)

*Topic(s):* Uniform Circular Motion
*Answer:* **a) 4s, b) 1.6 m/s, c) 2.5 m/s^2, d) velocity is left, acceleration is down**

# Chapter II.4: Newton's Laws

## *Illustrations*

### Illustration 4.1: Newton's First Law and Reference Frames

*Topic(s):* Projectile Motion, Relative Motion, Newton's First Law
*Purpose:*. Introduction to reference frames.
*Suggested Use:* Reading assignment or show in class as concepts are discussed.

### Illustration 4.2: Free-Body Diagrams

*Topic(s):* Newton's 2nd Law
*Purpose:* To introduce free-body diagrams.
*Suggested Use:* Reading Assignment

### Illustration 4.3: Newton's Second Law and Force

*Topic(s):* Newton's 2nd Law, Graphical Representations of Motion
*Purpose:* To develop connections between force, velocity and acceleration.
*Suggested Use:* This is a good one for students to interact with as it makes connections between their actions, motion, and graphical representations.

### Illustration 4.4: Mass on an Incline

*Topic(s):* Newton's 2nd Law
*Purpose:*. To demonstrate how forces on an object are related to mass and the angle of the incline the object is on.
*Suggested Use:* Reading assignment or in class demonstration. When used in class, have students predict the FBD or motion as variables are changed then view. Discussions based in the students' predictions can then follow.

### Illustration 4.5: Pull Your Wagons

*Topic(s):* Newton's 2nd Law
*Purpose:* To introduce students to problem solving with Newton's law
*Suggested Use:* Reading assignment or as a predict/explain exercise used in class.

### Illustration 4.6: Newton's Third Law, Contact Forces

*Topic(s):* Newton's 3rd Law, Graphical Representations of Motion
*Purpose:* To introduce Newton's 3rd law.
*Suggested Use:* Reading assignment or in class demonstration with students making predictions and then trying to explain the result.

## *Explorations*

### Exploration 4.1 – Vectors for a Box on an Incline

*Topic(s):* Newton's 2nd Law

*Answers*: a) 10N at –180 degrees, b) 5 m/s^2

### Exploration 4.2 – Change the 2 Forces Applied

*Topic(s):* Newton's 2nd Law

### Exploration 4.3 – Change the Force Applied to Get to the Goal

*Topic(s):* Newton's 2nd Law

### Exploration 4.4 – Set the Force on a Hockey Puck

*Topic(s):* Newton's 2nd Law

### Exploration 4.5 – Space Probe with Multiple Engines

*Topic(s):* Newton's 2nd Law

### Exploration 4.6 – Putted Golf Ball Breaks Toward the Hole

*Topic(s):* Newton's 2nd Law
*Answers:* c) 0.05 N, d) 0.047 N, 0.038 N, 0.023 N

### Exploration 4.7 – Atwood's Machine

*Topic(s):* Newton's 2nd Law
*Answer*: b) a = (Mg-mg)/(M+m)

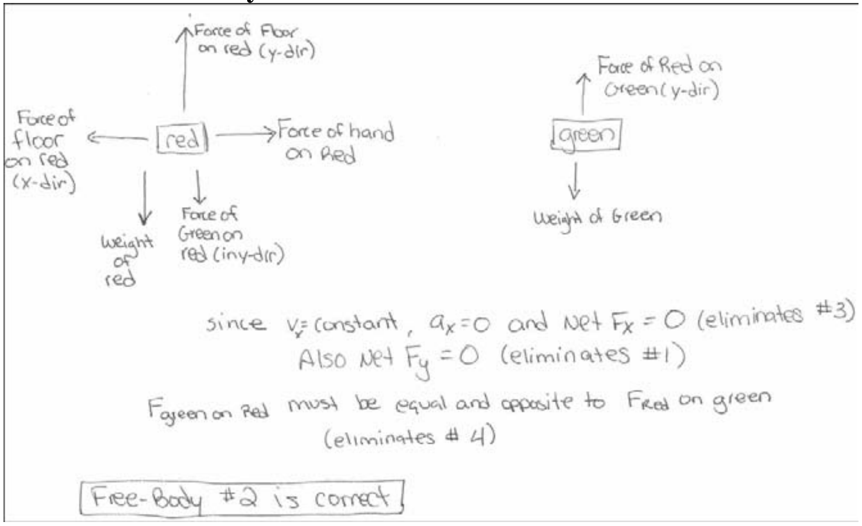
### Exploration 4.8 – Enter a Formula for the Force Applied

*Topic(s):* Newton's 2nd Law

## Problems

### P.4.1 (Level 2)

*Topic(s):* Newton's 2nd Law
*Answer:* **Free-Body #2**



since $v_x$= constant, $a_x = 0$ and Net $F_x = 0$ (eliminates #3)
Also Net $F_y = 0$ (eliminates #1)

$F_{green\ on\ Red}$ must be equal and opposite to $F_{Red\ on\ green}$ (eliminates # 4)

Free-Body #2 is correct

## P.4.2 (Level 1)

*Topic(s):* Newton's 2nd Law
*Answer:* **a) 0, b) 0, c) only that it is non-zero since there would be no thrust or drag if v=0., d) Net Force = 0 implies net a=0 not that v = 0.**

## P.4.3 (Level 2)

*Topic(s):* Newton's 2nd Law
*Answer:* **Animation doesn't correctly depict Newton's Laws.**
a = 0.5 ms^2 throughout

## P.4.4 (Level 2)

*Topic(s):* Newton's 2nd Law
*Answer:* **F = 50 N from t=0 to t=5.6s and F = 60 N from t=5.6s to t=10s.**
a = 0.5 m/s^2 through out, F=ma

## P.4.5 (Level 1)

*Topic(s):* Newton's 2nd Law
*Answer:* **a) 0.098 N downward, b) 0.6287 N upward, c) 0.7267 N upward, d) 0.002 N upward, v=0, e) 0.1 N upward, f) 0, g) 0.098 N upward, g) velocity is constant**

## P.4.6 (Level 2)

*Topic(s):* Newton's 2nd Law
*Answer:* **a) 0.044 N, b) 0**
ax=0.76 m/s^2, ay=0.44 m/s^2, a=0.88 m/s^2, then use F=ma

## P.4.7 (Level 1)

*Topic(s):* Newton's 2nd Law
*Answer:* **a) acceleration increases when force increases, b) acceleration decreases as angle increases, c) a=(Fcosθ)/m, d) mg - F sinθ**

## P.4.8 (Level 1)

*Topic(s):* Newton's 2nd Law
*Answer:* **a) normal force, c) N=mg+ma, d) a=-g**

## P.4.9 (Level 1)

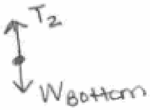
*Topic(s):* Newton's 2nd Law
*Answer:* **a) 4>5=2=1>3>6, b) 4>5=2=1>3>6, c) a1=a2=a5=0 a4=4m/s^2 a6=-9.8 m/s^2, a3 = -4 m/s^2 d) T1=T2=T5=98N T3=58 N T4=138 N T6=0**

## P.4.10 (Level 3)

*Topic(s):* Newton's 2nd Law
*Answer:* **b) 41.2 N, c) 20.6 N, d) 30.9 N, e) 20.6 N**

a) Top

Bottom



b) combined

$\Sigma F = T_1 - 2(mg) = 2ma$ $a = \frac{\Delta v}{\Delta t} = \frac{0.5 m/s}{1s} = 0.5 m/s^2$

$T_1 - 2(2kg)g = 2(2kg)(0.5 \frac{m}{s^2})$

$\boxed{T_1 = 41.2 N}$

c) $\Sigma F_{BOT} = T_2 - W = ma$

$T_2 - (2kg)g = (2kg)(0.5 m/s^2)$

$\boxed{T_2 = 20.6 N}$

d) as in part b) $\Sigma F = T_1 - 2(mg) - m_{cable} g = (2m + m_{cable}) a$

$T_1 = 2(2kg)g + (1kg)g + (2 \cdot 2kg + 1kg)(0.5 m/s^2)$

$T_1 = 51.5 N$

Then $\Sigma F_{Top} = T_1 - W_{Top} - T_2 = ma$

$51.5N - (2kg)g - T_2 = (2kg)(0.5 m/s^2)$

$\boxed{T_2 = 30.9 N}$

e) $\Sigma F_{BOT} = T_2' - W = ma$

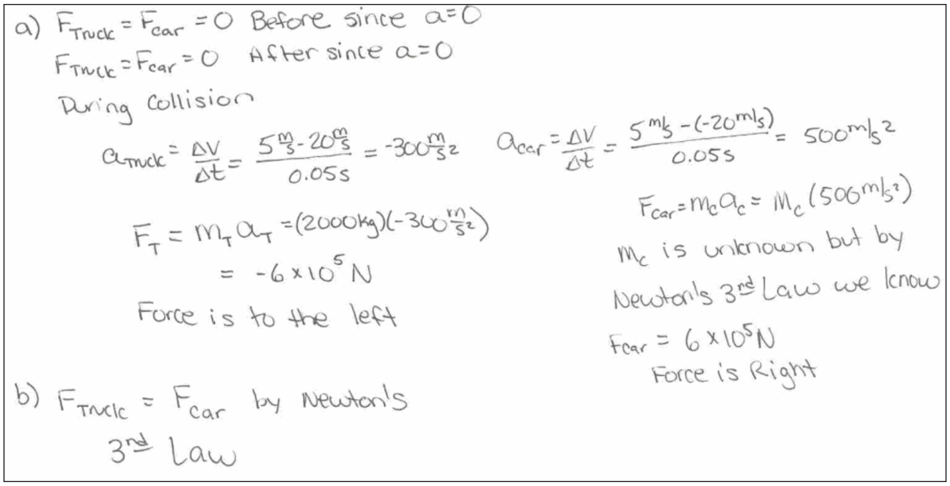
$T_2' - (2kg)g = (2kg)(0.5 m/s^2)$ $\boxed{T_2' = 20.6N}$

## P.4.11 (Level 2)

*Topic(s):* Newton's 2nd Law
*Answer:* **a) 3.3 N, b) 0.5 kg**
a = 3.3 m/s^2, T=McartXa then Wmass-T-MmassXa

## P.4.12 (Level 1)

*Topic(s):* Newton's 2nd Law
*Answer:* **Force on truck is equal and opposite to the force on the car.**



a) $F_{Truck} = F_{car} = 0$ Before since $a=0$

$F_{Truck} = F_{car} = 0$ After since $a=0$

During collision

$a_{Truck} = \frac{\Delta V}{\Delta t} = \frac{5\frac{m}{s} - 20\frac{m}{s}}{0.05s} = -300\frac{m}{s^2}$  $a_{car} = \frac{\Delta V}{\Delta t} = \frac{5 m/s - (-20 m/s)}{0.05s} = 500 m/s^2$

$F_T = m_T a_T = (2000kg)(-300\frac{m}{s^2})$

$= -6 \times 10^5 N$

Force is to the left

$F_{car} = m_c a_c = m_c (500 m/s^2)$

$m_c$ is unknown but by Newton's 3rd Law we know

$F_{car} = 6 \times 10^5 N$

Force is Right

b) $F_{Truck} = F_{car}$ by Newton's 3rd Law

## P.4.13 (Level 1)

*Topic(s):* Newton's 2nd Law

*Answer:* **2 and 4**

# Chapter II.5: Newton's Laws 2

## *Illustrations*

### Illustration 5.1: Static and Kinetic Friction

*Topic(s):* Friction
*Purpose:*. Introduction to friction.
*Suggested Use:* Reading Assignment

### Illustration 5.2: Uniform Circular Motion: $F_c$ and $a_c$

*Topic(s):* Uniform Circular Motion
*Purpose:* To introduce uniform circular motion
*Suggested Use:* Use as a visual aid in class to explain why there is an acceleration associated with circular motion. Would also be valuable as a reading assignment.

### Illustration 5.3: The Ferris Wheel

*Topic(s):* Uniform Circular Motion
*Purpose:* To introduce uniform circular motion.
*Suggested Use:* Reading assignment

### Illustration 5.4: Springs and Hooke's law

*Topic(s):* Springs
*Purpose:*. To introduce springs and Hooke's law.
*Suggested Use:* Reading assignment.

### Illustration 5.5: Air Friction

*Topic(s):* Friction
*Purpose:* To introduce air resistance.
*Suggested Use:* Us as a visual aid in class when discussing the effect of air friction on motion. Have the students consider the situation and make predictions for the graphs and motion before viewing and discussing. Would also be valuable as a reading assignment.

## *Explorations*

### Exploration 5.1 – Circular Motion

*Topic(s):* Uniform Circular Motion

### Exploration 5.2 – Force an Object around a Circle

*Topic(s):* Uniform Circular Motion

### Exploration 5.3 – Spring Force

*Topic(s):* Springs
*Answer(s):* b) 0.34 kg, c) 1.8 m

## Exploration 5.4 – Circular Motion and a Spring Force

*Topic(s):* Uniform Circular Motion, Springs

## Exploration 5.5 – Enter a Formula for the Force

*Topic(s):* Friction

## Exploration 5.6 – Air Friction

*Topic(s):* Friction

## Exploration 5.7 – Enter a Formula, $F_x$ and $F_y$, for the Force

*Topic(s):* Variable Forces

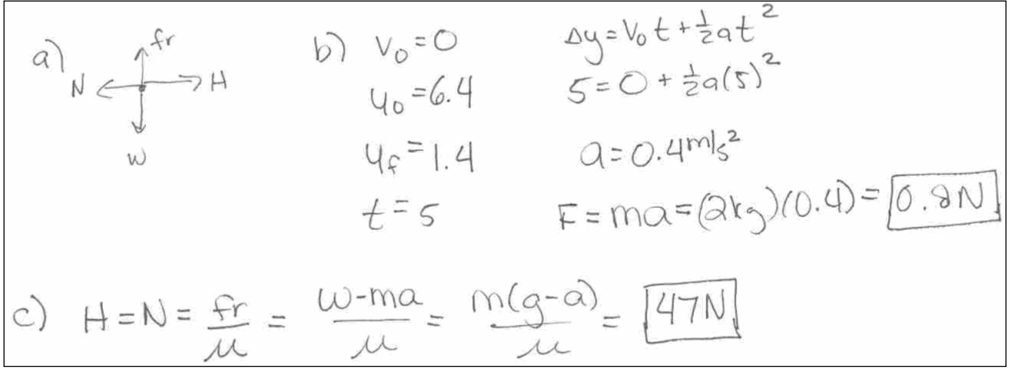
# *Problems*

## P.5.1 (Level 2)

*Topic(s):* Newton's 2nd law, Friction
*Answer:* **b) 0, c) 98 N, d) Book will slow down and come to a stop.**

## P.5.2 (Level 2)

*Topic(s):* Newton's 2nd law, Friction
*Answer:* **b) 0.8 N, c) 47 N**



b) $v_0 = 0$ $\quad \Delta y = v_0 t + \frac{1}{2}at^2$

$y_0 = 6.4$ $\quad 5 = 0 + \frac{1}{2}a(5)^2$

$y_f = 1.4$ $\quad a = 0.4 m/s^2$

$t = 5$ $\quad F = ma = (2kg)(0.4) = \boxed{0.8N}$

c) $H = N = \frac{fr}{\mu} = \frac{w - ma}{\mu} = \frac{m(g-a)}{\mu} = \boxed{47N}$

## P.5.3 (Level 2)

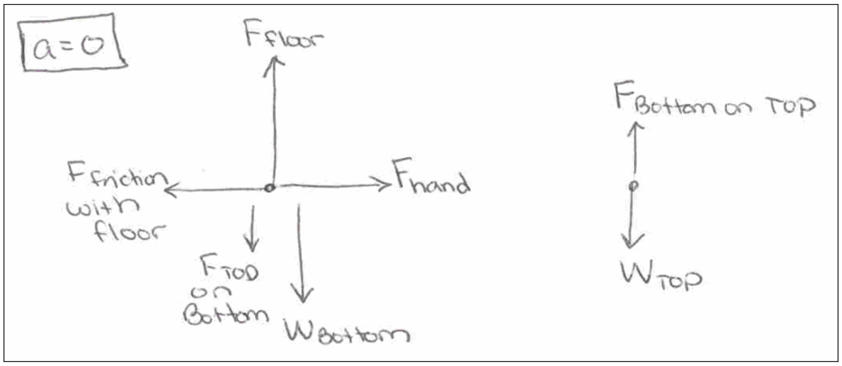
*Topic(s):* Newton's 2nd law, Friction
*Answer:* **b) 0, c) 49 N**

## P.5.4 (Level 2)

*Topic(s):* Newton's 2nd law, Friction
*Answer:* **0.15**
a = 0 then 1.5 m/s^2

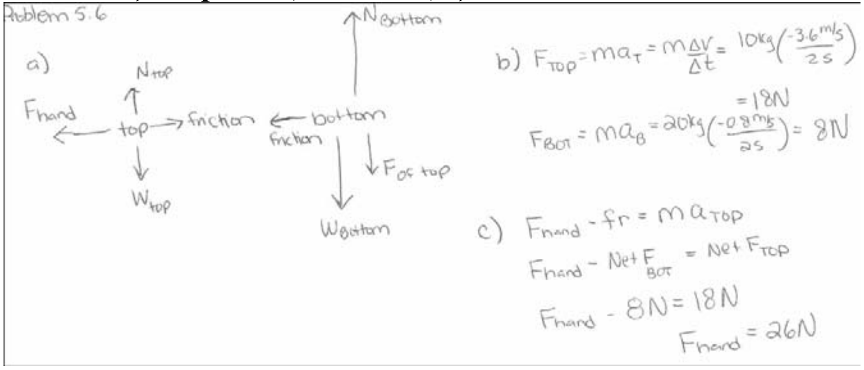
## P.5.5 (Level 2)

*Topic(s):* Newton's 2nd law, Friction
*Answer:* **#2 is correct**



## P.5.6 (Level 2)

*Topic(s):* Newton’s 2nd law, Friction
*Answer:* **b) Ftop=18N, Fbot=8N, c) 26N**



b) $F_{TOP} = ma_T = m\frac{\Delta v}{\Delta t} = 10kg\left(\frac{-3.6 m/s}{2s}\right) = 18N$

$F_{BOT} = ma_B = 20kg\left(\frac{-0.8 m/s}{2s}\right) = 8N$

c) $F_{hand} - fr = ma_{TOP}$

$F_{hand} - Net\,F_{BOT} = Net\,F_{TOP}$

$F_{hand} - 8N = 18N$

$F_{hand} = 26N$

## P.5.7 (Level 2)

*Topic(s):* Newton’s 2nd law, Friction
*Answer:* **b) 0, c) 0.5**

## P.5.8 (Level 2)

*Topic(s):* Newton’s 2nd Law, Uniform Circular Motion
*Answer:* **b) weight and tension, c) 14.8 m/s^2, d) 1480 N**

## P.5.9 (Level 1)

*Topic(s):* Newton’s 2nd Law, Uniform Circular Motion
*Answer:* **a) friction, b) blue**

## P.5.10 (Level 1)

*Topic(s):* Newton’s 2nd Law, Uniform Circular Motion
*Answer:* **a) 1.5 m/s^2, c) 0.15 N**

## P.5.11 (Level 1)

*Topic(s):* Newton’s 2nd Law, Uniform Circular Motion

**Answer:** *a) 1.2 m/s^2, c) 0.13*

## P.5.12 (Level 1)

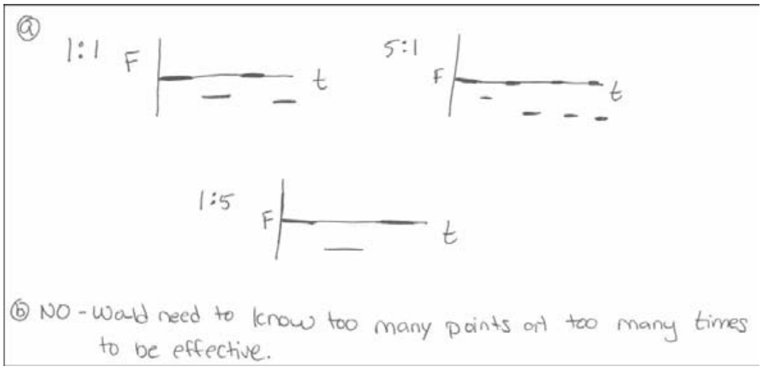
*Topic(s):* Springs
*Answer:* **a) x = -9.9m to x = 8.1m, b) 13m, c) 6 N/m**

## P.5.13 (Level 1)

*Topic(s):* Springs
*Answer:* **1.5 N/m**

## P.5.14 (Level 2)

*Topic(s):* Springs, Non-Constant Forces
*Answer:*



ⓑ NO - Would need to know too many points at too many times to be effective.

# Chapter II.6: Work

## *Illustrations*

### Illustration 6.1: Dot Products

*Topic(s):* Dot Product
*Purpose:*. Introduction to the dot product.
*Suggested Use:* Reading Assignment

### Illustration 6.2: Constant Forces

*Topic(s):* Friction, Newton's 2nd Law, Work-Energy Theorem
*Purpose:* To introduce concepts related to work.
*Suggested Use:* Reading assignment

### Illustration 6.3: Force and Displacement

*Topic(s):* Springs, Non-Constant Forces, Work
*Purpose:* To introduce work for non-constant forces.
*Suggested Use:* Reading assignment or in class discussions to help students connect the concepts of integration and area calculation to work.

### Illustration 6.4: Springs

*Topic(s):* Springs
*Purpose:*. To introduce springs and their relation to work.
*Suggested Use:* Reading assignment or as with Illustration 6.3, used in class to help students visually connected mathematics with the physical concepts.

### Illustration 6.5: Circular Motion

*Topic(s):* Uniform Circular Motion, Work
*Purpose:* To introduce concepts related to circular motion and work.
*Suggested Use:* Reading assignment or to support an in-class discussion of work. In addition to taking students through the ideas in the written portion of the illustration, it would also be interesting to have students predict and explain the motion for various values of the initial velocity. For example, what will happen over a long period of time if the initial velocity is such that the bob never makes it to the top? What if the velocity is large enough that it gets to the top with some velocity remaining?

## *Explorations*

### Exploration 6.1 – An Operational Definition of Work

*Topic(s):* Work

### Exploration 6.2 – The Two-block Push

*Topic(s):* Work

## Exploration 6.3 – The Gravitational Force and Work

*Topic(s):* Work

## Exploration 6.4 – Change the Direction of the Force Applied

*Topic(s):* Work

## Exploration 6.5 – Circular Motion and Work

*Topic(s):* Work and Uniform Circular Motion

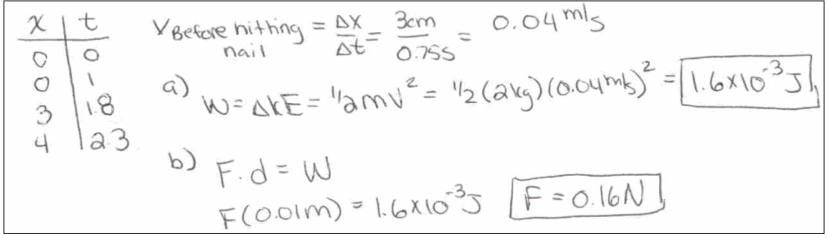
## Exploration 6.6 – Forces, Path Integrals, and Work

*Topic(s):* Work

## *Problems*

### P.6.1 (Level 2)

*Topic(s):* Work, Energy
*Answer:* **a) 1.6 x 10^(-3) J, b) 0.16 N**



| x | t |
|---|---|
| 0 | 0 |
| 0 | 1 |
| 3 | 1.8 |
| 4 | 2.3 |

$$V_{\text{Before hitting nail}} = \frac{\Delta x}{\Delta t} = \frac{3\text{cm}}{0.75\text{s}} = 0.04\,\text{m/s}$$

a) $$W = \Delta KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2\text{kg})(0.04\,\text{m/s})^2 = \boxed{1.6\times10^{-3}\,\text{J}}$$

b) $$F \cdot d = W$$
$$F(0.01\text{m}) = 1.6\times10^{-3}\,\text{J} \quad \boxed{F = 0.16\text{N}}$$

### P.6.2 (Level 2)

*Topic(s):* Work, Energy
*Answer:* **a) –16 J, b) 20 N**

### P.6.3 (Level 1)

*Topic(s):* Work, Energy
*Answer:* **a) 0, b) 49 N**

### P.6.4 (Level 1)

*Topic(s):* work
*Answer:* **1=2=3=4 for all**

### P.6.5 (Level 1)

*Topic(s):* Work, Newton's 2nd Law
*Answer:* **a) 0, b) 0, c) 0, d) 17 J**

### P.6.6 (Level 2)

*Topic(s):* Work-Energy
*Answer:* **5.5 m/s^2**

## P.6.7 (Level 1)

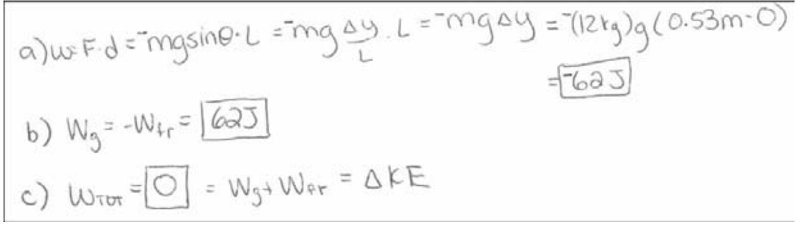
*Topic(s):* Work
*Answer:* **a) +, b) 0, c) 0, d) -, e) 0**

## P.6.8 (Level 2)

*Topic(s):* Work - Energy
*Answer:* **a) 59 J, b) –59 J, c) 0**

## P.6.9 (Level 2)

*Topic(s):* Work - Energy
*Answer:* **a) -62 J, b) 62 J, c) 0**



$$a)\ w = F\cdot d = -mg\sin\theta\cdot L = -mg\frac{\Delta y}{L}\cdot L = -mg\Delta y = -(12\,kg)g(0.53m - 0)$$

$$= \boxed{-62\,J}$$

$$b)\ W_g = -W_{fr} = \boxed{62\,J}$$

$$c)\ W_{TOT} = \boxed{0} = W_g + W_{fr} = \Delta KE$$

## P.6.10 (Level 2)

*Topic(s):* Work
*Answer:* **a) 6>3=3>5=0>2=4, b) 4>2>0=5=6>1>3, c) 6>3=4>1=2=5=0, d) W1=W3=196J, W2=W4=-196J, W5=0, W6=480J, e) W1=-196J, W2=196J, W3=-116J, W4=276J, W5=W6=0, f) W1=W2=W5=0, W3=W4=80J, W6=480J**

## P.6.11 (Level 2)

*Topic(s):* Newton's 2nd Law, Work
*Answer*: **a) –3.3J, b) 3.9J, c) 3.3J, d) 0, e) 0, f) 3.9J, g) 3.9J**

## P.6.12 (Level 1)

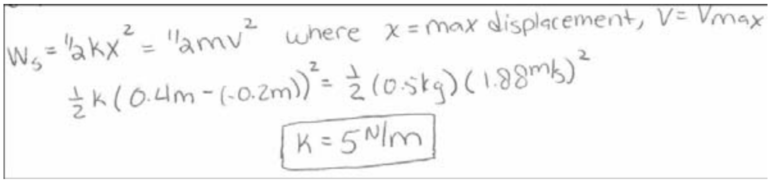
*Topic(s):* Springs, Work
*Answer:* **Graph 3**

## P.6.13 (Level 1, uses calculus)

*Topic(s):* Springs, Work
*Answer:* **a) 0, b) 0, c) 0, d) 25 N, e) 25 N, f) The force is not constant, it varies with displacement.**

## P.6.14 (Level 2)

*Topic(s):* Springs, Work
*Answer:* **4.9 N/m**



$W_s = 1/2\,kx^2 = 1/2\,mv^2$ where $x$ = max displacement, $V = V_{max}$

$$\frac{1}{2}k(0.4m - (-0.2m))^2 = \frac{1}{2}(0.5kg)(1.88\,m/s)^2$$

$$\boxed{k = 5\,N/m}$$

# Chapter II.7: Energy

## *Illustrations*

### Illustration 7.1: Choice of System

*Topic(s):* Energy Conservation
*Purpose:*. Introduction to energy conservation and choice of system.
*Suggested Use:* Reading assignment or in class when discussing energy. This illustrations provides a good way to introduce the energy bar representation to students.

### Illustration 7.2: Representations of Energy

*Topic(s):* Energy Conservation
*Purpose:* Introduction to energy bar graphs.
*Suggested Use:* Reading assignment or as an aid for in-class discussions. As with Illustration 7.1, this is a good way to acclimate students to the energy bar representation.

### Illustration 7.3: Potential Energy Diagrams

*Topic(s):* Energy Conservation
*Purpose:* Introduction to energy diagrams.
*Suggested Use:* Reading assignment or as another opportunity to discuss representations of energy. In addition to following the ideas presented in the narrative of the illustration, you could first show students the Physlet and then ask them what is represented by each bar/line/color.

### Illustration 7.4: External Forces and Energy

*Topic(s):* Energy Conservation
*Purpose:*. To changes in energy with external forces.
*Suggested Use:* Reading assignment. This is an illustration that students need to interact with individually to really benefit.

### Illustration 7.5: A Block on an Incline

*Topic(s):* Energy Conservation
*Purpose:* To introduce problem solving using energy conservation.
*Suggested Use:* Reading assignment or in class demonstration following narrative of the illustration. Also, would be beneficial to show students the Physlet first and ask them what is represented by each arrow/color/bar.

## *Explorations*

### Exploration 7.1 – Push a Cart Around

*Topic(s):* Energy Conservation

### Exploration 7.2 – Choice of Zero for Potential Energy

*Topic(s):* Energy Conservation

### Exploration 7.3 – Elastic Collision

*Topic(s):* Energy Conservation

### Exploration 7.4 – A Ball Hits a Mass Attached to a Spring

*Topic(s):* Energy Conservation

### Exploration 7.5 – Drag the Ball to Determine PE(x)

*Topic(s):* Energy Conservation

### Exploration 7.6 – Different Interactions

*Topic(s):* Energy Conservation

### Exploration 7.7 – Exploring Potential Energy Functions

*Topic(s):* Energy Conservation

## *Problems*

### P.7.1 (Level 1)

*Topic(s):* Energy Conservation
*Answer:* **5.4 m/s**

### P.7.2 (Level 2)

*Topic(s):* Energy Conservation
*Answer:* **a) 59 J, b) –59 J, c) 0**

### P.7.3 (Level 2)

*Topic(s):* Energy Conservation
*Answer:* **a) -59 J, b) 59 J, c) 0**

### P.7.4 (Level 2)

*Topic(s):* Energy Conservation
*Answer:* **a) -59 J, b) 59 J, c) 0**

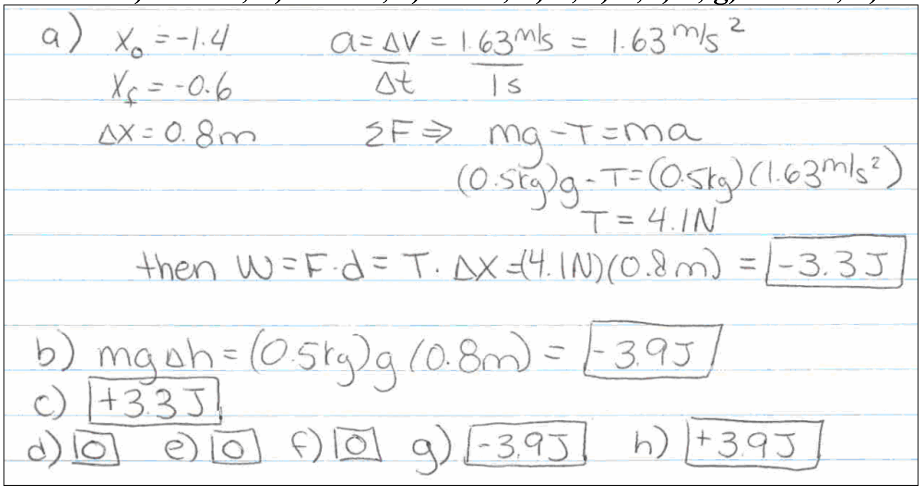
### P.7.5 (Level 2)

*Topic(s):* Energy Conservation, Newton's 2nd Law
*Answer:* **a) 2=4>5>1=3>6, b) 4>2>5=6>3>1, c) 6>3=4>1=2=5,d) Anim1=Anim3= -196 J, Anim2=Anim4 = 196 J, Anim5= 0, Anim6= -490J, e) Anim1= -196J, anim2= 196J, anim5=anim6= 0, anim3= -116J, anim4= 276J, f) anim1=anim2=anim5= 0, anim3=anim4= 80J, anim6= 490 J**

## P.7.6 (Level 2)

*Topic(s):* Energy Conservation, Work, Newton's 2nd Law

*Answer:* **a) –3.27J, b) –3.95J, c) 3.27J, d) 0, e) 0, f) 0, g) –3.95J, h) 3.95J**



a) $x_o=-1.4$ $a=\frac{\Delta v}{\Delta t}=\frac{1.63 m/s}{1s}=1.63 m/s^2$

$x_f=-0.6$

$\Delta x=0.8m$ $\Sigma F \Rightarrow mg-T=ma$

$(0.5kg)g-T=(0.5kg)(1.63 m/s^2)$

$T=4.1N$

then $W=F\cdot d=T\cdot\Delta x=(4.1N)(0.8m)=\boxed{-3.3J}$

b) $mg\Delta h=(0.5kg)g(0.8m)=\boxed{-3.9J}$

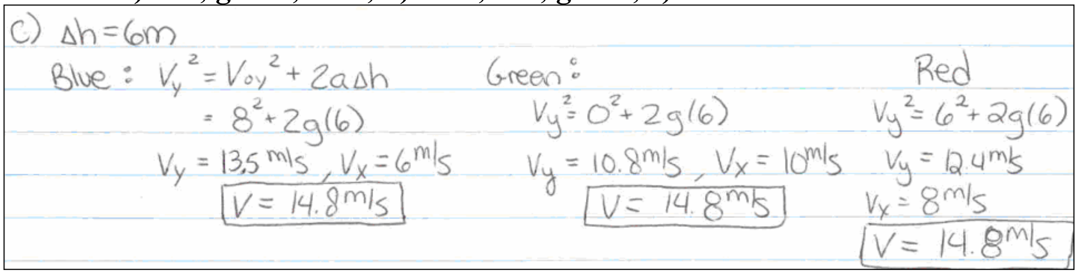
c) $\boxed{+3.3J}$

d) $\boxed{0}$ e) $\boxed{0}$ f) $\boxed{0}$ g) $\boxed{-3.9J}$ h) $\boxed{+3.9J}$

## P.7.7 (Level 2)

*Topic(s):* Energy Conservation

*Answer:* **a) red, green, blue, b) blue, red, green, c) 14.8 m/s for all**



c) $\Delta h=6m$

Blue: $V_y^2=V_{oy}^2+2a\Delta h$
$=8^2+2g(6)$
$V_y=13.5 m/s$, $V_x=6 m/s$
$\boxed{V=14.8 m/s}$

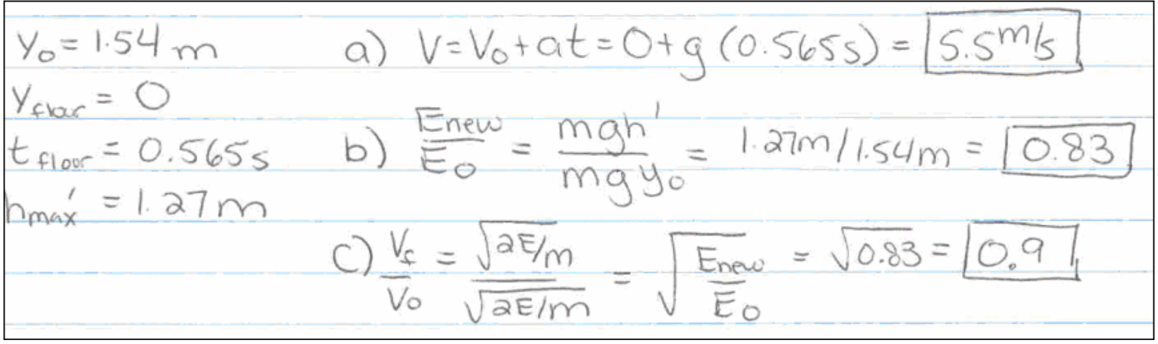
Green:
$V_y^2=0^2+2g(6)$
$V_y=10.8 m/s$, $V_x=10 m/s$
$\boxed{V=14.8 m/s}$

Red
$V_y^2=6^2+2g(6)$
$V_y=12.4 m/s$
$V_x=8 m/s$
$\boxed{V=14.8 m/s}$

## P.7.8 (Level 2)

*Topic(s):* Energy Conservation

*Answer:* **a) 5.5 m/s, b) Ef = 0.83 Eo, c) 0.9**



$Y_o=1.54m$

$Y_{floor}=0$

$t_{floor}=0.565s$

$h'_{max}=1.27m$

a) $V=V_o+at=0+g(0.565s)=\boxed{5.5 m/s}$

b) $\frac{E_{new}}{E_o}=\frac{mgh'}{mgy_o}=1.27m/1.54m=\boxed{0.83}$

c) $\frac{V_f}{V_o}=\frac{\sqrt{2E/m}}{\sqrt{2E/m}}=\sqrt{\frac{E_{new}}{E_o}}=\sqrt{0.83}=\boxed{0.9}$

## P.7.9 (Level 2)

*Topic(s):* Center of Mass

*Answer:* **a) 0, b) 1.25 kg, c) 8.1 J, d) –8.1 J**

## P.7.10 (Level 1)

*Topic(s):*
*Answer:* **a) 5 x 10^(-4) J, b) 0, 1.8 x 10^(-4) J, 3.2 x 10^(-4) J, 4.2 x 10^(-4) J, 4.8 x 10^(-4) J, 5 x 10^(-4) J**

## P.7.11 (Level 1)

*Topic(s):* Energy Conservation, 1-D Collisions
*Answer:* **a) Interval 1 = ball moves at constant speed right, interval 2 = ball hits spring, interval 3 = ball is stationary, spring recoils, interval 4 = spring hits ball, interval 5 = ball moves left at constant speed. B) energy is always conserved**

## P.7.12 (Level 1)

*Topic(s):* Energy Conservation, 1-D Collisions
*Answer:* **a) energy is always conserved, b) 90%**

# Chapter II.8: Momentum

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## *Illustrations*

### Illustration 8.1: Force and Impulse

*Topic(s):* Momentum
*Purpose:*. Connecting force with momentum.
*Suggested Use:* Reading assignment or as an in-class demonstration.

### Illustration 8.2: The Difference between Impulse and Work

*Topic(s):* Momentum, Work
*Purpose:* To connect impulse and work.
*Suggested Use:* Reading assignment or as an in-class demonstration.

### Illustration 8.3: Hard and Soft Collisions and the Third Law

*Topic(s):* Momentum, 1-D Collisions
*Purpose:* To connect momentum conservation to the 3rd law.
*Suggested Use:* Reading assignment or as an in-class demonstration when discussing types of collisions.

### Illustration 8.4: Relative Velocity in Collisions

*Topic(s):* Momentum, 1-D Collisions
*Purpose:*. Discussion of collisions.
*Suggested Use:* Reading assignment or as an in-class demonstration to show that the relative velocity between two objects in a collision is the same before and after the collision.

### Illustration 8.5: The Zero-momentum Frame

*Topic(s):* Momentum, 1-D Collisions
*Purpose:* To introduce reference frames with momentum.
*Suggested Use:* Reading assignment or as an in-class demonstration when transitioning from momentum to center of mass discussions.

### Illustration 8.6: Microscopic View of a Collision

*Topic(s):* Momentum, 1-D Collisions
*Purpose:* To show collisions on a molecular level.
*Suggested Use:* Reading assignment or in-class demonstration.

### Illustration 8.7: Center of Mass and Gravity

*Topic(s):* Center of Mass
*Purpose:* To introduce center of mass.
*Suggested Use:* Reading assignment.

### Illustration 8.8: Moving Objects and Center of Mass

*Topic(s):* Center of Mass
*Purpose:* To center of mass in moving systems.
*Suggested Use:* Reading assignment.

## *Explorations*

### Exploration 8.1 – Understanding Conservation Laws

*Topic(s):* Conservation Laws in General

### Exploration 8.2 – An Elastic Collision

*Topic(s):* Momentum, 1-D Collisions

### Exploration 8.3 – An Inelastic Collision with Unknown Masses

*Topic(s):* Momentum, 1-D Collisions

### Exploration 8.4 – Elastic and Inelastic Collisions and Δp

*Topic(s):* Momentum, 1-D Collisions

### Exploration 8.5 – 2 and 3 Ball Collisions

*Topic(s):* Momentum, 1-D Collisions

### Exploration 8.6 – An Explosive Collision

*Topic(s):* Momentum, 1-D Collisions

### Exploration 8.7 – A Bouncing Ball

*Topic(s):* Momentum, 1-D Collisions, Energy

## *Problems*

### P.8.1 (Level 1)

*Topic(s):* Momentum
*Answer:* **a) 12 kg-m/s, b) yes, c) not, d) yes, e) no**

### P.8.2 (Level 1)

*Topic(s):* Momentum, 1-D Collisions
*Answer:* **a) ball, b)flower pot, c) less**

### P.8.3 (Level 1)

*Topic(s):* Momentum, 1-D Collisions
*Answer:* **a) KE is conserved in bottom cart, b) top, c) no, d) KE depends on v^2 and is therefore independent of direction. Momentum is a vector and will change when direction changes.**

## P.8.4 (Level 1)

*Topic(s):* Momentum, 1-D Collisions
*Answer:* **1.04 kg**

## P.8.5 (Level 1)

*Topic(s):* Momentum, 1-D Collisions
*Answer:* **1401 kg**

## P.8.6 (Level 1)

*Topic(s):* Momentum, 1-D Collisions, Energy Conservation
*Answer:* **1, 2, and 3**

## P.8.7 (Level 1)

*Topic(s):* Momentum, 1-D Collisions, Energy Conservation
*Answer:* **E and p conserved for all**

## P.8.8 (Level 1)

*Topic(s):* Momentum, 1-D Collisions, Energy Conservation
*Answer:* **a) –1.35 kg-m/s, b) 1.35 kg-m/s, c) 0, d) 1.7 J**

## P.8.9 (Level 2)

*Topic(s):* Momentum, 1-D Collisions, Energy Conservation
*Answer:* **yes**
From momentum conservation mblue/mred=2

## P.8.10 (Level 1)

*Topic(s):* Momentum, 1-D Collisions, Energy Conservation
*Answer:* **Inelastic**

## P.8.11 (Level 2)

*Topic(s):* Momentum, 1-D Collisions, Energy Conservation
*Answer:* **a) equal for all values of m since momentum is always conserved, b) equal when masses are equal, ay > ab when my > mb, ay< ab when my < mb because of Newton's 3rd law which leads to ay = (mb/my) ab**

## P.8.12 (Level 1)

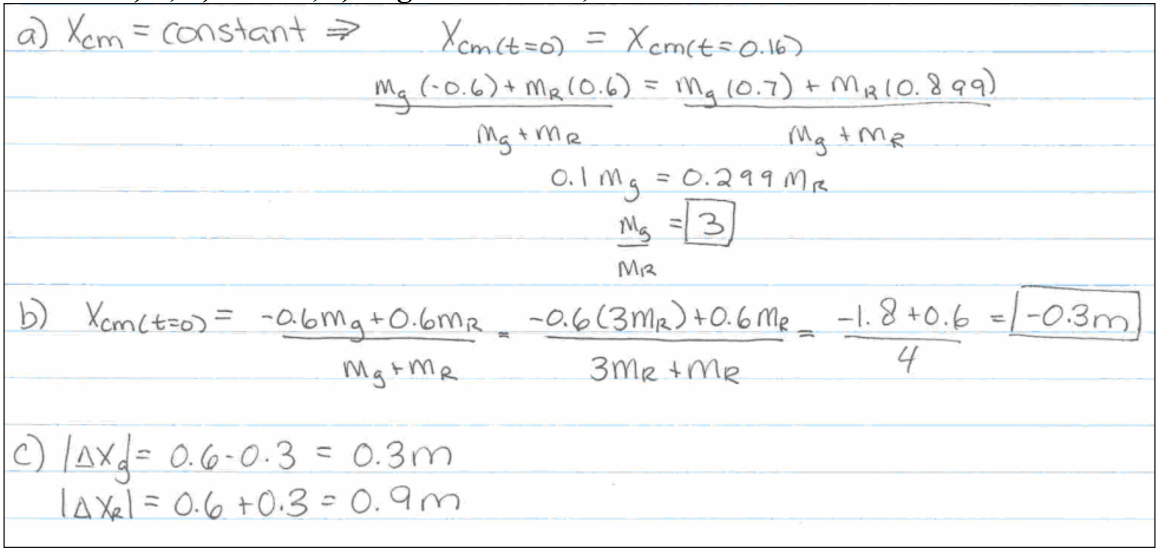
*Topic(s):* Momentum, 2-D Collisions
*Answer:* **In terms of magnitudes …**
**a) poblue1=pogreen1=poblue2=pogreen2=pogreen3=pogreen4=poblue5=14.14 kg m/s, poblue3=0, poblue4=28.28 kg m/s, pogreen5=141.4 kg m/s, b) pfblue1=pfgreen1=pfblue2=pfgreen2=pfblue3=14.14 kg m/s, pfgreen3=0, pfblue4=21.08 kg m/s, pfgreen4=19.44 kg m/s, pfblue5=28.2 kg m/s, pfgreen5=118.5 kg m/s, c) mom is conserved for all.**

## P.8.13 (Level 2)

*Topic(s):* Center of Mass
*Answer:* **0.5 m**

## P.8.14 (Level 2)

*Topic(s):* Center of Mass
*Answer:* **a) 3, b) –0.3 m, c) Δxgreen = 0.3 m, Δxred = 0.9m**



a) $X_{cm} = \text{constant} \Rightarrow \quad X_{cm(t=0)} = X_{cm(t=0.16)}$

$$\frac{m_g(-0.6)+m_R(0.6)}{m_g+m_R} = \frac{m_g(0.7)+m_R(0.899)}{m_g+m_R}$$

$$0.1\,m_g = 0.299\,m_R$$

$$\frac{m_g}{m_R} = \boxed{3}$$

b) $$X_{cm(t=0)} = \frac{-0.6m_g+0.6m_R}{m_g+m_R} = \frac{-0.6(3m_R)+0.6m_R}{3m_R+m_R} = \frac{-1.8+0.6}{4} = \boxed{-0.3m}$$

c) $|\Delta x_g| = 0.6-0.3 = 0.3m$

$|\Delta x_R| = 0.6+0.3 = 0.9m$

# Chapter II.9: Reference Frames

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## *Illustrations*

### Illustration 9.1: Newton's First Law and Reference Frames

*Topic(s):* References Frames, Newton's 1st Law
*Purpose:*. Introduction to reference frames.
*Suggested Use:* Reading assignment or to direct an in-class discussion. The narrative in the illustration provides leading questions to direct the student discussion.

### Illustration 9.2: Reference Frames

*Topic(s):* Reference Frames
*Purpose:* Introduction to reference frames.
*Suggested Use:* Reading assignment or to lead students in a discussion of reference frames and measurements. There are several problems that are similar to this illustration so discussing it would probably help students to interpret the assigned problems.

### Illustration 9.3: The Zero-momentum Frame

*Topic(s):* Momentum, Reference Frames, 1-D Collisions
*Purpose:* To connect momentum conservation and reference frames.
*Suggested Use:* Reading assignment or as an in-class demonstration.

### Illustration 9.4: Rotating Reference Frames

*Topic(s):* Uniform Circular Motion, Reference Frames
*Purpose:*. To connect reference frame considerations with circular motion.
*Suggested Use:* Reading assignment or as an in-class aid for discussion.

## *Explorations*

### Exploration 9.1 – Compare Momentum in Different Frames

*Topic(s):* Reference Frames, Momentum, 1-D Collisions

### Exploration 9.2 – Compare Energy in Different Frames

*Topic(s):* Momentum, 1-D Collisions, Energy, Reference Frames

### Exploration 9.3 – Compare Relative Motion in Different Frames

*Topic(s):* Reference Frames

### Exploration 9.4 – Compare Motion in Accelerating Frames

*Topic(s):* Momentum, 1-D Collisions, Energy, Reference Frames

## Exploration 9.5 – Two Airplanes with Different Land Speeds

*Topic(s):* Reference Frames

## *Problems*

### P.9.1 (Level 1)

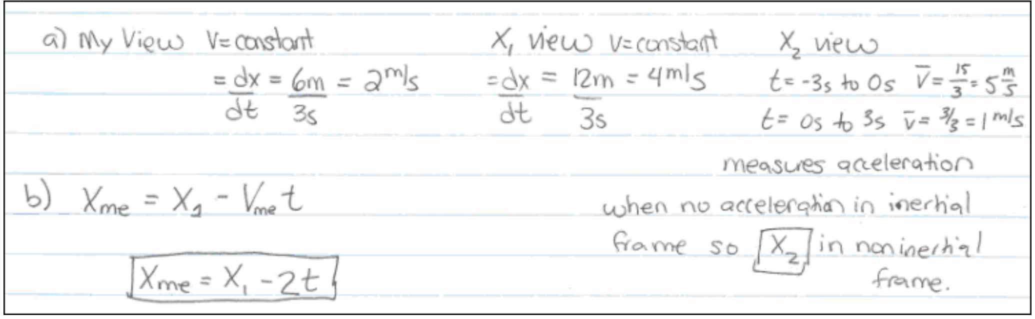
*Topic(s):* Reference Frames
*Answer:* **-20m/s**

### P.9.2 (Level 1)

*Topic(s):* Reference Frames
*Answer:* **a) 3.6 m/s, b) 3.9 m/s, c) 21 degrees**

### P.9.3 (Level 2)

*Topic(s):* Reference Frames
*Answer:* **a) 2 measures star to be accelerating, 1 is in an inertial frame, b) xme = x1 – 2t**



a) My View V=constant $= \frac{dx}{dt} = \frac{6m}{3s} = 2 m/s$

$X_1$ view V=constant $= \frac{dx}{dt} = \frac{12m}{3s} = 4 m/s$

$X_2$ view
$t = -3s$ to $0s$ $\bar{V} = \frac{15}{3} = 5 \frac{m}{s}$
$t = 0s$ to $3s$ $\bar{v} = 3/3 = 1 m/s$

measures acceleration when no acceleration in inertial frame so $\boxed{X_2}$ in noninertial frame.

b) $X_{me} = X_1 - V_{me} t$

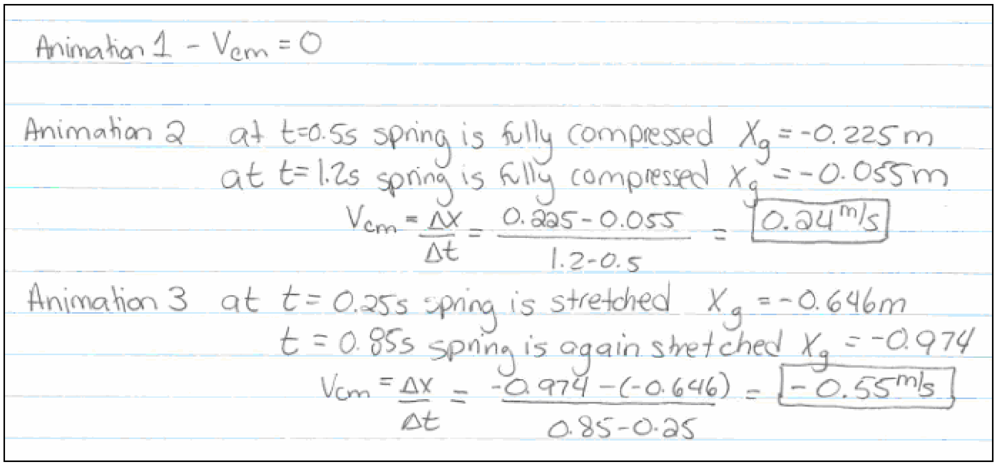
$\boxed{X_{me} = X_1 - 2t}$

### P.9.4 (Level 2)

*Topic(s):* Reference Frames
*Answer:* **a) 7 m/s, b) x2 = x1 –7t**

### P.9.5 (Level 2)

*Topic(s):* Reference Frames
*Answer:* **anim1 = 0, anim2 = 0.24 m/s, anim3 = -0.55 m/s**



Animation 1 - $V_{cm} = 0$

Animation 2 at t=0.5s spring is fully compressed $X_g = -0.225m$

at t=1.2s spring is fully compressed $X_g = -0.055m$

$$V_{cm} = \frac{\Delta x}{\Delta t} = \frac{0.225-0.055}{1.2-0.5} = \boxed{0.24 m/s}$$

Animation 3 at t=0.25s spring is stretched $X_g = -0.646m$

t=0.85s spring is again stretched $X_g = -0.974$

$$V_{cm} = \frac{\Delta x}{\Delta t} = \frac{-0.974-(-0.646)}{0.85-0.25} = \boxed{-0.55 m/s}$$

## P.9.6 (Level 2)

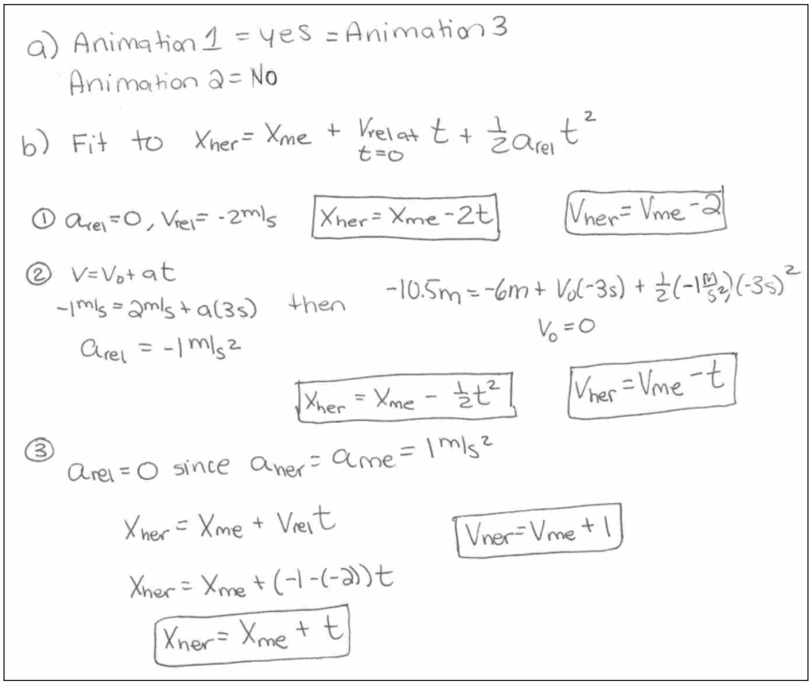
*Topic(s):* Reference Frames
*Answer:* **xg = ½ (-3m/s^2) t^2**

## P.9.7 (Level 1)

*Topic(s):* Reference Frames
*Answer:* **a) anim1 (red=8m/s, green= -8m/s), anim2 (red= 8 m/s, green = -4m/s), anim3 (red=0, green = -6m/s), b) anim1 = 16 m/s, anim2 = 12 m/s, anim3 = 6 m/s, c) no, d) anim1 = 8m/s, anim2 = 8m/s, anim3 = 0, e) anim1 = -8m/s, anim2 = -4 m/s, anim3 = -6m/s**

## P.9.8 (Level 3)

*Topic(s):* Reference Frames
*Answer:*



a) Animation 1 = yes = Animation 3

Animation 2 = No

b) Fit to $X_{her} = X_{me} + V_{rel\,at\,t=0}\, t + \frac{1}{2} a_{rel} t^2$

① $a_{rel} = 0$, $V_{rel} = -2 m/s$ $\boxed{X_{her} = X_{me} - 2t}$ $\boxed{V_{her} = V_{me} - 2}$

② $V = V_0 + at$

$-1 m/s = 2 m/s + a(3s)$ then $-10.5m = -6m + V_0(-3s) + \frac{1}{2}(-1\frac{m}{s^2})(-3s)^2$

$a_{rel} = -1 m/s^2$ $V_0 = 0$

$\boxed{X_{her} = X_{me} - \frac{1}{2}t^2}$ $\boxed{V_{her} = V_{me} - t}$

③ $a_{rel} = 0$ since $a_{her} = a_{me} = 1 m/s^2$

$X_{her} = X_{me} + V_{rel} t$ $\boxed{V_{her} = V_{me} + 1}$

$X_{her} = X_{me} + (-1-(-2))t$

$\boxed{X_{her} = X_{me} + t}$

## P.9.9 (Level 3)

*Topic(s):* Reference Frames
*Answer:* **0.16**

$$V_{Air} = \frac{\Delta L}{\Delta t}_{BOTTOM} = \frac{L}{1.32}$$

$$|V_{Air}| + |V_{wind}| = \frac{L}{1.14} \quad \text{(from top outbound)}$$

$$|V_{Air}| - |V_{wind}| = \frac{L}{1.59} \quad \text{(from top inband)}$$

combining

$$|V_{Air}| = \frac{L}{1.14} - |V_{wind}| = \frac{L}{1.59} + |V_{wind}|$$

$$0.25L = 2|V_{wind}|$$

$$\frac{0.124L}{L/1.32} \doteq \frac{|V_{wind}|}{|V_{Air}|} = \boxed{0.16}$$

# Chapter II.10: Rotations about a Fixed Axis

## *Illustrations*

### Illustration 10.1: Coordinates for Circular Motion

*Topic(s):* Rotational Kinematics
*Purpose:*. To introduce circular motion.
*Suggested Use:* Reading Assignment

### Illustration 10.2: Motion about a Fixed Axis

*Topic(s):* Angular Kinematics
*Purpose:* To connect angular and linear quantities.
*Suggested Use:* Reading assignment

### Illustration 10.3: Moment of Inertia, Rotational Energy, and Angular Momentum

*Topic(s):* Rotational Newton's Laws, Rotational Energy, Rotational Momentum
*Purpose:* To discuss moment of inertia, rotational energy, and angular momentum.
*Suggested Use:* Reading assignment

## *Explorations*

### Exploration 10.1 – Constant Angular Velocity Equation

*Topic(s):* Rotational Kinematics

### Exploration 10.2 – Constant Angular Acceleration Equation

*Topic(s):* Rotational Kinematics

### Exploration 10.3 – Torque and Moment of Inertia

*Topic(s):* Rotational Newton's Laws

### Exploration 10.4 – Torque on Pulley Due to the Tension of Two Strings

*Topic(s):* Rotational Newton's Laws

## *Problems*

### P.10.1 (Level 1)

*Topic(s):* Rotational Kinematics
*Answer:* **2.1 rad**

## P.10.2 (Level 1)

*Topic(s):* Rotational Kinematics
*Answer:* **a) 5.9 m/s, velocity is tangent to the merry-go-round, b) 4.7 rad/s, velocity is counter-clock-wise**

## P.10.3 (Level 1)

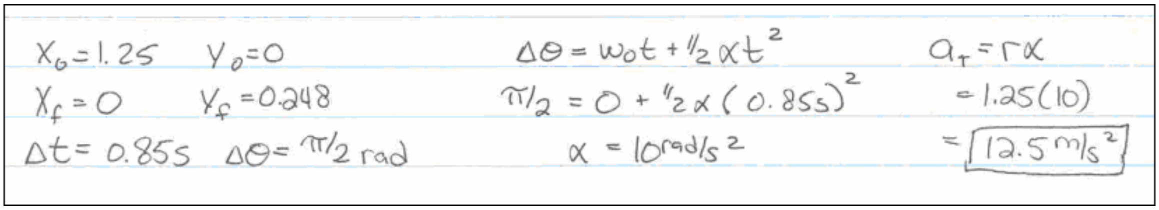
*Topic(s):* Rotational Kinematics
*Answer:* **a) the speeds are equal, b) 4.7 rad/s**

## P.10.4 (Level 1)

*Topic(s):* Rotational Kinematics
*Answer:* **-197 rad/s^2**

## P.10.5 (Level 2)

*Topic(s):* Rotational Kinematics
*Answer:* **12.5 m/s^2**



$X_0 = 1.25$ $Y_0 = 0$ $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $a_T = r\alpha$

$X_f = 0$ $Y_f = 0.248$ $\pi/2 = 0 + \frac{1}{2}\alpha(0.85s)^2$ $= 1.25(10)$

$\Delta t = 0.85s$ $\Delta\theta = \pi/2 \text{ rad}$ $\alpha = 10 \text{rad/s}^2$ $= \boxed{12.5 \text{m/s}^2}$

## P.10.6 (Level 2)

*Topic(s):* Rotational Kinematics
*Answer:* **-6 rad/s^2**

## P.10.7 (Level 2)

*Topic(s):* Rotational Kinematics, Rotational Newton's Laws
*Answer:* **-1.35 N-m**

## P.10.8 (Level 1)

*Topic(s):* Rotational Kinematics
*Answer:* **Animation 1**

## P.10.9 (Level 2)

*Topic(s):* Rotational Energy
*Answer:* **0.01**

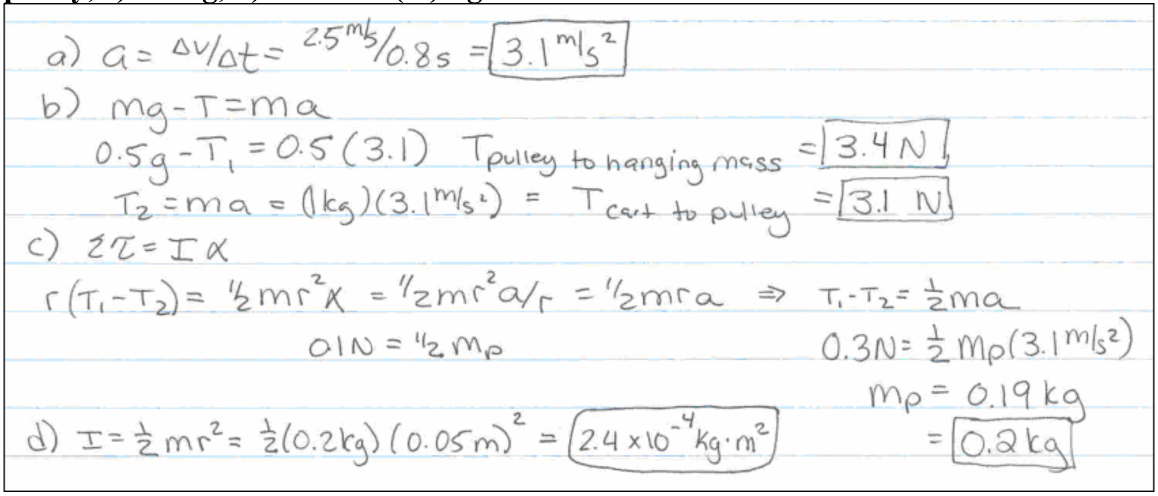
## P.10.10 (Level 2)

*Topic(s):* Rotational Energy, Rotational Newton's Laws
*Answer:* **a) 2, b) 1, c) 1, d) 1, e) T = m(g-a), f) torque = Rm(g-a), g) I = R^2 T/a**

## P.10.11 (Level 2)

*Topic(s):* Rotational Energy, Rotational Newton's Laws

*Answer:* **a) 3.1 m/s^2, b) 3.4 N for tension in pulley to hanging mass, 3.1 N in cart to pulley, c) 0.2 kg, d) 2.4 x 10^(-4) kg-m^2**



a) $a = \Delta v/\Delta t = 2.5\,\text{m/s} / 0.8\,\text{s} = \boxed{3.1\,\text{m/s}^2}$

b) $mg - T = ma$

$0.5g - T_1 = 0.5(3.1)$ $\quad T_{\text{pulley to hanging mass}} = \boxed{3.4\,\text{N}}$

$T_2 = ma = (1\,\text{kg})(3.1\,\text{m/s}^2) = T_{\text{cart to pulley}} = \boxed{3.1\,\text{N}}$

c) $\Sigma\tau = I\alpha$

$r(T_1 - T_2) = \frac{1}{2}mr^2\alpha = \frac{1}{2}mr^2 a/r = \frac{1}{2}mra \Rightarrow T_1 - T_2 = \frac{1}{2}ma$

$01\,\text{N} = \frac{1}{2}m_p$ $\quad 0.3\,\text{N} = \frac{1}{2}m_p(3.1\,\text{m/s}^2)$

$m_p = 0.19\,\text{kg}$

$= \boxed{0.2\,\text{kg}}$

d) $I = \frac{1}{2}mr^2 = \frac{1}{2}(0.2\,\text{kg})(0.05\,\text{m})^2 = \boxed{2.4\times10^{-4}\,\text{kg}\cdot\text{m}^2}$

## P.10.12 (Level 2)

*Topic(s):* Rotational Energy
*Answer:* **6.1 kg-m^2**

## P.10.13 (Level 2)

*Topic(s):* Rotational Energy
*Answer:* **0.9 kg**

## P.10.14 (Level 1)

*Topic(s):* Angular Momentum
*Answer:* **L1 = L2 = L3 = L4 = 28 kg-m^2/s**

# Chapter II.11: General Rotations

## *Illustrations*

### Illustration 11.1: Cross Product

*Topic(s):* Cross Products
*Purpose:*. Introduction to the cross product.
*Suggested Use:* Reading Assignment

### Illustration 11.2: Rolling Motion

*Topic(s):* Rotational Motion
*Purpose:* To introduce rolling motion.
*Suggested Use:* Reading assignment or as in-class visual aid.

### Illustration 11.3: Translational and Rotational Kinetic Energy

*Topic(s):* Rotational Energy
*Purpose:* To introduce rotational energy.
*Suggested Use:* Reading assignment or as an in-class visual aid when introducing rotational kinetic energy. Ask students to predict the shape of the two graphs before viewing and discuss discrepancies. Also focus their attention on the question at the end of the illustration, “Why do you think that the energy vs. time graphs curve, while the energy vs. position graphs are straight lines?”

### Illustration 11.4: Angular Momentum and Area

*Topic(s):* Rotational Momentum
*Purpose:*. To introduce angular momentum as it relates to area.
*Suggested Use:* Reading assignment as an in-class visual aid. This is a great visual to explain how angular momentum works when an object is moving in a straight line.

### Illustration 11.5: Conservation of Angular Momentum

*Topic(s):* Rotational Momentum
*Purpose:* To introduce conservation of angular momentum
*Suggested Use:* Reading assignment.

## *Explorations*

### Exploration 11.1 – Torque

*Topic(s):* Cross Products, Torque

### Exploration 11.2 – Non-uniform Circular Motion

*Topic(s):* Torque, Circular Motion

### Exploration 11.3 – Rolling down an Incline

*Topic(s):* Rotational Energy

### Exploration 11.4 – Moment of Inertia and Angular Momentum

*Topic(s):* Rotational Newton's Laws, Energy, and Momentum

### Exploration 11.5 – Conservation of Angular Momentum

*Topic(s):* Angular Momentum

## *Problems*

### P.11.1 (Level 2)

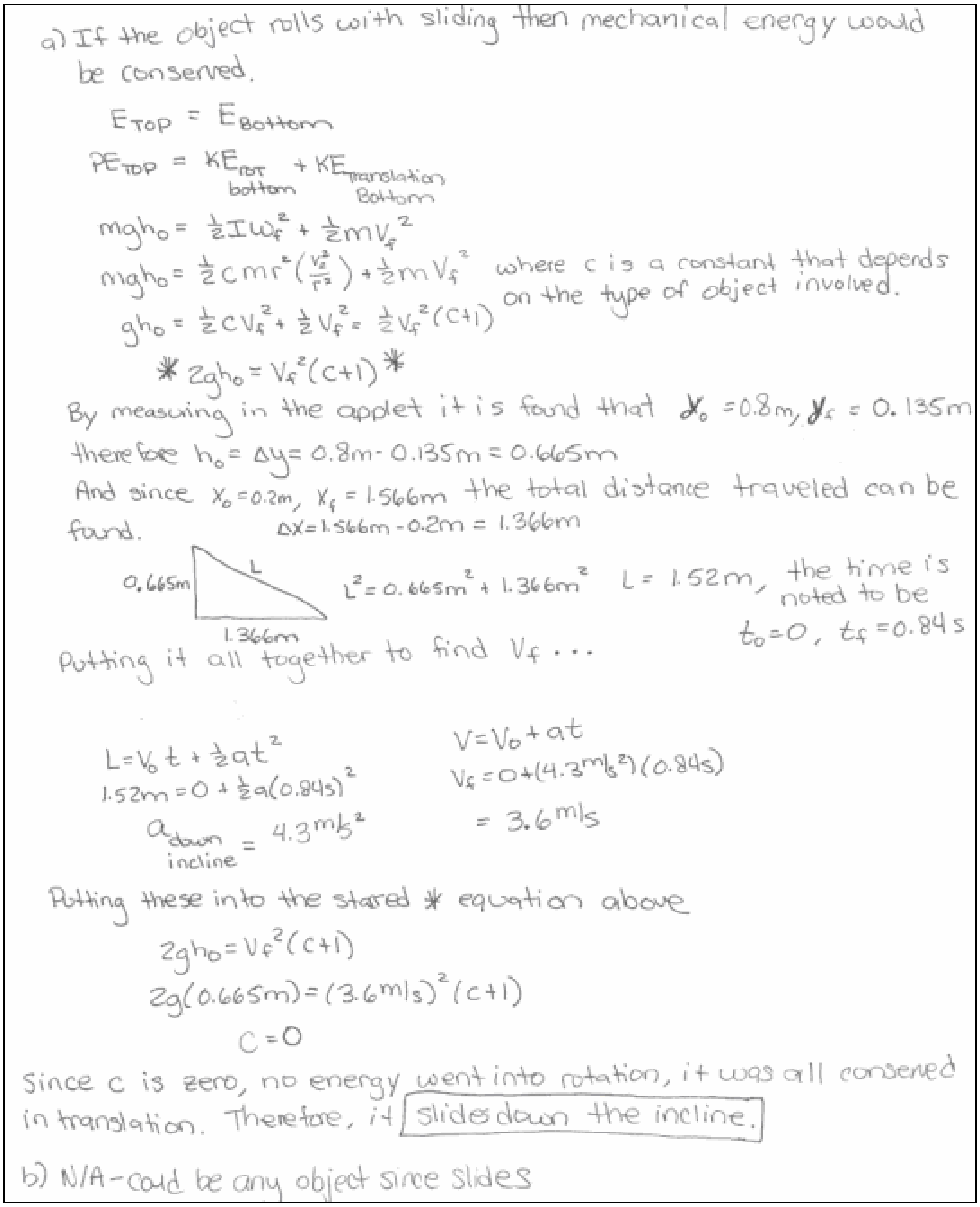
*Topic(s):* Rotational Motion
*Answer:* **5.7 m/s**

### P.11.2 (Level 1)

*Topic(s):* Rotational Motion
*Answer:* **Animation 2**

### P.11.3 (Level 2)

*Topic(s):* Rotational Energy
*Answer:* **a) slides down the incline, b) could be any object**



a) If the object rolls with sliding then mechanical energy would be conserved.

$E_{TOP} = E_{Bottom}$

$PE_{TOP} = KE_{\substack{rot \\ bottom}} + KE_{\substack{translation \\ Bottom}}$

$mgh_o = \frac{1}{2} I \omega_f^2 + \frac{1}{2} m V_f^2$

$mgh_o = \frac{1}{2} c m r^2 \left(\frac{V_f^2}{r^2}\right) + \frac{1}{2} m V_f^2$ where c is a constant that depends on the type of object involved.

$gh_o = \frac{1}{2} c V_f^2 + \frac{1}{2} V_f^2 = \frac{1}{2} V_f^2 (C+1)$

$* \; 2gh_o = V_f^2 (C+1) \; *$

By measuing in the applet it is found that $y_o = 0.8m$, $y_f = 0.135m$

therefore $h_o = \Delta y = 0.8m - 0.135m = 0.665m$

And since $X_o = 0.2m$, $X_f = 1.566m$ the total distance traveled can be found. $\Delta X = 1.566m - 0.2m = 1.366m$

$L^2 = 0.665m^2 + 1.366m^2$ $L = 1.52m$, the time is noted to be $t_o = 0$, $t_f = 0.84s$

Putting it all together to find $V_f$ . . .

$L = V_o t + \frac{1}{2} a t^2$

$1.52m = 0 + \frac{1}{2} a (0.84s)^2$

$a_{\substack{down \\ incline}} = 4.3 m/s^2$

$V = V_o + at$

$V_f = 0 + (4.3 m/s^2)(0.84s)$

$= 3.6 m/s$

Putting these into the stared * equation above

$2gh_o = V_f^2 (C+1)$

$2g(0.665m) = (3.6 m/s)^2 (c+1)$

$C = 0$

Since c is zero, no energy went into rotation, it was all consered in translation. Therefore, it $\boxed{\text{slides down the incline.}}$

b) N/A - could be any object since slides

## P.11.4 (Level 2)

*Topic(s):* Rotational Energy

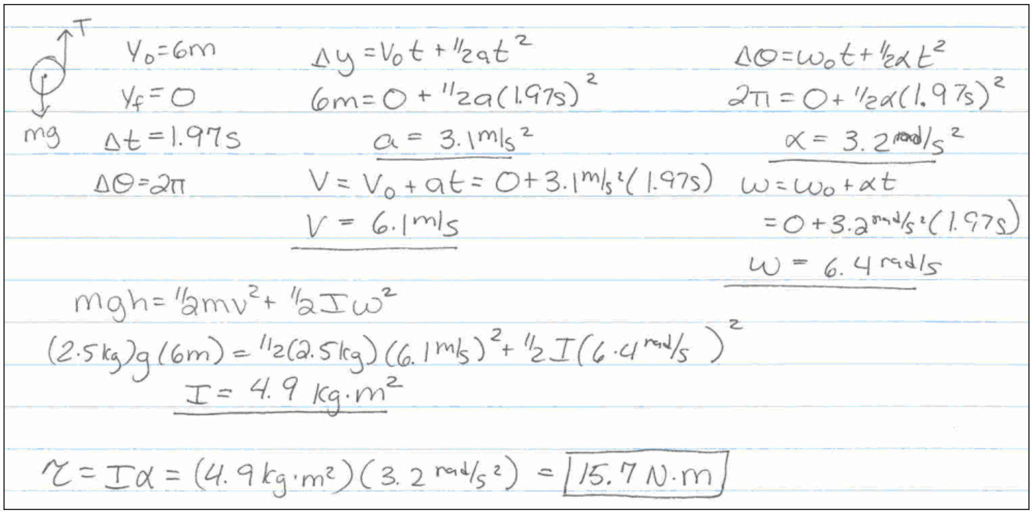
*Answer:* **Rolls, Disk**

a = 3 m/s^2, v at bottom = 3 m/s, some energy goes into rotation

## P.11.5 (Level 2)

*Topic(s):* Rotational Energy, Torque, Rotational Kinematics

*Answer:* **15.7 N-m**



$y_0=6m$

$y_f=0$

$\Delta t=1.97s$

$\Delta\theta=2\pi$

$\Delta y=v_0t+\frac{1}{2}at^2$

$6m=0+\frac{1}{2}a(1.97s)^2$

$\underline{a=3.1m/s^2}$

$V=V_0+at=0+3.1m/s^2(1.97s)$

$\underline{V=6.1m/s}$

$\Delta\theta=\omega_0t+\frac{1}{2}\alpha t^2$

$2\pi=0+\frac{1}{2}\alpha(1.97s)^2$

$\underline{\alpha=3.2rad/s^2}$

$\omega=\omega_0+\alpha t$

$=0+3.2rad/s^2(1.97s)$

$\underline{\omega=6.4rad/s}$

$mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$

$(2.5kg)g(6m)=\frac{1}{2}(2.5kg)(6.1m/s)^2+\frac{1}{2}I(6.4rad/s)^2$

$\underline{I=4.9\ kg\cdot m^2}$

$\tau=I\alpha=(4.9kg\cdot m^2)(3.2rad/s^2)=\boxed{15.7N\cdot m}$

## P.11.6 (Level 2)

*Topic(s):* Rotational Energy
*Answer:* **0.17 kg**

## P.11.7 (Level 2)

*Topic(s):* Angular Momentum
*Answer:* **76kg-m^2/s for all**

## P.11.8 (Level 1)

*Topic(s):* Conservation of Angular Momentum
*Answer:* **54 kg**

## P.11.9 (Level 1)

*Topic(s):* Angular Momentum, Rotational Energy
*Answer:* **KE and Angular Momentum**

## P.11.10 (Level 2)

*Topic(s):* Angular Momentum
*Answer:* **5 kg-m^2/s before and after for all animations**

# Chapter II.12: Gravitation

## *Illustrations*

### Illustration 12.1: Projectile and Satellite Orbits

*Topic(s):* Gravitation
*Purpose:*. Introduction to orbits.
*Suggested Use:* This is a great visual aid to help students understand orbiting bodies. It can be used to show that an orbit body is always "falling" under the influence of gravity and the velocity dependence of an orbit.

### Illustration 12.2: Orbits and Planetary Mass

*Topic(s):* Gravitation
*Purpose:* Discussion of orbiting bodies.
*Suggested Use:* Reading assignment

### Illustration 12.3: Circular and Non-Circular Motion

*Topic(s):* Circular Motion
*Purpose:* To discuss circular motion and orbits.
*Suggested Use:* Reading assignment

### Illustration 12.4: Angular Momentum and Area

*Topic(s):* Angular Momentum
*Purpose:*. To discuss angular momentum and its relationship to orbiting bodies.
*Suggested Use:* Reading assignment or as an in-class visual to connect angular momentum to the area swept out.

### Illustration 12.5: Kepler's Second Law

*Topic(s):* Kepler's Laws
*Purpose:* To introduce Kepler's 2nd Law.
*Suggested Use:* Reading assignment.

### Illustration 12.6: Heliocentric vs. Geocentric

*Topic(s):* Circular Motion
*Purpose:* To discuss reference frames and circular motion.
*Suggested Use:* This illustration provides a great visualization of the difference perspective can make on the appearance of an orbit. As students view the animation they can be lead into a discussion of heliocentric vs. geocentric orientations and the effect orientations would have had on the world.

## *Explorations*

### Exploration 12.1 – Different $x_o$ or $v_o$ for Planetary Orbits

*Topic(s):* Uniform Circular Motion

### Exploration 12.2 – Set Both $x_o$ and $v_o$ for Planetary Orbits

*Topic(s):* Kepler's Laws

### Exploration 12.3 – Properties of Elliptical Orbits

*Topic(s):* Kepler's Laws

### Exploration 12.4 – Angular Momentum and Energy

*Topic(s):* Angular Momentum, Kepler's Laws, Energy

## *Problems*

### P.12.1 (Level 1)

*Topic(s):* Newton's Law of Gravitation
*Answer:* **a) Animation 3, b) 10 kg**

### P.12.2 (Level 2)

*Topic(s):* Newton's Law of Gravitation, Circular Motion
*Answer:* **3.4 x 10^(15) kg**

### P.12.3 (Level 2)

*Topic(s):* Newton's Law of Gravitation, Circular Motion
*Answer:* **a) 0, b) G Mp [ Ms/R^2 + Mp/(2R)^2] = Mp 4 Pi^2 R/T^2 where R is the distance from a planet to the star, c) 2.4 x 10^(25) kg**

### P.12.4 (Level 2)

*Topic(s):* Kepler's Laws
*Answer:* **a) 1.5 x 10^(23) kg, b) 2.5 x 10^6 km**

### P.12.5 (Level 2)

*Topic(s):* Kepler's Laws, Angular Momentum, Energy Conservation
*Answer:* **a) perhaps, b) if the system is only the planet and the star then the orbit is not physical as energy and angular momentum are not conserved, however, the motion could be explained if there were another mass nearby.**

### P.12.6 (Level 2)

*Topic(s):* Kepler's Laws
*Answer:* **Red does not have the same T^2/R^3 ratio as the others**

## P.12.7 (Level 2)

*Topic(s):* Kepler's Laws, Circular Motion
*Answer:* **2.5 x 10^3 m/s**

## P.12.8 (Level 2)

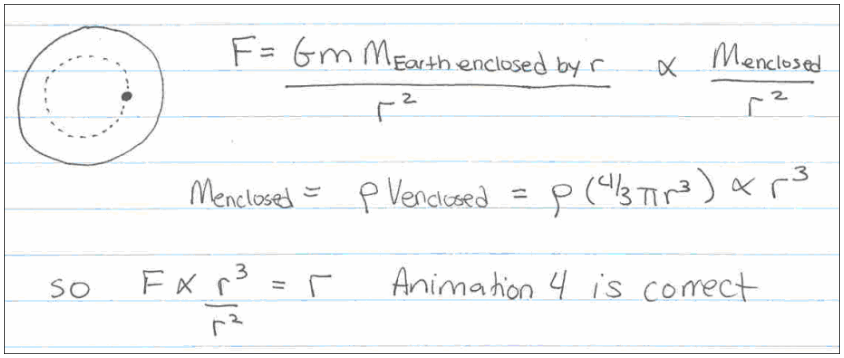
*Topic(s):* Kepler's Laws
*Answer:* **9 x 10^31 kg**

## P.12.9 (Level 2)

*Topic(s):* Reference Frames
*Answer:* **6 m/s^2**

## P.12.10 (Level 2 - Calculus)

*Topic(s):* Gravitation
*Answer:* **Animation 4**



$$F = \frac{G m M_{\text{Earth enclosed by } r}}{r^2} \propto \frac{M_{\text{enclosed}}}{r^2}$$

$$M_{\text{enclosed}} = \rho V_{\text{enclosed}} = \rho\left(\tfrac{4}{3}\pi r^3\right) \propto r^3$$

so $F \propto \frac{r^3}{r^2} = r$ Animation 4 is correct

# Chapter II.13: Statics

## *Illustrations*

### Illustration 13.1: Equilibrium on a Ramp

*Topic(s):* Normal Force, Distributed Load
*Purpose:* Identify the conditions for tipping for a block in equilibrium on a ramp.
*Suggested Use:* Demonstration in class when solving the problem of a tipping block on a ramp or when discussing "where the normal force acts" of a ramp on a block.

### Illustration 13.2: Center of Mass and Gravity

*Topic(s):* Center of Mass, Center of Gravity
*Purpose:* To introduce the concept of center of mass.
*Suggested Use:* To demonstrate that the ratio of the distances (from each particle to the center of mass) for a two-particle system is proportional to the ratio of the masses of the particles.

### Illustration 13.3: Force and Torque in Equilibrium

*Topic(s):* Conditions of Equilibrium.
*Purpose:* To find the force and torque needed for an object to be in equilibrium.
*Suggested Use:* Reading assignment to introduce students to the conditions of equilibrium; although the correct answer can be obtained using trial and error, follow-up questions and exercises can be used to verify that they can calculate the correct answer as well.

### Illustration 13.4: The Diving Board Problem

*Topic(s):* Conditions of Equilibrium
*Purpose:*. To help students visualize how the forces on a diving board depend on the location of the load on the board.
*Suggested Use:* An exercise to be performed before doing the typical "diving board problem" where students calculate the forces on a diving board when a person stands on the end.

## *Explorations*

### Exploration 13.1 – Balance a Mobile

*Topic(s):* Torque, Newton's Laws

### Exploration 13.2 – Static Friction on a Horizontal Beam

*Topic(s):* Torque, Newton's Laws

### Exploration 13.3 – Distributed Load

*Topic(s):* Torque, Newton's Laws

## Exploration 13.4 – The Stacking of Bricks

*Topic(s):* Torque, Newton's Laws, Center of Mass

## *Problems*

### P.13.1 (Level 2)

*Topic(s):* Newton's Laws, Torque
*Answer:* **a) 14°, b) 0.25**

### P.13.2 (Level 2)

*Topic(s):* Newton's Laws, Torque
*Answer:* **a) Tright = 6.3 N, Tleft = 18N, b) right = 13 N, left = 21 N**

### P.13.3 (Level 1)

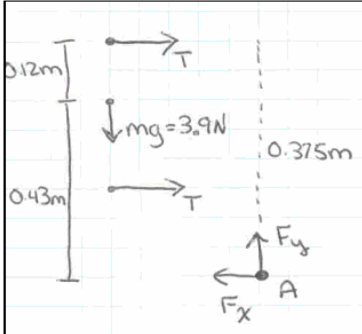
*Topic(s):* Newton's Laws, Torque, Center of Mass
*Answer:* **a) 8054 N upward, b) 1.9 m from left axle**

### P.13.4 (Level 2)

*Topic(s):* Newton's Laws, Torque
*Answer:* **a) 33 N, b) 0.34, c) attach the rope lower on the box**

### P.13.5 (Level 3)

*Topic(s):* Newton's Laws, Torque
*Answer:* **a) 3.3 N, b) 6.6 N in x and 3.9 N in y**



$\Sigma F_y = F_y - mg = 0 \quad F_y = mg = (0.4\text{kg})g = 3.9N$

$\Sigma F_x = T + T - F_x = 0 \quad F_x = 2T$

$\Sigma \tau_A = (0.375m)T - (3.9N)(0.43m) + (0.135m)T = 0$

$(0.51m)T = 1.68N\,m$

$T = 3.3N$

so $F_x = 2T = 6.6N$

$\boxed{\begin{array}{l} a)\ T = 3.3N \\ b)\ F_x = 6.6N,\ F_y = 3.9N \end{array}}$

### P.13.6 (Level 2)

*Topic(s):* Newton's Laws, Torque
*Answer:* **a) 3.3, b) Fy = W/2, Fx = 3.1 W, c) 0.16**

### P.13.7 (Level 2)

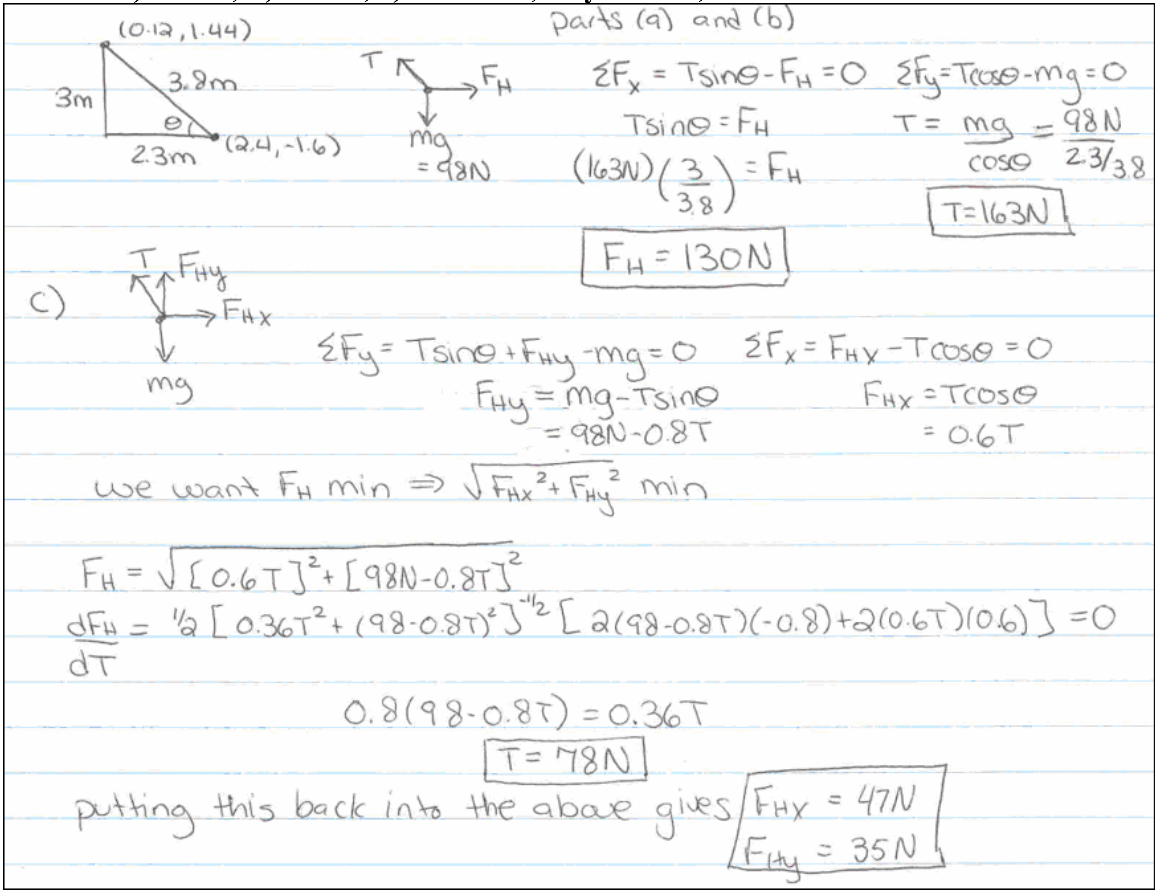
*Topic(s):* Newton's Laws, Torque
*Answer:* **a) 70 N left and 7.4 N up, b) 70 N right and 27 N down, c) on black rod (70 N right and 7.4 N up), on blue rod (70 N left and 7.4 N down)**

## P.13.8 (Level 2)

*Topic(s):* Newton's Laws, Torque
*Answer:* **a) 0.6 m to right of pivot, b) 1600 N, c) 2.9 m to left of pivot**

## P.13.9 (Level 3 - Calculus)

*Topic(s):* Newton's Laws, Torque
*Answer:* **a) 130 N, b) 163 N, c) T = 78 N, Fhy = 35 N, Fhx = 47 N**



parts (a) and (b)

$\Sigma F_x = T\sin\theta - F_H = 0$

$T\sin\theta = F_H$

$(163N)\left(\frac{3}{3.8}\right) = F_H$

$\boxed{F_H = 130N}$

$\Sigma F_y = T\cos\theta - mg = 0$

$T = \frac{mg}{\cos\theta} = \frac{98N}{2.3/3.8}$

$\boxed{T = 163N}$

$\Sigma F_y = T\sin\theta + F_{Hy} - mg = 0$

$F_{Hy} = mg - T\sin\theta$
$= 98N - 0.8T$

$\Sigma F_x = F_{Hx} - T\cos\theta = 0$

$F_{Hx} = T\cos\theta$
$= 0.6T$

we want $F_H$ min $\Rightarrow \sqrt{F_{Hx}^2 + F_{Hy}^2}$ min

$$F_H = \sqrt{[0.6T]^2 + [98N - 0.8T]^2}$$

$$\frac{dF_H}{dT} = \tfrac{1}{2}\left[0.36T^2 + (98 - 0.8T)^2\right]^{-1/2}\left[2(98 - 0.8T)(-0.8) + 2(0.6T)(0.6)\right] = 0$$

$$0.8(98 - 0.8T) = 0.36T$$

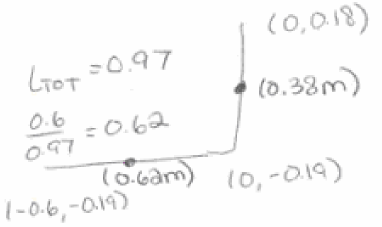
$$\boxed{T = 78N}$$

putting this back into the above gives $\boxed{\begin{array}{l} F_{Hx} = 47N \\ F_{Hy} = 35N \end{array}}$

## P.13.10 (Level 3)

*Topic(s):* Newton's Laws, Torque, Center of Mass
*Answer:* **a) 0.39 N-m clockwise, b) 1.96 N up, c) 2.39 N-m clockwise, 6.96 N up, d) apply the force further out along the lever, e) 16 N**

First find cm of lever.



$L_{TOT} = 0.97$

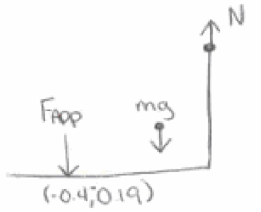
$\frac{0.6}{0.97} = 0.62$

$$X_{cm} = \frac{0.62m(-0.3)+0.38m(0)}{m} = -0.186$$

$$Y_{cm} = \frac{0.62m(-0.19)+0.38m(0)}{m} = -0.118$$

let cm be at X = -0.2, y = -0.12

Now



a) $\tau_{mg} = (0.2)mg = (0.2)(0.2kg)g = 0.39 N\cdot m$ ccw

So $\tau_{friction}$ must be $\boxed{0.39\ N\cdot m \text{ clockwise}}$

Since Net $\tau = 0$

b) $F_{\text{Axle on Lever}} = -F_{\text{lever on Axle}} = mg = \boxed{1.96N \text{ up}}$

c) $\tau_{mg} + \tau_{F_{App}} = 0.39N\cdot m$ ccw + $(0.4m)(5N)$ ccw = $\boxed{2.39N\cdot m \text{ ccw}}$

Net $F = mg + F_{App} = \boxed{6.96N \text{ up}}$

d) $\boxed{\text{Apply the force further out along the Lever}}$

e) $\tau_{App} + \tau_{mg} = \tau_{fr}$

$F(0.6m) + 0.39N\cdot m = 10\ N\cdot m$

$F = \boxed{16N}$

## P.13.11 (Level 2)

*Topic(s):* Newton's Laws, Torque
*Answer:* **a) 1.8 N, perpendicular to the ramp, located directly below the cm at x = 0.23 m and y = -0.2 m.**

## P.13.12 (Level 1)

*Topic(s):* Center of Mass
*Answer:* **-0.7 m**

## P.13.13 (Level 2)

*Topic(s):* Torque, Newton's Laws
Answer: **15.2 kg**

# Chapter II.14: Static Fluids

## *Illustrations*

### Illustration 14.1: Pressure in a Liquid

*Topic(s):* pressure
*Purpose:* To demonstrate the change in pressure as a function of depth.
*Suggested Use:* Have students calculate the pressure at various points in the water container and check their calculations with the animation. Some students will be surprised that to calculate the pressure at point A, the depth below the surface is 10-m (from the top of the water level) not 1-m, the distance to the container above it. Have students explain why this is so (if the pressure were much lower at point A, water would flow from the center part of the container to point A, equalizing the pressure). In the context of this illustration, have students explain why we use pressure instead of force in talking about fluids.

### Illustration 14.2: Pascal's Principle

*Topic(s):* pressure, Pascal's principle, work
*Purpose:* To illustrate the use of Pascal's principle in hydraulic lifts.
*Suggested Use:* Students can determine the work done (1.96J) and the amount the green block moves up in the animation (0.1 cm) to explain why you can't see it on the animation. Can discuss why, when using lifts of any type, you must move the low force lever (or whatever the mechanism is) many times (this is, of course, true for any lift: not simply hydraulic ones). Note that the change in pressure with depth (for a change in depth of 10-cm) if the liquid is oil is negligible (pressure change is less than 100 Pa so it requires an increase of 0.1% in the force to compensate).

### Illustration 14.3: Buoyant Force

*Topic(s):* Buoyant force
*Purpose:* To illustrate the cause of the buoyant force.
*Suggested Use*: Students can quickly realize that whatever the fraction of the water density the object is (e.g. 0.4 at the beginning), that is the percentage of the object submerged under the water as the object floats. To help them understand the buoyant force, though, it is useful to have them calculate the pressure differences between the top and the bottom of the floating block as suggested in the text of the Illustration to help make the connection between buoyancy and pressure as a function of depth. It is also easy for students to see that the water displaced (in this case into an adjoining container) is equal to the volume of the object under water and they can calculate the weight of this displaced water which gives them the easiest way to calculate the buoyant force. Finally, as they move a floating block, it will undergo simple harmonic motion (since the pressure, which determines the force on the object, varies linearly with depth creating a linear restoring force).

## Illustration 14.4: Pumping Water up from a Well

*Topic(s):* pressure
*Purpose:* To illustrate an application of pressure as a function of depth.
*Suggested Use:* Many students think that suction systems (pumps, straws, etc.) do not have any limits (ask students how long a straw they could drink from) simply because they just "suck" the liquid out of something. Students do not realize that for the suction to work, it requires pressure down (from the atmosphere) on the liquid. By explicitly showing the pressure at the pump for deeper wells, students should be able to explain limits on pumping wells with suction pumps and predict the limits for a less dense or more dense liquid.

# *Explorations*

## Exploration 14.1

*Topic(s):* Buoyant force
a) 6.4x10-5 $m^3$
*c) 630 $kg/m^3$*

## Exploration 14.2

*Topic(s):* Buoyant force

## Exploration 14.3

*Topic(s):* Buoyant force
*a) $5x10^5$ N*
*b) $5x10^5$ N; 790 $kg/m^3$*
*c) 80% submerged*
*j) $\Delta p=0.3x10^5$ so $F=5\ x10^5$ N*

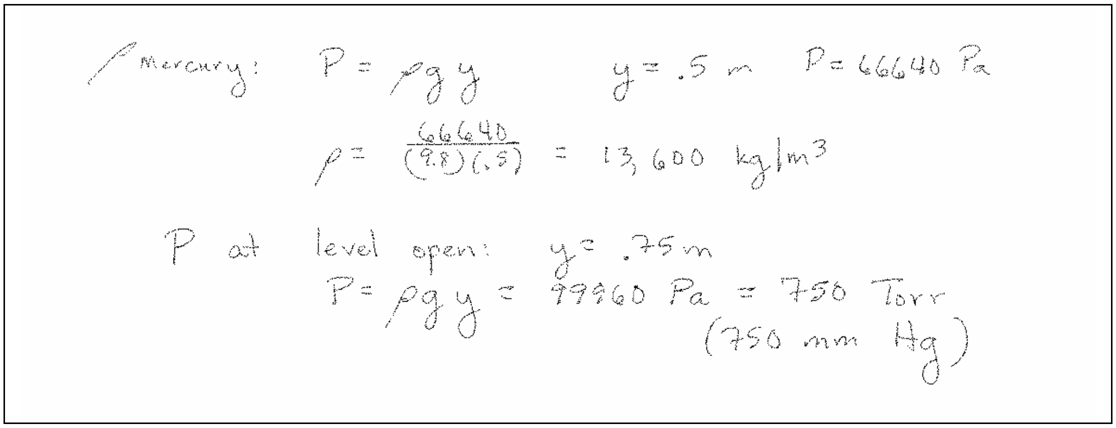
# *Problems*

## P.14.1 (Level 1)

*Topic(s):* pressure, Pascal's principle
*Answer:*
a) **7 N**. $\text{diameter}_{\text{left}}$=0.02m; $\text{diameter}_{\text{right}}$=0.15m; p=F/A is same on both sides.
b) **56 cm**.Volume of liquid displaced equal.

## P.14.2 (Level 2)

*Topic(s):* pressure
*Answer:*
**99960 Pa=750 Torr**. $p=\rho gh$. Density of mercury=$13.6x10^3$ $kg/m^3$ and height of mercury column is 75 cm (7.5 units) from surface.



Mercury: $P = \rho g y$ $y = .5$ m $P = 66640$ Pa

$$\rho = \frac{66640}{(9.8)(.5)} = 13{,}600 \text{ kg/m}^3$$

P at level open: $y = .75$ m

$P = \rho g y = 99960$ Pa $= 750$ Torr

(750 mm Hg)

## P.14.3 (Level 2)

*Topic(s):* Buoyant force

*Answer:*

**$1.7 \times 10^3$ to $1.9 \times 10^3$ kg/m$^3$**. mg=19N, m=1.9-kg. $F_b$=10-N to 11-N so V=.001m$^3$.

## P.14.4 (Level 2)

*Topic(s):* Buoyant force

*Answer:*

a) **1.47 N, 0.9 N, 0.9 N**. $V_{oil\_displacement}$=.05x.007x.2=7x10$^{-5}$m$^3$. Fb=$gV_{oil}\rho_{oil}$=.65.

b) **90 Pa, 550 Pa**

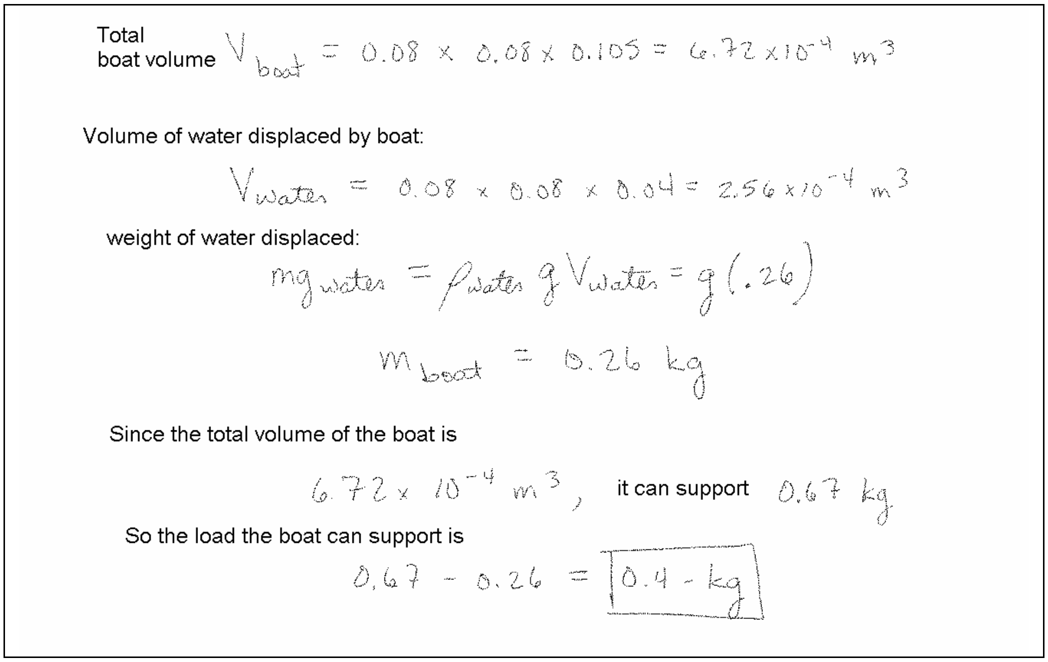
c) **Δp is the same between top and bottom of the brick and that, not the pressure on the top (or bottom) alone, determines $F_b$**.

## P.14.5 (Level 2)

Topic(s): Buoyant force

*Answer:* **0.4-kg**.

V in water=2.56x10$^{-4}$m$^3$ so weight of water displaced is 0.26-kg. "Boat" is 0.26-kg, but total volume is .08x.08x.105=6.72x10$^{-4}$m$^3$ so can support total of 0.67-kg.



Total boat volume $V_{boat} = 0.08 \times 0.08 \times 0.105 = 6.72 \times 10^{-4}\ m^3$

Volume of water displaced by boat:

$$V_{water} = 0.08 \times 0.08 \times 0.04 = 2.56 \times 10^{-4}\ m^3$$

weight of water displaced:

$$mg_{water} = \rho_{water}\, g\, V_{water} = g(.26)$$

$$m_{boat} = 0.26\ kg$$

Since the total volume of the boat is

$6.72 \times 10^{-4}\ m^3$, it can support $0.67\ kg$

So the load the boat can support is

$$0.67 - 0.26 = \boxed{0.4 - kg}$$

## P.14.6 (Level 2)

*Topic(s):* Buoyant force

*Answer:*

**740 kg/m$^3$**. $F_b$=2N=ρgV. Block V=2.75x10$^{-4}$m$^3$.

## P.14.7 (Level 2)

*Topic(s):* Buoyant force

*Answer:*

**Animation 1a & 1b:** In the free-fall case, $F_b$=0 on block because there is no pressure differential in because no weight of water to support (weight of water displaced is 0). In upward acceleration case, force diagram of block gives $F_b$>mg and the increased $F_b$ comes from the increased apparent weight of the block—or increased pressure differential.

## P.14.8 (Level 3)

*Topic(s):* Buoyant force

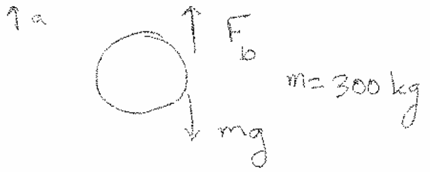
*Answer:*

**0.9 kg/m$^3$**. a=0.46m/s$^2$. $F_b$-mg=ma and m=300+$\rho_{inside}$V. V=(4/3)πr$^3$=905m$^3$.

Balloon volume $= \frac{4}{3}\pi r^3$

$r = 6\ m$

$\underline{V = 905\ m^3}$



| $t$ | $y$ (bottom of balloon) |
|---|---|
| 0 | -11.2 |
| .02 | ~-11.2 |
| 4 | -7.5 |
| 7 | 0.2 |
| 10 | 11.9 |

$v_0 = 0$

$y = \overset{0}{\cancel{v_0 t}} + \frac{1}{2}at^2$

$(-7.5 - -11.2) = \frac{1}{2}a(4)^2$

$a = 0.46\ m/s^2$

Checking:

$(11.9 - -11.2) = \frac{1}{2}a(10)^2$

$\underline{a = 0.46\ m/s^2}$

$F_b = \rho_{\text{outside air}} V_{\text{balloon}}\, g = 11.5\ kN$

$F_b - m_T g = m_T a$

$m_T = 300 + m_{\text{air inside}}$

$F_b = m_T(10.3)$

$m_T = 1.1 \times 10^3\ kg$

$m_{air} = 810\ kg$

$\rho_{\text{air inside}} = m_{air}/V_{\text{balloon}} = \boxed{0.9\ kg/m^3}$

## P.14.9 (Level 2)

*Topic(s):* Buoyant force

*Answer:*

**Animation 2**. (Mass of ice supported)*g is equal to weight of water displaced by ice so when ice turns into water, takes same volume.

## P.14.10 (Level 3)

*Topic(s):* Buoyant force

Answer:

**Animation B**. As oil is pumped out, do not have additional $F_b$ due to displaced oil so block sinks some.

# Chapter II.15: Fluids in Motion

## *Illustrations*

### Illustration 15.1: The Continuity Equation

*Topic(s):* continuity equation, Bernoulli's equation
*Purpose:* To demonstrate the continuity equation for fluids.
*Suggested Use:* Have students calculate the volume flow rate in the wide and narrower regions. Use Bernoulli's equation to explain why the pressure drops in the narrower region (see Exploration 15.1 for a further exploration of this.)

### Illustration 15.2: Bernoulli's law: Flow from Opening

*Topic(s):* Bernoulli's equation, projectile motion
*Purpose:* To illustrate an application of Bernoulli's equation to fluid flow from a reservoir.
*Suggested Use:* Change values of the height of the opening (leak) in the side of the container and have students predict and then explain why the velocity (and distance the water travels) changes the way it does.

### Illustration 15.3: Ideal and Viscous Fluid Flow

*Topic(s):* viscosity, Bernoulli's equation
*Purpose:* To illustrate the difference between ideal and viscous fluid flow.
*Suggested Use*: Students should quickly notice the difference between the two cases as well as the difference between the narrower tube and the wider tube when there is viscosity. As an extension, students can find the volume flow rate ($Av=0.23m^3/s$) and use the equation to find that $\eta=1.23$ Pa-s.

### Illustration 15.4: Airplane lift

*Topic(s):* Bernoulli's equation
*Purpose:* To illustrate an application of Bernoulli's equation for airplane lift.
*Suggested Use:* Quantitatively compare the speeds of the air going above and below the wing. Students should be able to explain why the different speeds result in a lift and the direction of the lift force. It is worth noting, that for this example, the lift is quite small and students can come up with ways to improve the lift and discuss the trade-offs.

## *Explorations*

### Exploration 15.1

*Topic(s):* Bernoulli's equation, continuity equation
*a)-b)* $v_{wide}=240mm/s$*;* $v_{w/2mm}=360mm/s$*;* $Av=\pi 7.3x10^3mm^3/s$ *for both 2-mm*
*c)-e)* $v_{w/8mm}=3225$ *mm/s; Av same as above*
*f)-j)acceleration to right; F to right so p higher in wider region*
*l)* $P+1/2\rho v^2=25944Pa$ *(be sure to change to MKS units) for all regions*

## Exploration 15.2

*Topic(s):* Bernoulli's equation, conservation of energy, continuity equation

*a)-f) Left: Volume=6.28m$^3$; v=2m/s; Right: Volume=6.28m$^3$; v=8m/s; Av=25.1m$^3$/s for both.*

*g)-m) $P_{left}$=2.7x10$^5$Pa, $F_{left}$=33.9x10$^5$N, $W_{left}$=16.96x10$^5$J ; $P_{right}$=1.52x10$^5$Pa, $F_{right}$=4.78x10$^5$N, $W_{right}$=9.55x10$^5$J; ΔW=7.4x10$^5$J*

*n)-p) KE=1/2ρVv$^2$; ΔKE=1.88x10$^5$J; ΔPE=ρVgΔh=5.54x10$^5$J; Volume=Avt*

## Exploration 15.3

*Topic(s):* Bernoulli's equation, projectile motion

*a) y=5: $P_{bottom}$=1.5x10$^5$Pa*

*b) v=9.9m/s*

*c) falls 9m so t=1.36s; x=vt=13.4 which agrees.*

# *Problems*

## P.15.1 (Level 1)

*Topic(s):* continuity equation

*Answer:*

**Animation 3**

## P.15.2 (Level 2)

*Topic(s):* Bernoulli's equation

*Answer:*

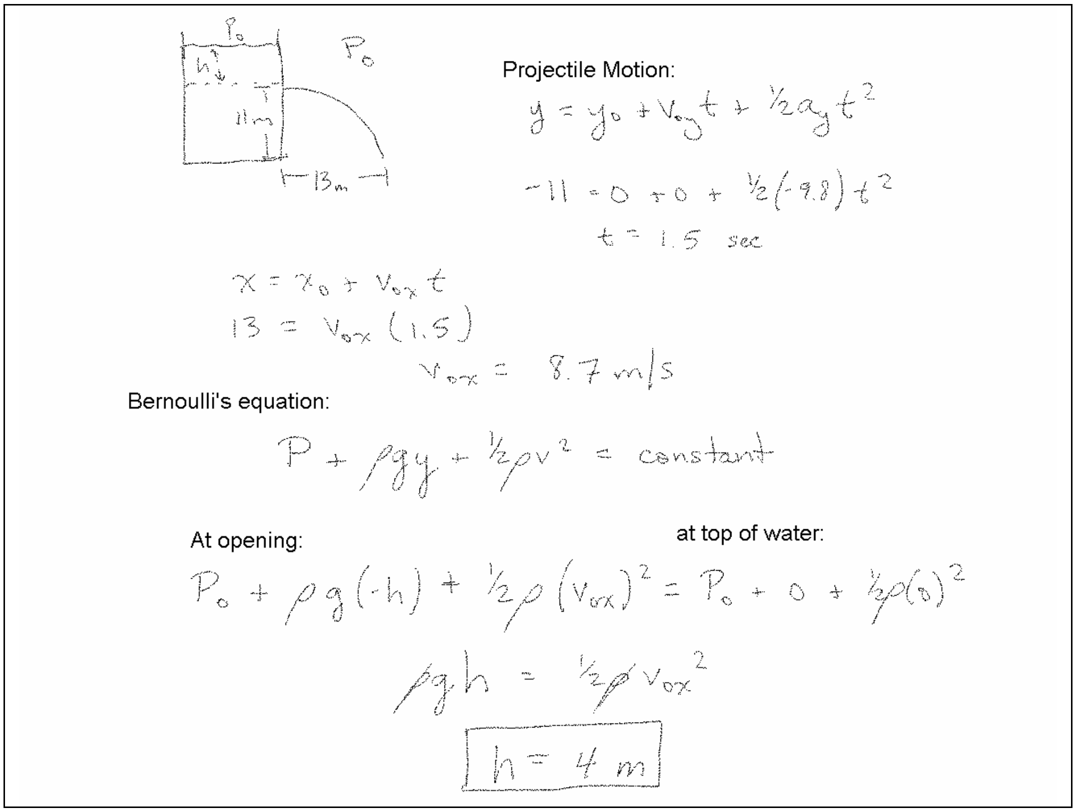
**850 kg/m$^3$**. Can use pressure marker in vertical tube (v is the same there) so ΔP=ρgΔh.

## P.15.3 (Level 2)

*Topic(s):* Bernoulli's equation, projectile motion

*Answer:*

**4-m above the opening (y=11m)**. Water falls 11-m; t=1.5s so $v_x$=8.7 m/s. Use P+ρgy+1/2ρv$^2$=constant and pressure on top of water and outside is the same.



Projectile Motion:

$$y = y_0 + v_{oy}t + \frac{1}{2}a_y t^2$$

$$-11 = 0 + 0 + \frac{1}{2}(-9.8)t^2$$

$$t = 1.5 \text{ sec}$$

$$x = x_0 + v_{ox}t$$

$$13 = v_{ox}(1.5)$$

$$v_{ox} = 8.7 \text{ m/s}$$

Bernoulli's equation:

$$P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

At opening: at top of water:

$$P_0 + \rho g(-h) + \frac{1}{2}\rho(v_{ox})^2 = P_0 + 0 + \frac{1}{2}\rho(0)^2$$

$$\cancel{\rho} g h = \frac{1}{2}\cancel{\rho} v_{ox}^2$$

$$\boxed{h = 4 \text{ m}}$$

## P.15.4 (Level 2)

*Topic(s):* Bernoulli's equation, projectile motion

*Answer:*

**$1.5\text{x}10^5$ Pa.** Water falls 9-m; so t=1.36s so $v_x$=14.8m/s. Use P+mgy+1/2ρ$v^2$=constant.

## P.15.5 (Level 2)

Topic(s): Bernoulli's equation, projectile motion

*Answer:*

**ρ=800 kg/$m^3$**. For h=5m: Water falls 9-m; so t=1.36s so $v_x$=13.1m/s. Use P+ρgy+1/2ρ$v^2$=constant.

## P.15.6 (Level 2)

*Topic(s):* Bernoulli's equation, continuity equation

*Answer:*

a) **28.4 m.** In widest region (r=8m) v=1m/s, length=4m; middle region (r=5m) v=2.5m/s, length=10.2m; Volume constant and in narrowest region (r=3)

b) **7.1 m/s.** Av=constant

c) **$1.83\text{x}10^5$Pa.** P +1/2ρ$v^2$=constant

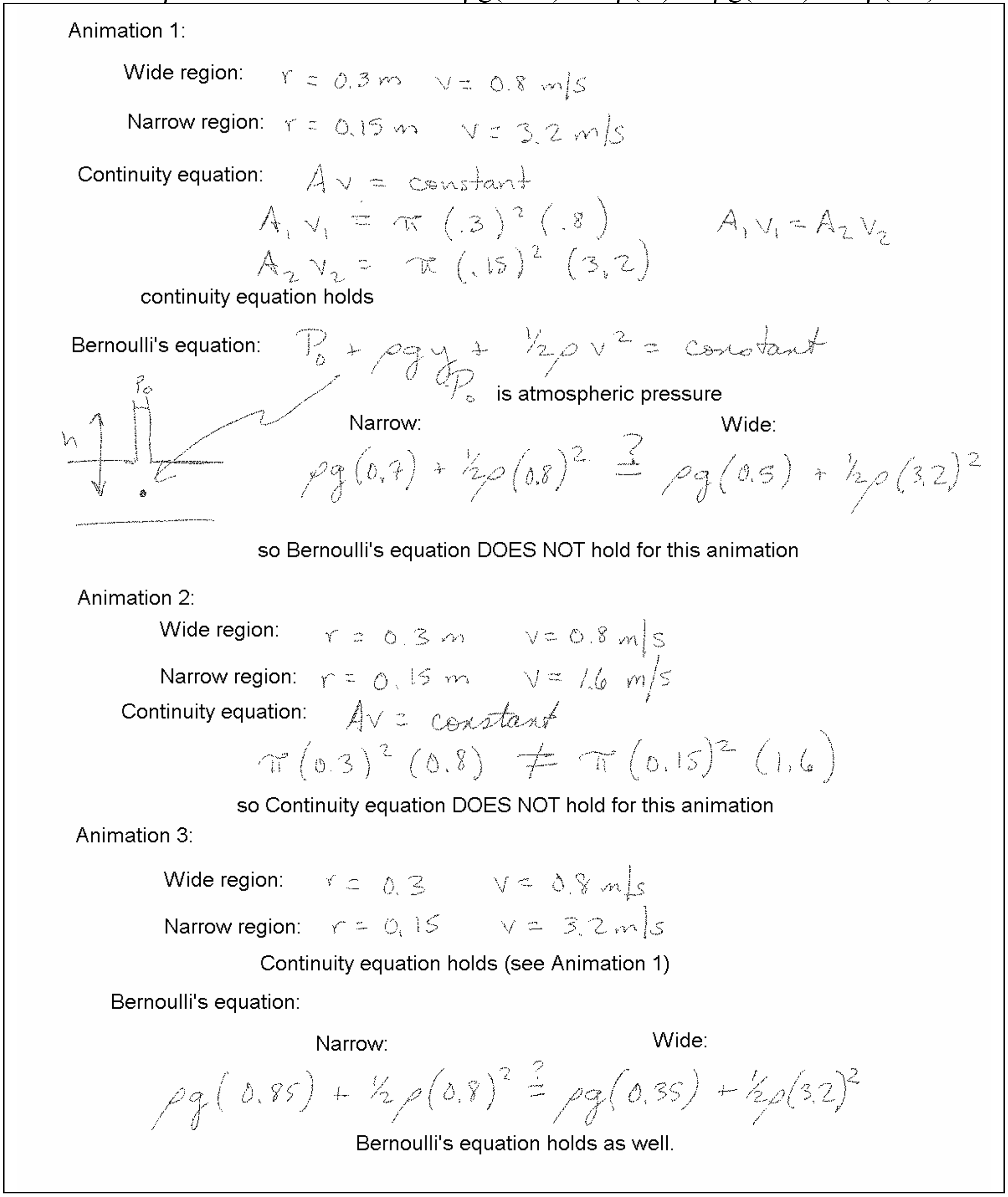
## P.15.7 (Level 3)

*Topic(s):* Bernoulli's equation, continuity equation

*Answer:*

**Animation 3 is physically possible.**

**Animation 1: Bernoulli's equation violated.** Wide region (r=3) v=0.8m/s; Narrow (r=1.5) v=3.2m/s so continuity (Av=constant) holds. P in pipe equals the ρgh in vertical tubes and $\rho g(0.7)+1/2\rho(.8)^2 \neq \rho g(0.5)+1/2\rho(3.2)^2$.
**Animation 2: Continuity equation violated**. Wide region (r=3) v=0.8m/s; Narrow (r=1.5) v=1.6m/s.
Animation C: Wide region (r=3) v=0.8m/s; Narrow (r=1.5) v=3.2m/s so continuity holds. $P+1/2\rho v^2=$constant is also true: $\rho g(0.85)+1/2\rho(.8)^2 = \rho g(0.35)+1/2\rho(3.2)^2$ .



Animation 1:

Wide region: $r = 0.3\ m$ $v = 0.8\ m/s$

Narrow region: $r = 0.15\ m$ $v = 3.2\ m/s$

Continuity equation: $Av = \text{constant}$

$$A_1 v_1 = \pi(.3)^2(.8) \qquad A_1 v_1 = A_2 v_2$$
$$A_2 v_2 = \pi(.15)^2(3.2)$$

continuity equation holds

Bernoulli's equation: $P_o + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$

$P_o$ is atmospheric pressure

Narrow: Wide:

$$\rho g(0.7) + \frac{1}{2}\rho(0.8)^2 \overset{?}{=} \rho g(0.5) + \frac{1}{2}\rho(3.2)^2$$

so Bernoulli's equation DOES NOT hold for this animation

Animation 2:

Wide region: $r = 0.3\ m$ $v = 0.8\ m/s$

Narrow region: $r = 0.15\ m$ $v = 1.6\ m/s$

Continuity equation: $Av = \text{constant}$

$$\pi(0.3)^2(0.8) \neq \pi(0.15)^2(1.6)$$

so Continuity equation DOES NOT hold for this animation

Animation 3:

Wide region: $r = 0.3$ $v = 0.8\ m/s$

Narrow region: $r = 0.15$ $v = 3.2\ m/s$

Continuity equation holds (see Animation 1)

Bernoulli's equation:

Narrow: Wide:

$$\rho g(0.85) + \frac{1}{2}\rho(0.8)^2 \overset{?}{=} \rho g(0.35) + \frac{1}{2}\rho(3.2)^2$$

Bernoulli's equation holds as well.

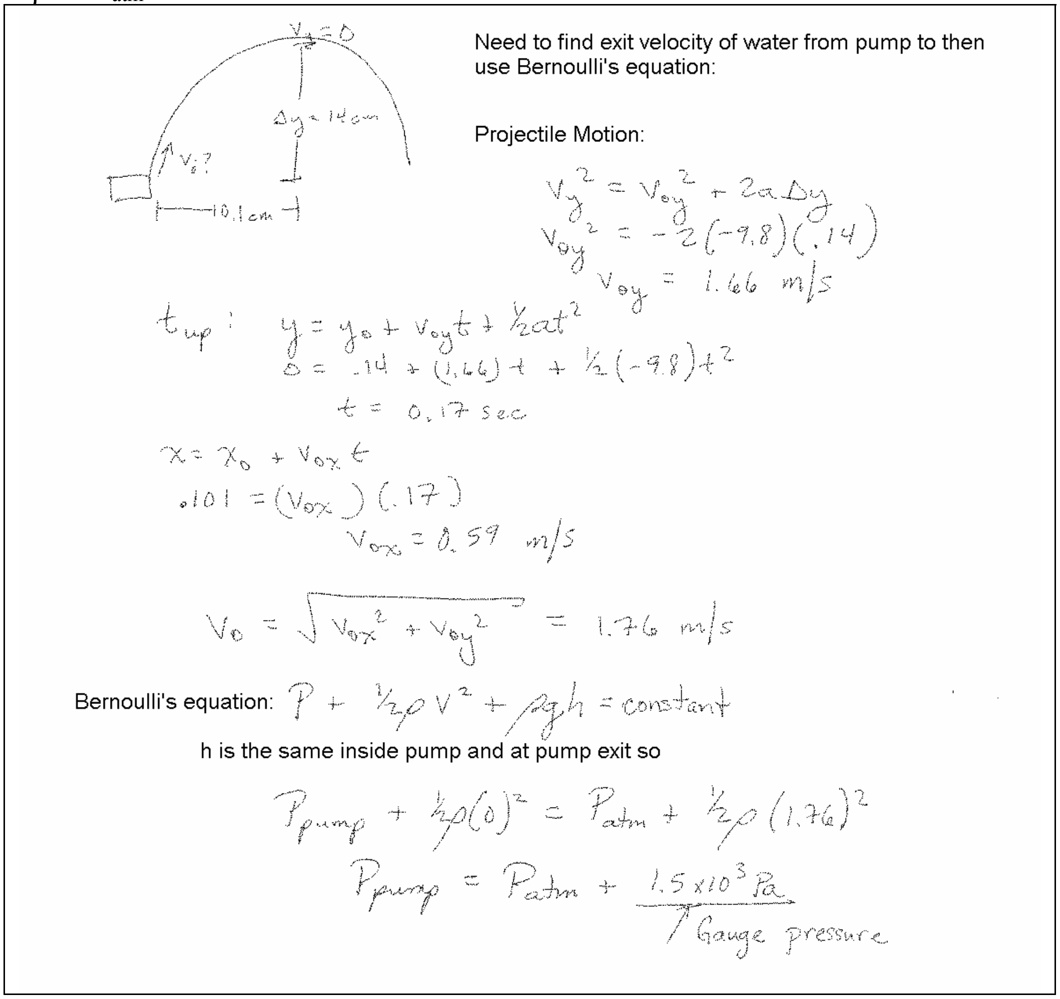
## P.15.8 (Level 1)

*Topic(s):* Bernoulli's equation

*Answer:*
a) **The water flowing between the boats flows faster than the water between the boat and back (narrower region). This means a lower pressure region between boats so the boats move together.**
b) **same thing happens**

## P.15.9 (Level 3)

*Topic(s):* Bernoulli’s equation, projectile motion
*Answer:*
**$1.5\text{x}10^3$ Pa.** Water maximum height of 0.141m so $v_{initial\_y}$=1.66m/s; $t_{up}$=0.17sec; $v_{initial\_x}$=0.59m/s for an initial v=1.76m/s out of pump: $P_{pump}=P_{atm}+1/2\rho v^2=P_{atm}+1.5\text{x}10^3$Pa.



Need to find exit velocity of water from pump to then use Bernoulli's equation:

Projectile Motion:

$$v_y^2 = v_{oy}^2 + 2a\Delta y$$
$$v_{oy}^2 = -2(-9.8)(.14)$$
$$v_{oy} = 1.66 \text{ m/s}$$

$t_{up}$ : $y = y_0 + v_{oy}t + \frac{1}{2}at^2$

$$0 = .14 + (1.66)t + \frac{1}{2}(-9.8)t^2$$
$$t = 0.17 \text{ sec}$$

$$x = x_0 + v_{ox}t$$
$$.101 = (v_{ox})(.17)$$
$$v_{ox} = 0.59 \text{ m/s}$$

$$V_0 = \sqrt{v_{ox}^2 + v_{oy}^2} = 1.76 \text{ m/s}$$

Bernoulli's equation: $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

h is the same inside pump and at pump exit so

$$P_{pump} + \frac{1}{2}\rho(0)^2 = P_{atm} + \frac{1}{2}\rho(1.76)^2$$
$$P_{pump} = P_{atm} + \underbrace{1.5\times10^3 \text{ Pa}}_{\text{Gauge pressure}}$$

## P.15.10 (Level 2)

*Topic(s):* Bernoulli’s equation
*Answer:*
**$\eta$=1.1 Pa-s.** v=320 cm/s=3.2m/s; R=0.15m; $Av=\pi R^4\Delta P/8\eta L$; $\Delta P=\rho g\Delta h$ at the two vertical tubes; L=0.7m (distance between tubes).

# Chapter II.16: Periodic Motion

## *Illustrations*

### Illustration 16.1: Representations of Simple Harmonic Motion

*Topic(s):* Periodic Motion
*Purpose:*. Introduction periodic motion representations.
*Suggested Use:* This illustration visually connects three representations of SHM including showing how a mass on a turntable can be in SHM. It would be a great demonstration when introducing the topic to help students make conceptual connections.

### Illustration 16.2: The Simple Pendulum and Spring Motion

*Topic(s):* Periodic Motion
*Purpose:* Introduction periodic motion representations.
*Suggested Use:* Reading assignment or used in-class when introducing SHM. Illustration is similar to Illustration 16.3.

### Illustration 16.3: Energy and Simple Harmonic Motion

*Topic(s):* Periodic Motion, Energy
*Purpose:* To introduce energy considerations in periodic motion.
*Suggested Use:* This illustration is similar to Illustration 16.2. Good visual demonstration when introducing concepts.

### Illustration 16.4: Forced and Damped Motion

*Topic(s):* Periodic Motion, Damping
*Purpose:*. To discuss damping in periodic motion.
*Suggested Use:* Reading assignment.

### Illustration 16.5: The Fourier Series, Qualitative Features

*Topic(s):* Periodic Motion, Fourier Series
*Purpose:* To introduce non sinusoidal periodic functions.
*Suggested Use:* Reading assignment or as an in-class visual when introducing Fourier Series.

### Illustration 16.6: The Fourier Series, Quantitative Features

*Topic(s):* Periodic Motion, Fourier Series
*Purpose:* To introduce Fourier series.
*Suggested Use:* This is a good illustration for students to do themselves as it involves a bit of “play”. Would be good to assign as a reading assignment in preparation for a class discussion on Fourier Series.

## Explorations

### Exploration 16.1 – Spring and Pendulum Motion

*Topic(s):* Periodic Motion, Springs

### Exploration 16.2 – Pendulum Motion and Energy

*Topic(s):* Periodic Motion, Energy

### Exploration 16.3 – Simple Harmonic Motion with and without Damping

*Topic(s):* Periodic Motion, Springs, Damping

### Exploration 16.4 – Pendulum Motion, Forces, and Phase Space

*Topic(s):* Periodic Motion, Phase Space Representation

### Exploration 16.5 – Driven Motion and Resonance

*Topic(s):* Periodic Motion, Springs, Driving Forces

### Exploration 16.6 – Damped and Forced Motion

*Topic(s):* Periodic Motion, Springs, Damping

### Exploration 16.7 – A Chain of Oscillators

*Topic(s):* Periodic Motion, Springs, Damping, Driving Forces

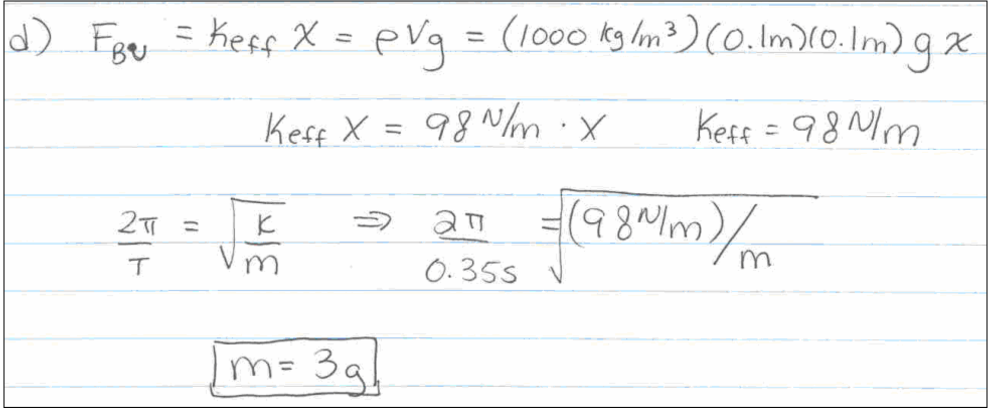
## Problems

### P.16.1 (Level 1)

*Topic(s):* Periodic Motion, Springs
*Answer:* **a) x = -9.8 m to x = 8.3 m, b) x = 13 m, c) 6 N/m, d) 0.95 kg, e) x=-9.8m to 8.3m**

### P.16.2 (Level 1)

*Topic(s):* Periodic Motion, Springs
*Answer:* **a) Anim 1, b) 6.3 s and 0.16 Hz, c) –1.6 Cos (2Pi(0.16)t), d) 2 N/m**



d) $F_{Bu} = k_{eff}\,x = \rho V g = (1000\ \mathrm{kg/m^3})(0.1\mathrm{m})(0.1\mathrm{m})\,g\,x$

$K_{eff}\,x = 98\ \mathrm{N/m}\cdot x \qquad K_{eff} = 98\ \mathrm{N/m}$

$\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow \frac{2\pi}{0.35\mathrm{s}} = \sqrt{(98\ \mathrm{N/m})/m}$

$\boxed{m = 3g}$

## P.16.3 (Level 1)

*Topic(s):* Periodic Motion, Springs
*Answer:* **a) Anim 3, b) 1.6 Sin(2Pi*0.16t), c) 1.3 J**

## P.16.4 (Level 1)

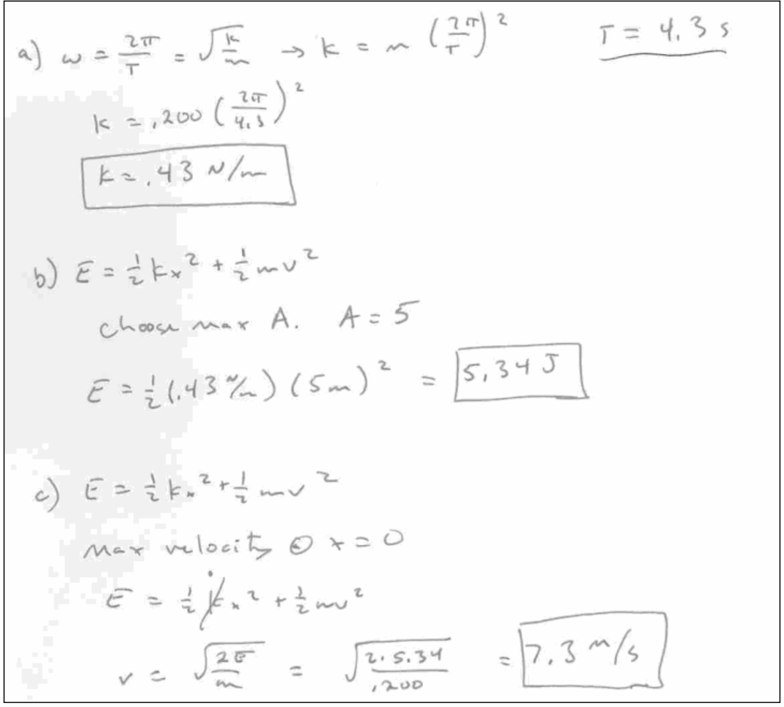
*Topic(s):* Periodic Motion, Springs
*Answer:* **No, this is most easily seen in Anim 5 which shows that the force is not a linear restoring force as required by SHM**

## P.16.5 (Level 1)

*Topic(s):* Periodic Motion, Springs
*Answer:* **0.5 N/m**

## P.16.6 (Level 2)

*Topic(s):* Periodic Motion, Springs, Energy
*Answer:* **a) 0.43 N/m, b) 5.3 J, c) 7.3 m/s**



a) $\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \rightarrow k = m\left(\frac{2\pi}{T}\right)^2$ $\underline{T = 4.3\ s}$

$k = .200\left(\frac{2\pi}{4.3}\right)^2$

$\boxed{k = .43\ N/m}$

b) $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

choose max A. A = 5

$E = \frac{1}{2}(.43\ N/m)(5m)^2 = \boxed{5.34\ J}$

c) $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

max velocity @ x = 0

$E = \frac{1}{2}\cancel{k}x^2 + \frac{1}{2}mv^2$

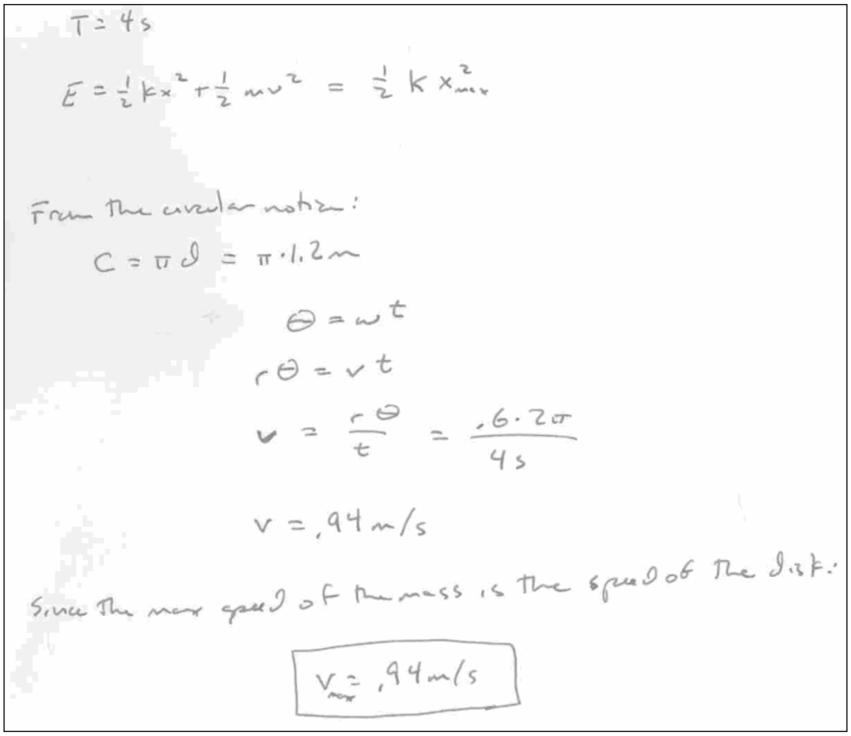
$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\cdot 5.34}{.200}} = \boxed{7.3\ m/s}$

## P.16.7 (Level 1)

*Topic(s):* Periodic Motion
*Answer:* **Animation 3**

## P.16.8 (Level 2)

*Topic(s):* Periodic Motion
*Answer:* **0.9 m/s**



$T = 4s$

$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2$

From the circular motion:

$C = \pi d = \pi \cdot 1.2m$

$\theta = \omega t$

$r\theta = vt$

$v = \frac{r\theta}{t} = \frac{.6 \cdot 2\pi}{4s}$

$v = .94 m/s$

Since the max speed of the mass is the speed of the disk:

$\boxed{v_{max} = .94 m/s}$

## P.16.9 (Level 2)

*Topic(s):* Periodic Motion
*Answer:* **a) Anim 1, b) Anim 3, c) Anim 2, d) –Cos(1.8t), e) Sin(1.8t), f) Cos(1.8t), g) 1.8-1.8x (found by using energy conservation)**

## P.16.10 (Level 2)

*Topic(s):* Periodic Motion
*Answer:* **3.3 m/s^2**
L=3m, T=6s

## P.16.11 (Level 2 - Calculus)

*Topic(s):* Periodic Motion
*Answer:* **Anim 4**
The acceleration is proportional to r

## P.16.12 (Level 3)

*Topic(s):* Periodic Motion, Buoyant Force
*Answer:* **a) The weight must equal the buoyant force, b) weight of cube – buoyant force, c) 0.035 s, d) 3g**
Hint for (d) – Keff = Fbuoyant/x=density*g*area=98N, then use m = k*(T/2Pi)^2.

# Chapter II.17: Waves

## *Illustrations*

### Illustration 17.1: Wave Types

*Topic(s):* Wave Motion
*Purpose:*. Introduction to the types of waves.
*Suggested Use:* This is a good illustration to use when first introducing wave types because it provides a strong visual representation. If used in class, draw students attention to the red dot and perhaps have them predict both its motion and the shape of the graph before showing them.

### Illustration 17.2: Wave Functions

*Topic(s):* Wave Motion
*Purpose:* Introduction mathematical representations of waves.
*Suggested Use:* Reading assignment.

### Illustration 17.3: Superposition of Pulses

*Topic(s):* Wave Motion, Superposition
*Purpose:* To introduce wave superposition.
*Suggested Use:* A must for in-class use This is a Physlet representation of the classic superposition animation that is often shown when discussing interference of waves and pulses.

### Illustration 17.4: Superposition of Traveling Waves

*Topic(s):* Wave Motion, Superposition, Standing Waves
*Purpose:*. To show the effects of wave superposition.
*Suggested Use:* Reading assignment, In-Class Demo. If used in-class have students predict what the superposition will look like once the two waves meet. Most students will be surprised by the result.

### Illustration 17.5: Resonant Behavior on a String

*Topic(s):* Wave Motion, Superposition, Resonance
*Purpose:* To show resonant behavior.
*Suggested Use:* Reading assignment, In-Class Demo. This makes a great visualization for resonance behavior and is effective in the classroom.

### Illustration 17.6: Plucking a String

*Topic(s):* Harmonics
*Purpose:* To introduce harmonics.
*Suggested Use:* Reading assignment.

### Illustration 17.7: Group and Phase Velocity

*Topic(s):* Wave Motion, Group and Phase Velocity
*Purpose:* To introduce the concepts of group and phase velocity.
*Suggested Use:* Reading assignment. This is a good illustration to have students look at on their own as there is a lot of "playing around" that they can do with it.

## *Explorations*

### Exploration 17.1 – Superposition of Two Pulses

*Topic(s):* Wave Motion, Superposition

### Exploration 17.2 – Measure the Properties of a Wave

*Topic(s):* Wave Motion

### Exploration 17.3 – Traveling Pulses and Barriers

*Topic(s):* Wave Motion, Reflection

### Exploration 17.4 – Superposition of Two Waves

*Topic(s):* Wave Motion, Superposition

### Exploration 17.5 – Superposition of Two Waves

*Topic(s):* Wave Motion, Superposition

### Exploration 17.6 – Make a Standing Wave

*Topic(s):* Wave Motion, Superposition, Standing Waves

## *Problems*

### P.17.1 (Level 1)

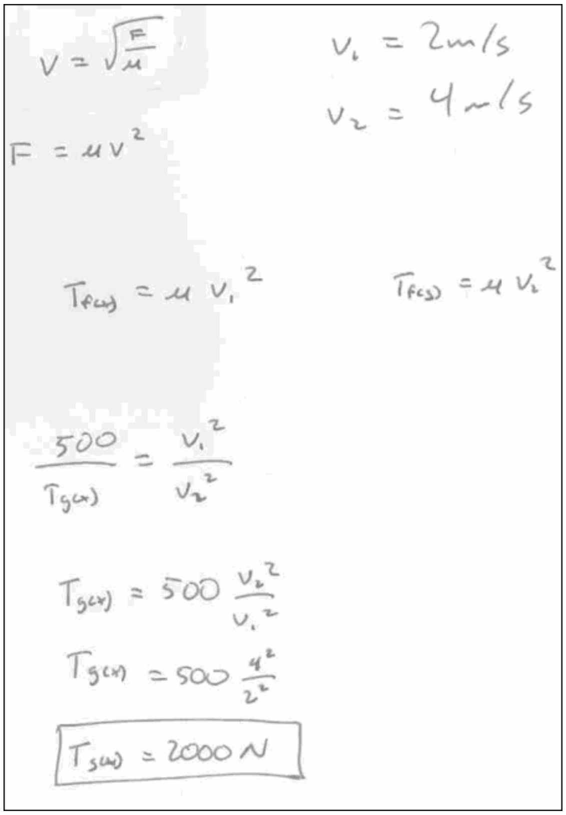
*Topic(s):* Wave Motion
*Answer:* **0.5 Hz**

### P.17.2 (Level 1)

*Topic(s):* Wave Motion
*Answer:* **8 cm/s**

### P.17.3 (Level 2)

*Topic(s):* Wave Motion
*Answer:* **2000 N**



$V = \sqrt{\frac{F}{\mu}}$

$v_1 = 2m/s$

$v_2 = 4m/s$

$F = \mu v^2$

$T_{f(x)} = \mu v_1^2$

$T_{f(s)} = \mu v_2^2$

$$\frac{500}{T_{g(x)}} = \frac{v_1^2}{v_2^2}$$

$$T_{g(x)} = 500 \frac{v_2^2}{v_1^2}$$

$$T_{g(x)} = 500 \frac{4^2}{2^2}$$

$$\boxed{T_{s(x)} = 2000 N}$$

## P.17.4 (Level 1)

*Topic(s):* Wave Motion, Superposition
*Answer:* **only 1 is true**

## P.17.5 (Level 1)

*Topic(s):* Wave Motion, Superposition
*Answer:* **only 4 is true**

## P.17.6 (Level 1)

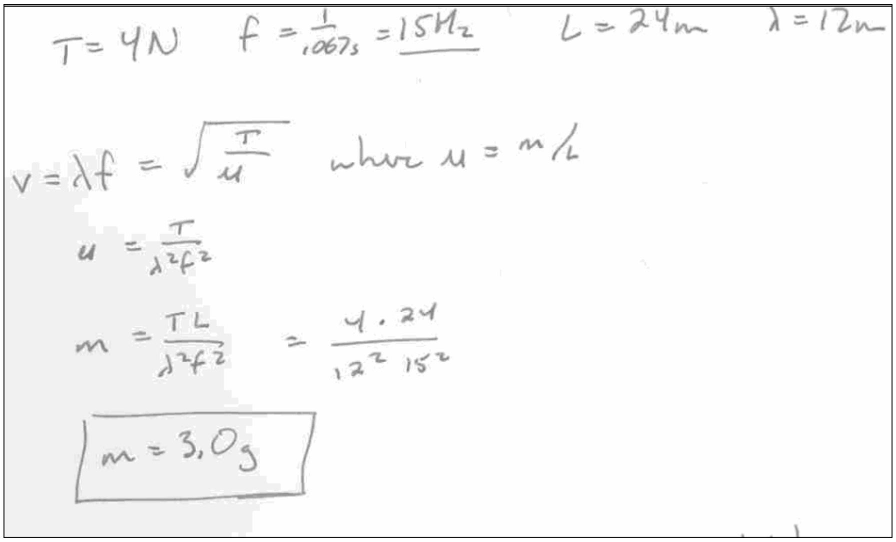
*Topic(s):* Wave Motion, Standing Waves
*Answer:* **only C is true**

## P.17.7 (Level 2)

*Topic(s):* Wave Motion, Standing Waves
*Answer:* **2 cm/s**
Use v = lamda*f

## P.17.8 (Level 2)

*Topic(s):* Wave Motion, Standing Waves
*Answer:* **3g**



$T = 4N \quad f = \frac{1}{.067s} = \underline{15Hz} \quad L = 24m \quad \lambda = 12m$

$v = \lambda f = \sqrt{\frac{T}{\mu}}$ where $\mu = m/L$

$\mu = \frac{T}{\lambda^2 f^2}$

$m = \frac{TL}{\lambda^2 f^2} = \frac{4 \cdot 24}{12^2\ 15^2}$

$\boxed{m = 3.0g}$

## P.17.9 (Level 1)

*Topic(s):* Wave Motion
*Answer:* **a) a = period, b = phase, c = amplitude, b) T = 3s, phase=1rad, A = 2.1 m**

## P.17.10 (Level 1)

*Topic(s):* Wave Motion
*Answer:* **a) 0.5 Hz, 2.65 cm, 2s, c) 4*Cos[2*Pi*(x/2.65+t/2)]**

## P.17.11 (Level 2)

*Topic(s):* Standing Waves, Wave Motion
*Answer:* **a) 4 m/s, c) 1.8 Cos (2*Pi/2*t), d) 2.5 Sin(Pi/4*x – Pi*t) + 2.5 Sin(Pi/4*x+Pi*t)**

## P.17.12 (Level 1)

*Topic(s):* Standing Waves, Wave Motion
*Answer:* **a) For F(x,t) … 0.7 m/s, 4m, 1/6 Hz, For g(x,t) … -0.7 m/s, 4m, 1/6 Hz, b) 4m, 1/6 Hx, 0 m/s**

## P.17.13 (Level 1)

*Topic(s):* Wave Motion
*Answer:* **y(x,t) = 2.3 Sin [2*Pi(x/7.2-t/3.8)]cm**

## P.17.14 (Level 1)

*Topic(s):* Standing Waves, Wave Motion
*Answer:* **a) 2 m/s, b) Sin[2*Pi(x/8-t/4)]+Sin[2*Pi(x/8+t/4)]**

## P.17.15 (Level 2)

*Topic(s):* Standing Waves, Wave Motion
*Answer:* **a) 0.5 Hx, 2s, 1.6 m, b) y(0,t) = 5Cos(Pi*t), y(2,t)=5Cos[2*Pi(2/1.6+t/2)], c) at x=0, v=-5Sin(Pi*t), at x=2 v=-5Sin(7.85+Pi*t)**

## P.17.16 (Level 2)

*Topic(s):* Wave Motion
*Answer:* **a) no, b) Sin(2*Pi*t/5.3), c) only the phase is different, d) too few markers can make the wavelength seem too long, d) Sin(2.3x-1.2t-1.3)**

## P.17.17 (Level 1)

*Topic(s):* Standing Waves, Wave Motion
*Answer:* **a) sketch is mirror image of the wave form, b) sketch is shifted in time**

# Chapter II.18: Sound

## *Illustrations*

### Illustration 18.1: Representations of 2-d Waves

*Topic(s):* Wave Motion
*Purpose:*. To show students how waves can be represented in two dimensions.
*Suggested Use:* Reading Assignment

### Illustration 18.2: Molecular View of a Sound Wave

*Topic(s):* Sound Waves
*Purpose:* To show students why sound is considered to be a wave.
*Suggested Use:* This is a great Physlet to show students during class when introducing sound waves. Use it to emphasize to students that a sound wave is merely a wave of molecular vibrations. It also clearly shows why sound is a longitudinal wave.

### Illustration 18.3: Interference in Time and Beats

*Topic(s):* Superposition, Beats
*Purpose:* To show the addition of sound waves.
*Suggested Use:* Reading assignment.

### Illustration 18.4: Doppler Effect

*Topic(s):* Doppler Effect
*Purpose:*. To visually demonstrate the relationship between sound sources and receivers when one and/or both are moving.
*Suggested Use:* This is another Physlet that makes a great in-class demonstration. It is helpful when explaining to students why there is a Doppler effect. Show the animations as you introduce the Doppler effect in order to build a conceptual understanding before mathematics are introduced.

### Illustration 18.5: The Location of a Supersonic Airplane

*Topic(s):* Sonic Boom
*Purpose:* To show why a supersonic airplane produces a sonic boom.
*Suggested Use:* Use this as an in-class demonstration when talking about sonic booms. It can help students to conceptually and intuitively the reasons behind the boom.

## *Explorations*

### Exploration 18.1 – Creating Sounds by Adding Harmonics

*Topic(s):* Superposition, Harmonics

### Exploration 18.2 – Creating Sounds by Adding Harmonics

*Topic(s):* Superposition, Harmonics, Electronic Music

### Exploration 18.3 – A Microphone between Two Loudspeakers

*Topic(s):* Beats

### Exploration 18.4 – Doppler Effect and the Velocity of the Source

*Topic(s):* Doppler Effect

### Exploration 18.5 – An Ambulance Drives by with its Siren on

*Topic(s):* Doppler Effect

## *Problems*

### P.18.1 (Level 1)

*Topic(s):* Sound Waves
*Answer:* **0.35 m/s**

### P.18.2 (Level 1)

*Topic(s):* Sound Waves
*Answer:* **Anim 1**

### P.18.3 (Level 1)

*Topic(s):* Superposition
*Answer:* **When two are more sources are present there is interference which leads to dead spots.**

### P.18.4 (Level 1)

*Topic(s):* Sound Waves
*Answer:* **3.4 m/s**
T = 0.01s, f=100 Hz

### P.18.5 (Level 2)

*Topic(s):* Sound Waves
*Answer:* **71 Hz**
T = 0.02 s, f = 50 Hz

### P.18.6 (Level 1)

*Topic(s):* Beats
*Answer:* **4 Hz**

### P.18.7 (Level 2)

*Topic(s):* Doppler Effect
*Answer:* **a) Ambulance Driver, b) Woman, c) Man**

### P.18.8 (Level 2)

*Topic(s):* Doppler Effect

*Answer:* **a) 2 and 3, b) All travel at the speed of sound.**

## P.18.9 (Level 1)

*Topic(s):* Doppler Effect
*Answer:* **Animation 3**

## P.18.10 (Level 2)

*Topic(s):* Doppler Effect
*Answer:* **47 Hz**

$X_0 = 2.42$

$X_{t=0.4} = -1.59$

So $V = \frac{\Delta X}{\Delta t} = 10\,\text{m/s}$

$f_1 = (1 + \frac{V_0}{V}) f$

$f_2 = (1 - \frac{V_0}{V}) f$

$$\Delta f = f_2 - f_1 = f\left[1 - \frac{V_0}{V} - 1 - \frac{V_0}{V}\right] = -\frac{2fV_0}{V}$$

$$|\Delta f| = \frac{21800\,\text{Hz})(10\,\text{m/s})}{343\,\text{m/s}} = \boxed{47\,\text{Hz}}$$

## P.18.11 (Level 1)

*Topic(s):* Standing Waves
*Answer*: **a) 0, -2cm, 2cm, antinode, b) –0.1cm, 0.1cm, 0.2cm, -0.2cm, node c) 0.2 cm, d) 800 Hz, e) 1.6 m/s**

## P.18.12 (Level 1)

*Topic(s):* Harmonics
*Answer:* **a) 3rd Harmonic, b) 2000 Hz, c) 666 Hz**

## P.18.13 (Level 2)

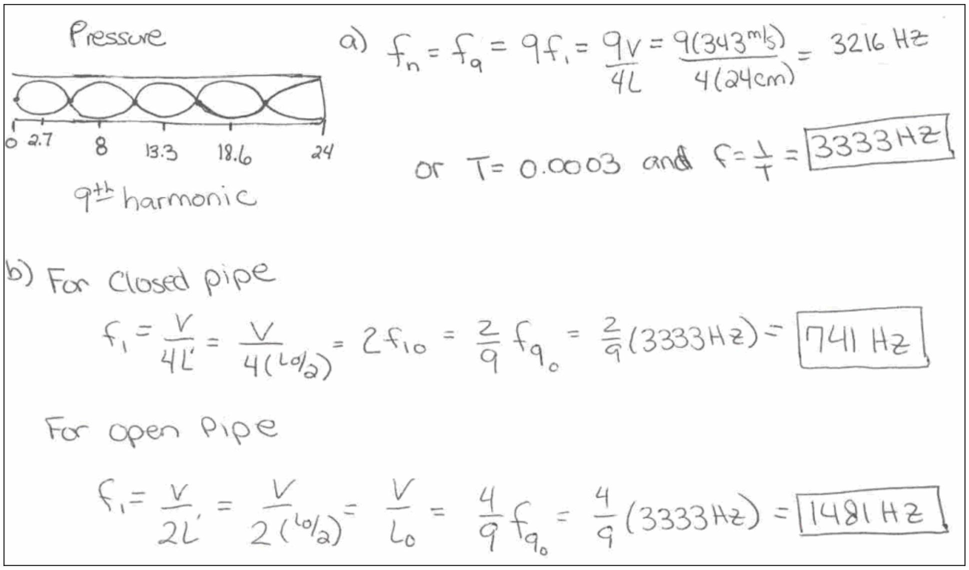
*Topic(s):* Harmonics
*Answer:* **a) 5th Harmonic, b) 333 Hz, c) 533 Hz, d) 32 m/s**

## P.18.14 (Level 2)

*Topic(s):* Harmonics
*Answer:* **a) 7th Harmonic, b) 250 Hz, c) 36 Hz, d) 35 m/s**

## P.18.15 (Level 2)

*Topic(s):* Harmonics
*Answer:* **a) 3333 Hz, b) 741 Hz for closed pipe and 1481 Hz open end pipe**



a) $f_n = f_9 = 9f_1 = \frac{9v}{4L} = \frac{9(343 m/s)}{4(24cm)} = 3216 Hz$

or $T = 0.0003$ and $f = \frac{1}{T} = \boxed{3333 Hz}$

b) For Closed pipe

$$f_1 = \frac{V}{4L'} = \frac{V}{4(L_0/2)} = 2f_{1_0} = \frac{2}{9} f_{9_0} = \frac{2}{9}(3333 Hz) = \boxed{741 Hz}$$

For open Pipe

$$f_1 = \frac{V}{2L'} = \frac{V}{2(L_0/2)} = \frac{V}{L_0} = \frac{4}{9} f_{9_0} = \frac{4}{9}(3333 Hz) = \boxed{1481 Hz}$$

## P.18.16 (Level 1)

*Topic(s):* Harmonics
*Answer:* **a) 5th Harmonic, b) 19 cm**

# Chapter II.19: Heat

## *Illustrations*

### Illustration 19.1: Specific Heat

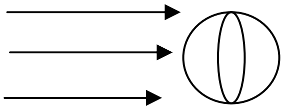
*Topic(s):* specific heat
*Purpose:* To demonstrate the relationship between heat, temperature and specific heat.
*Suggested Use:* Have students compare the high heat and low heat cases. Students should be able to predict what will happen to the change in temperature (for materials exposed to heaters for the same amount of time) as the mass increases. It is important for students to recognize that the temperature change depends on the heat added and the mass of the object, but that the specific heat capacity is a constant (at least for the problems they will encounter). As an extension, if students use $Q=Pt$, vary mass and $\Delta T$ (temp change) and $Q=mc\Delta T$ they will find that c=1000 J/kg*K

### Illustration 19.2: Heat Transfer, Conduction

*Topic(s):* conduction
*Purpose:* To demonstrate the effect of various parameters on the conductivity of a wall.
*Suggested Use:* Students can change the conductivity, thickness and temperature difference across a wall to see the power transfer across the wall. They can use this model to develop the relationship between power and these factors ($P \sim k\Delta T/(\text{thickness})$). As an extension, students can use $P=kA\Delta T/L$ and find that the cross-sectional area is 0.1.

### Illustration 19.3: Heat Transfer, Radiation

*Topic(s):* radiation, heat balance
*Purpose:* To illustrate transfer of heat by radiation applied to estimating planet temperatures.
*Suggested Use*: This very simple model allows students to see the temperature of various planets if the atmosphere of the planets is neglected. For this model, $P_{in}=P_{out}$ and the planet effectively absorbs radiation across an area of $\pi R^2$ (P/A)*Acosϕ



but radiates back out over the surface area of a sphere: $4\ \pi R^2$. The radiation from the sun drops off as $1/r^2$ and the number used for the Earth is the value at the top of the atmosphere. Students can see the predictions of this model and then get a sense of the impact of the atmosphere (and the Greenhouse effect). It provides an application of the ideas of heat transfer.

## *Explorations*

### Exploration 19.1

*Topic(s):* specific heat, work, energy

### Exploration 19.2

*Topic(s):* thermal expansion

### Exploration 19.3

*Topic(s):* calorimetry

### Exploration 19.4

*Topic(s):* conduction, heat balance

## *Problems*

### P.19.1 (Level 2)

*Topic(s):* specific heat, work, energy, calorimetry

*Answer:*

$3.04 \text{x} 10^3$ J/kg*K. $V=1\text{x}1\text{x}0.1=0.1\text{m}^3$; m=92-kg; $Q=cm\Delta T$
W=6860J so c=3043 J/kg*K.

### P.19.2 (Level 2)

*Topic(s):* friction, power, calorimetry

*Answer:*

$\mu=0.6$. $P=Q/t=cm\Delta T/t=11.8$*mass
$P_{\text{friction}}=F*v=\mu Nv=\mu mgv$; v of conveyor belt=2 m/s so $P=\mu g*2$.

### P.19.3 (Level 2)

*Topic(s):* thermal expansion

*Answer:*

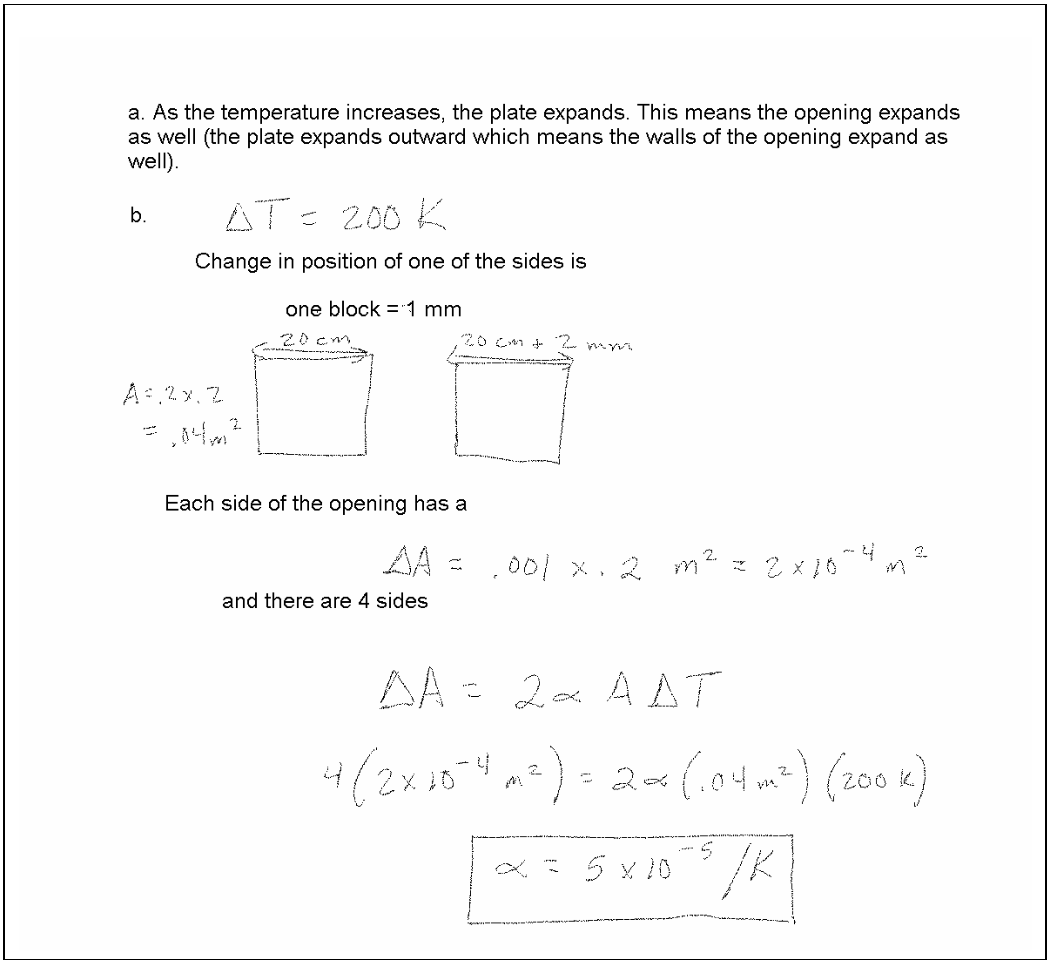
$\alpha=1.1\text{x}10^{-4}$/K. Note that the rod expands in both directions—measure $\Delta L$ for one end, but recognize that must double that to find total $\Delta L$—for L=20m and final T=1000, find a $\Delta L$ of 9.5 units x 0.1m for a total $\Delta L=1.8$m**.**

### P.19.4 (Level 3)

*Topic(s):* thermal expansion

*Answer:*

$\alpha=5\text{x}10^{-5}$/K. T is increasing (entire plate expands so hole gets bigger as well). From t=0 to t=2, hole expands 1 mm in each dimension. This gives a $\Delta A$ of .001mx0.2m for each side and there are **4** sides for a total $\Delta A=8\text{x}10^{-4}$m. Using, $\Delta A=2\alpha A\Delta T$ gives the value for $\alpha$.



a. As the temperature increases, the plate expands. This means the opening expands as well (the plate expands outward which means the walls of the opening expand as well).

b. $\Delta T = 200\ K$

Change in position of one of the sides is

one block = 1 mm

Each side of the opening has a

$$\Delta A = .001 \times .2\ m^2 = 2 \times 10^{-4}\ m^2$$

and there are 4 sides

$$\Delta A = 2\alpha A \Delta T$$

$$4(2 \times 10^{-4}\ m^2) = 2\alpha(.04\ m^2)(200\ K)$$

$$\boxed{\alpha = 5 \times 10^{-5}/K}$$

## P.19.5 (Level 2)

Topic(s): thermal expansion

*Answer:*

$\alpha=2\text{x}10^{-5}$/K. Change in temperature=1000K. Initial diameter of hole=19.6mm (r =9.8mm) and final diameter of hole=20mm (r =10 mm). A of hole is $\pi R^2$ and using $\Delta A=2\alpha A\Delta T$, gives value for $\alpha$.

## P.19.6 (Level 2)

*Topic(s):* calorimetry

*Answer:*

c=130 J/kg*K and L=22.8-kJ/kg (Material: Lead). For t=3.1 to 12.6, T=597K. Q=Pt=22,800 for that time and Q=mL so L=22.8-kJ/kg. From t=0 to t=3.1 $\Delta T=57$ and Q=2400*3.1 so c=130 J/kg*K.

## P.19.7 (Level 2)

*Topic(s):* calorimetry

*Answer:*

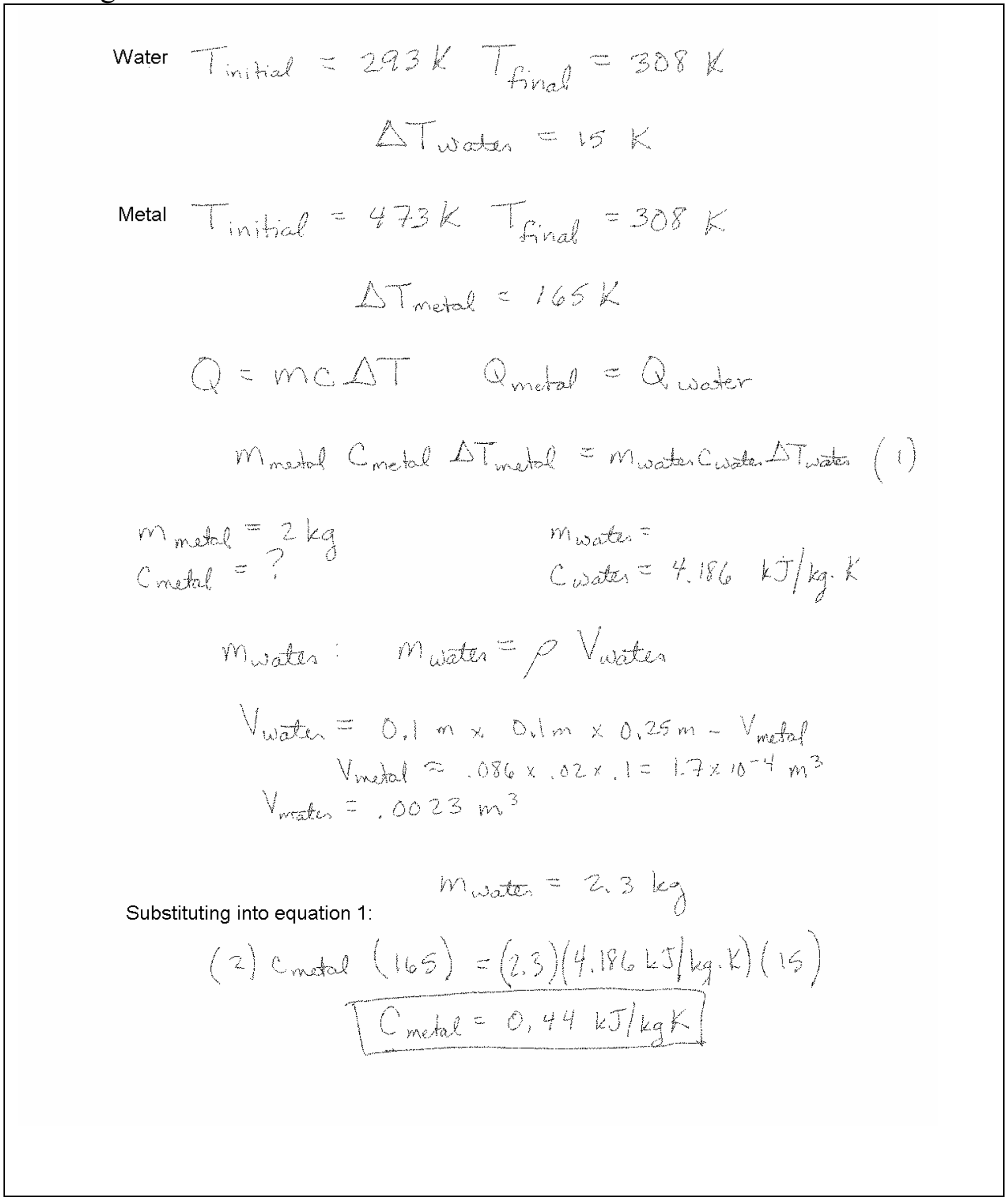
Specific heat of material 3<material 2< material 1. The final temperature of material one is the greatest and the final temperature of material three is the smallest. Since c is a measure of the heat required to change the temperature, material three requires the least heat from the water (water temp increases the least) and has the greatest change in temperature.

## P.19.8 (Level 2)

*Topic(s):* calorimetry

*Answer:*

c=0.44 kJ/kg-K.



Water $T_{initial} = 293\,K$ $T_{final} = 308\,K$

$$\Delta T_{water} = 15\,K$$

Metal $T_{initial} = 473\,K$ $T_{final} = 308\,K$

$$\Delta T_{metal} = 165\,K$$

$Q = mc\Delta T$ $Q_{metal} = Q_{water}$

$$m_{metal}\, C_{metal}\, \Delta T_{metal} = m_{water} C_{water} \Delta T_{water} \quad (1)$$

$m_{metal} = 2\,kg$ $m_{water} =$

$C_{metal} = ?$ $C_{water} = 4.186\ kJ/kg\cdot K$

$m_{water}$ : $m_{water} = \rho\, V_{water}$

$V_{water} = 0.1\,m \times 0.1\,m \times 0.25\,m - V_{metal}$

$V_{metal} \approx .086 \times .02 \times .1 = 1.7 \times 10^{-4}\,m^3$

$V_{water} = .0023\,m^3$

$m_{water} = 2.3\,kg$

Substituting into equation 1:

$$(2)\, C_{metal}\, (165) = (2.3)(4.186\,kJ/kg\cdot K)(15)$$

$$\boxed{C_{metal} = 0.44\ kJ/kgK}$$

## P.19.9 (Level 2)

*Topic(s):* conductivity

*Answer:*

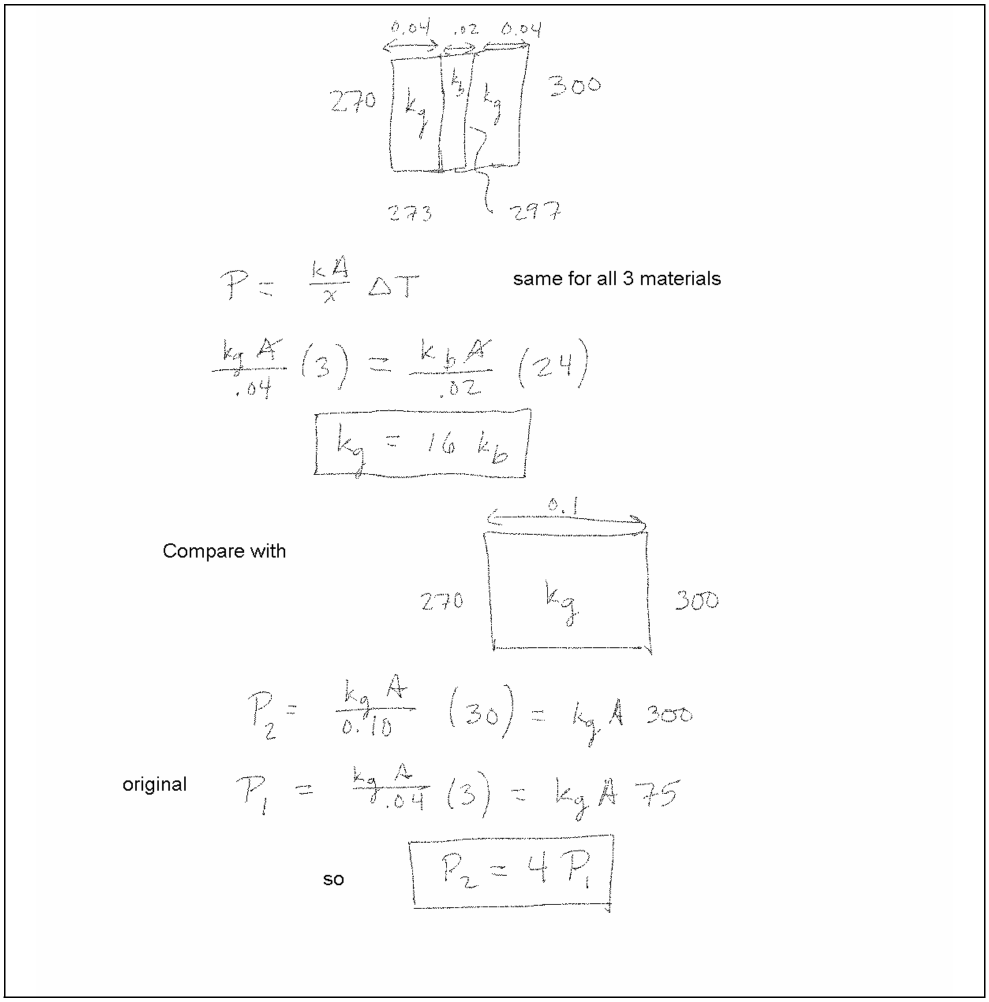
k-green>k-red. When Temp1=200 and Temp2=400, interface=320. Use P=(kA/x) ΔT and equate the power through green and red. x for the green material is 4 and 2 for the red.

## P.19.10 (Level 3)

*Topic(s):* conductivity

Answer:

k-green=16*k-blue. If you replace the window with glass (green) of same thickness (x=10), use 4 times as much power. Use P=(kA/x) ΔT. When outside temp=270, T1=273, T2=297. x-green=4 and x-red=2. Equate power across the materials:



$$P = \frac{kA}{x}\Delta T$$ same for all 3 materials

$$\frac{k_g \cancel{A}}{.04}(3) = \frac{k_b \cancel{A}}{.02}(24)$$

$$\boxed{k_g = 16 k_b}$$

Compare with

$$P_2 = \frac{k_g A}{0.10}(30) = k_g A\, 300$$

original $P_1 = \frac{k_g A}{.04}(3) = k_g A\, 75$

so $\boxed{P_2 = 4 P_1}$

## P.19.11 (Level 2)

*Topic(s):* conductivity, heat balance

Answer:

k=0.15 W/m*K. Room temp=0, P=2.6kW. P=(kA/x) ΔT. A=$\pi r^2 h$ where r=1.5m and h=5m and x=0.1m.

# Chapter II.20: Kinetic Theory and Ideal Gas Law

## *Illustrations*

### Illustration 20.1: Maxwell-Boltzmann Distribution

*Topic(s):* Maxwell-Boltzmann distribution, kinetic theory
*Purpose:* To show that gas particles have a speed distribution.
*Suggested Use:* Changing the temperature of the gas in the box allows students to quickly see the speed distribution and compare the histogram of the speeds with the Maxwell-Boltzmann distribution function as well as the average, rms and mean speeds.

### Illustration 20.2: Kinetic Theory, Temperature, and Pressure

*Topic(s):* kinetic theory, ideal gas law
*Purpose:* To connect microscopic quantities: kinetic energy and momentum with macroscopic quantities: temperature and pressure.
*Suggested Use:* Students should compare both the speeds and the kinetic energies of the blue and yellow particles. Have students predict what will happen as the temperature is increased. Finally, it is worth having students think about the assumptions in developing the ideal gas law. Here they can see the effect of finite particle size (since the ideal gas law assumes a point particle) where the effective volume is reduced because the particles are not point-like.

### Illustration 20.3: Thermodynamic Processes

*Topic(s):* pV diagrams, thermodynamic processes
*Purpose:* To connect pV diagrams with a microscopic model of gas particles in a container and provide a comparison between the typical thermodynamic processes that students will encounter.
*Suggested Use*: Allow students the chance to compare the different processes. What do they notice about them? Why do they think we pick out these particular ones (as opposed to the unnamed process)?

### Illustration 20.4: Evaporative Cooling

*Topic(s):* kinetic theory, Maxwell-Boltzmann distribution
*Purpose:* To demonstrate why evaporation is a cooling process.
*Suggested Use*: Have students explain why the temperature changes the way that it does when they allow particles to pass through the membrane. Why does this help explain why evaporation is a cooling process? Note that the membrane requires the use of a "Maxwell's demon."

## *Explorations*

### Exploration 20.1

*Topic(s):* kinetic theory, ideal gas law, Maxwell-Boltzmann distribution

## Exploration 20.2

*Topic(s):* kinetic theory, partial pressure

## Exploration 20.3

*Topic(s):* ideal gas law

## Exploration 20.4

*Topic(s):* equipartition theorem, degrees of freedom

## Exploration 20.5

*Topic(s):* work, pV diagram, thermodynamic processes, 1st law of thermodynamics

## Exploration 20.6

*Topic(s):* specific heat, 1st law of thermodynamics

## *Problems*

### P.20.1 (Level 2)

*Topic(s):* kinetic theory, ideal gas law

*Answer:*

**T=122. P=13.5.** <P>=.57 (ideal gas law: P=.54); all the same speed so T same: T=122; Ideal gas law: P=13.5.

### P.20.2 (Level 1)

*Topic(s):* kinetic theory

*Answer:*

**right wall T > left wall T**

### P.20.3 (Level 2)

*Topic(s):* kinetic theory, Maxwell-Boltzmann distribution

*Answer:*

**green > pink > yellow.** When T is same on both sides the KE is the same so look at Maxwell-Boltzmann distribution to compare the speeds and by extension (KE= ½ $mv^2$), the masses.

### P.20.4 (Level 2)

*Topic(s):* equipartition theorem, degrees of freedom, kinetic theory

*Answer:*

**pink=total energy; green=KE of atom; red=KE of molecule; black=translational KE of molecule; blue=rotational KE of molecule.** KE of atom+molecule=total KE and KE of translation and rotation of molecule=total KE of molecule. KE rotation (1 degree of freedom) < KE translation (2 degrees of freedom).

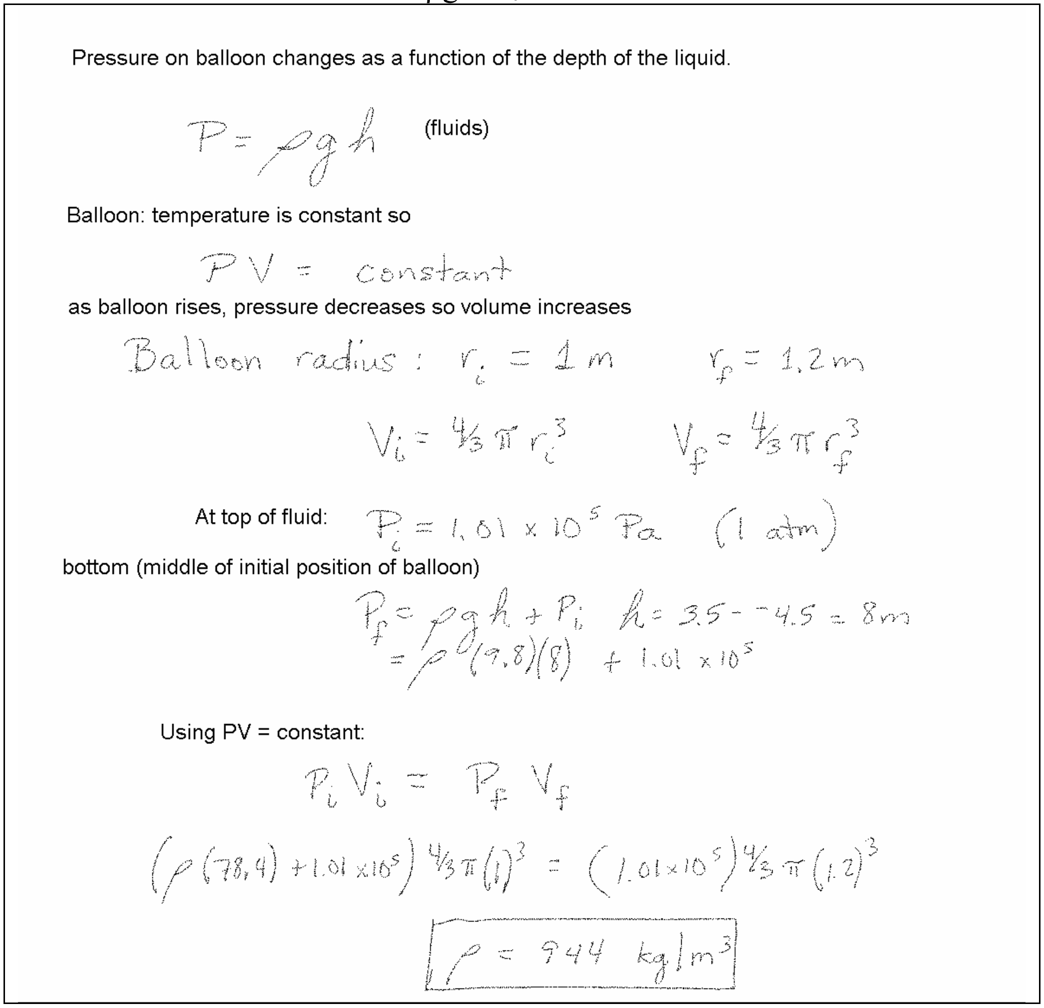
### P.20.5 (Level 1)

Topic(s): ideal gas law

*Answer:*
**If Px2 and V divides in ½, T should stay the same.**

## P.20.6 (Level 3)

*Topic(s):* ideal gas law, pressure
*Answer:*
**$\rho$=0.94x$10^3$ kg/m$^3$.** PV=constant. Balloon $r_i$=1 and $r_f$=1.2. $p_f$=1.01x$10^5$ Pa use $P_iV_i$= $P_fV_f$ and find $P_i$=1.74x$10^5$ Pa. and P=$\rho$gh+$P_0$.



Pressure on balloon changes as a function of the depth of the liquid.

$P = \rho g h$ (fluids)

Balloon: temperature is constant so

$PV = \text{constant}$

as balloon rises, pressure decreases so volume increases

Balloon radius : $r_i = 1\,m$ $r_f = 1.2\,m$

$V_i = \frac{4}{3}\pi r_i^3$ $V_f = \frac{4}{3}\pi r_f^3$

At top of fluid: $P_i = 1.01 \times 10^5\ Pa$ (1 atm)

bottom (middle of initial position of balloon)

$P_f = \rho g h + P_i$ $h = 3.5 - -4.5 = 8m$
$= \rho (9.8)(8) + 1.01 \times 10^5$

Using PV = constant:

$P_i V_i = P_f V_f$

$(\rho(78.4) + 1.01 \times 10^5)\frac{4}{3}\pi(1)^3 = (1.01\times10^5)\frac{4}{3}\pi(1.2)^3$

$\boxed{\rho = 944\ kg/m^3}$

## P.20.7 (Level 2)

*Topic(s):* ideal gas law, thermal expansion
*Answer:*
a) **$\beta$=1/100**.
b) **$\beta$=1/T.** Δh=4 and ΔT=200 so ΔV=βVΔT gives β=0.01; derivation: ΔV=($V_i$/$T_i$) ΔT so β=1/T=1/100.

## P.20.8 (Level 2)

*Topic(s):* ideal gas law, thermodynamic processes

*Answer:*
a) **W=48.5.** W=PΔV
b) **$T_f$=-150.** V/T=constant
c) **temperature scale=50°C.**

## P.20.9 (Level 1)

*Topic(s):* thermodynamic processes, 1st law of thermodynamics
*Answer:*
**Q=1386**. ΔT=0; W=1386.

## P.20.10 (Level 3)

*Topic(s):* ideal gas law, thermodynamic processes, 1st law of thermodynamics
Answer:
a) **-3652.J** W=PΔV and ΔV=V/2 so W=PV/2=nRT/2= **-**3652J.
b) **Q=-9130J.** V/T=constant; T=146.5K so ΔU=(3/2)nRΔT=-5478J and Q= W+ΔU.

Given: $n = 3$ moles $T_i = 20\,°C = 293\,K$

$p =$ constant

Volume decreases so temperature decreases:

$$V/T = \text{constant}$$

$V_f = \frac{1}{2} V_i$ since height of container goes from 8m to 4m so

$$T_f = T_i/2 = 147\,K$$

a. $W = p\Delta V$ since pressure is constant.

$$W = p(V_f - V_i) = p(\tfrac{1}{2}V_i - V_i) = -\tfrac{1}{2}pV_i$$

Ideal gas law: $pV = nRT$

so $\frac{1}{2}pV_i = \frac{1}{2}nRT_i$

$$W = -\tfrac{1}{2}nRT_i = -\tfrac{1}{2}(3\,\text{mol})(8.32\,J/\text{mol}\cdot K)(293\,K)$$

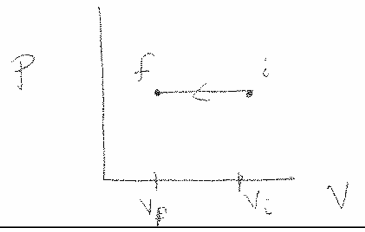
$$\boxed{W = -3.65\,kJ}$$

b. 1st law of thermodynamics: $\Delta U = Q - W$

$$\Delta U = \tfrac{3}{2}nR\Delta T = -(\tfrac{3}{2})(3)(8.32)(147) = -5.5\,kJ$$

$$Q = -5.5\,kJ + -3.65\,kJ = -9\,kJ$$

c. pV diagram:



## P.20.11 (Level 2)

*Topic(s):* thermodynamic processes. 1st law of thermodynamics

Answer:

a) Work: **1<3<2**.

b) ΔU; **1<3<2**.

c) Q: **1<3<2**.

## P.20.12 (Level 3)

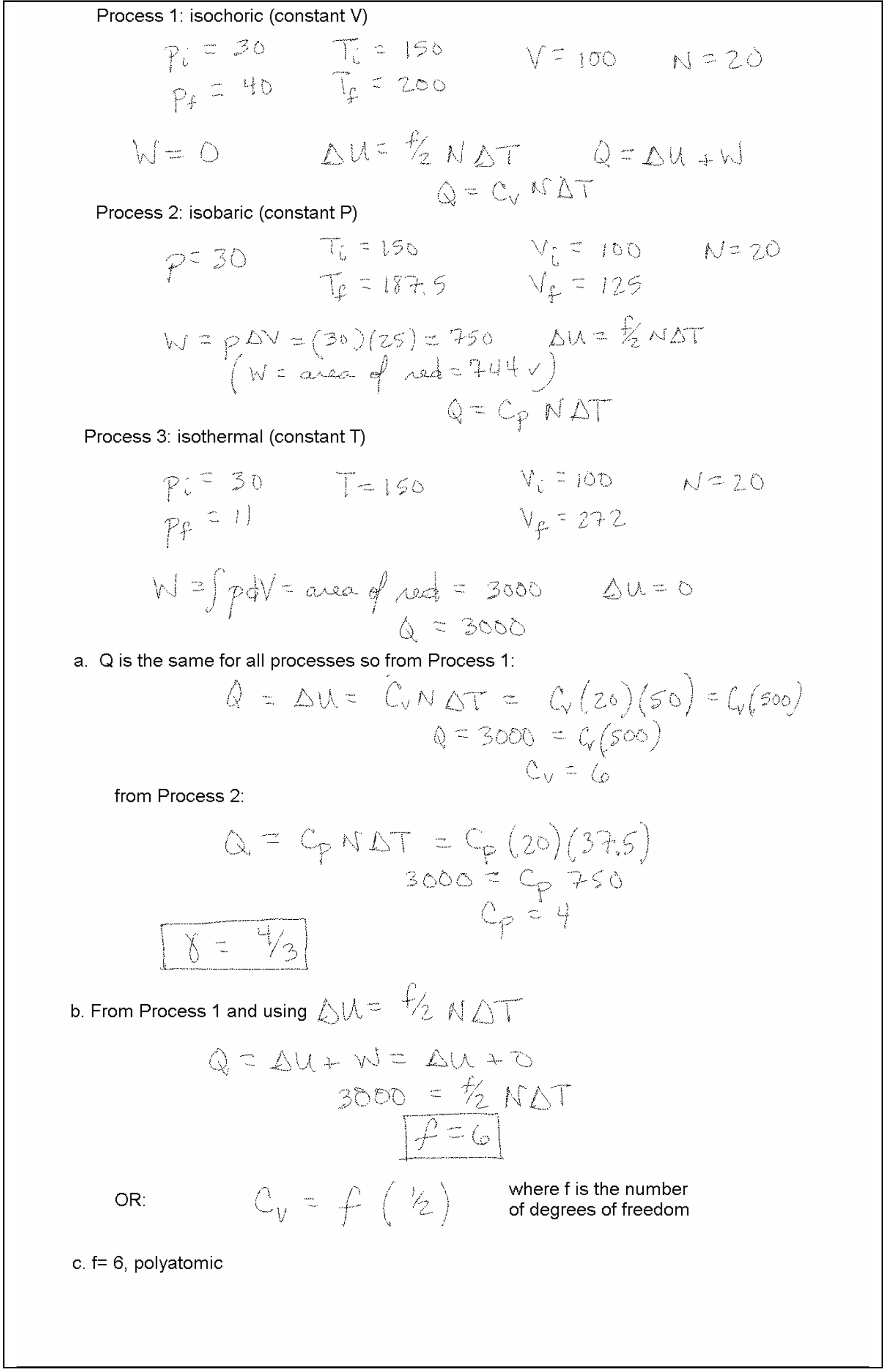
*Topic(s):* thermodynamic processes, 1st law of thermodynamics, equipartition of energy

Answer:

a) **$\gamma$=4/ 3.** 1: W=0; isochoric; 2: W=750; 3: W=3000=Q so from 1: Q=(f/2)NΔT=3000 so f=6 so $\gamma$=4/3.

b) **6**

c) **polyatomic**



Process 1: isochoric (constant V)

$p_i = 30$ $T_i = 150$ $V = 100$ $N = 20$

$p_f = 40$ $T_f = 200$

$W = 0$ $\Delta U = \frac{f}{2} N \Delta T$ $Q = \Delta U + W$

$Q = C_V N \Delta T$

Process 2: isobaric (constant P)

$p = 30$ $T_i = 150$ $V_i = 100$ $N = 20$

$T_f = 187.5$ $V_f = 125$

$W = p \Delta V = (30)(25) = 750$ $\Delta U = \frac{f}{2} N \Delta T$

(W = area of red = 744 ✓)

$Q = C_p N \Delta T$

Process 3: isothermal (constant T)

$p_i = 30$ $T = 150$ $V_i = 100$ $N = 20$

$p_f = 11$ $V_f = 272$

$W = \int p\,dV =$ area of red $= 3000$ $\Delta U = 0$

$Q = 3000$

a. Q is the same for all processes so from Process 1:

$$Q = \Delta U = C_V N \Delta T = C_V (20)(50) = C_V (500)$$

$$Q = 3000 = C_V (500)$$

$$C_V = 6$$

from Process 2:

$$Q = C_p N \Delta T = C_p (20)(37.5)$$

$$3000 = C_p \, 750$$

$$C_p = 4$$

$$\boxed{\gamma = 4/3}$$

b. From Process 1 and using $\Delta U = \frac{f}{2} N \Delta T$

$$Q = \Delta U + W = \Delta U + 0$$

$$3000 = \frac{f}{2} N \Delta T$$

$$\boxed{f = 6}$$

OR: $C_V = f \left(\frac{1}{2}\right)$ where f is the number of degrees of freedom

c. f= 6, polyatomic

# Chapter II.21: Engines and Entropy

## *Illustrations*

### Illustration 21.1: Carnot Engine

*Topic(s):* engines, thermodynamic processes
*Purpose:* To demonstrate a Carnot Engine, showing the heat absorbed and released along with the work done.
*Suggested Use:* Have students describe and/or identify the different steps, making sure they agree with the discussion in the text. Students should watch both the piston and the pV diagram for each step, making sure they understand the connection between the two. As an extension that provides practice for future problems, students can calculate the change in internal energy for each step as well as the work done and thus the heat absorbed and released to verify the numbers in the Illustration.

### Illustration 21.2: Entropy and Reversible/Irreversible Processes

*Topic(s):* entropy, reversible/irreversible processes, Maxwell-Boltzmann distribution
*Purpose:* To provide a qualitative picture of entropy associated with disorder and statistics.
*Suggested Use:* Students should describe the differences between the two animations (one highly ordered and one disordered) and then should quickly see which is more likely and identify which is a state of lower entropy. Also have students give examples of processes that result in an increase and a decrease in entropy.

### Illustration 21.3: Entropy and Heat Exchange

*Topic(s):* entropy
*Purpose:* To connect heat exchange (heat flows from hot to cold) to the second law of thermodynamics.
*Suggested Use*: Students should verify that energy is conserved in the two animations (tow objects in thermal contact: animation 1 the temperatures of the two objects get closer together while in animation 2, the temperatures of the two objects get further apart) so neither violates the 1st law of thermodynamics. Students should recognize that the difference between the two animations is the entropy change.

### Illustration 21.4: Engines and Entropy

*Topic(s):* entropy, engines, engine efficiency
*Purpose:* To connect engine efficiency with entropy.
*Suggested Use*: Students can observe both pV diagrams (to find the heat and work) and TS diagrams for the same Carnot engine. Students should verify that the change in entropy is zero for the Carnot engine cycle. As an extension, students can verify the engine efficiency equation by using the following data: Step 1: Q=2079; Step 3: Q=-1380; net W=698 so ε=.336.

## *Explorations*

### Exploration 21.1

*Topic(s):* engines, engine efficiency

### Exploration 21.2

*Topic(s):* engines, engine efficiency

### Exploration 21.3

*Topic(s):* entropy, stastistics

### Exploration 21.4

*Topic(s):* entropy, thermodynamic processes, pV diagrams

## *Problems*

### P.21.1 (Level 3)

*Topic(s):* engines, thermodynamic processes

*Answer:*

a. Step 1: isothermal; Step 2: isochoric; Step 3: adiabatic.

b. **$\gamma$=1.4.** Step 2: W=0.Q= $\Delta U=(f/2)N\Delta T$=-1513 find f=5 so $\gamma$=1.4.

c. Step 1: isothermal; W=Q=**1732**; Step 2: isochoric: W=**0**; Step 3: adiabatic: W= $\Delta U=(f/2)N\Delta T$**= -1513**.

d. **Net work=219.**

a. Thermodynamic processes

Step 1: isothermal

$P_i = 25$ $V_i = 100$ $T = 125$ $N = 20$

$P_f = 12.5$ $V_f = 200$

$W = Q = 1732$ $(\Delta U = 0)$

Step 2: isochoric

$P_i = 12.5$ $V = 200$ $T_i = 125$ $N = 20$

$P_f = 9.47$ $T_f = 94.7$

$W = 0$ $Q = -1513 = \Delta U$

Step 3: adiabatic

$P_i = 9.47$ $V_i = 200$ $T_i = 94.7$ $N = 20$

$P_f = 25$ $V_f = 100$ $T_f = 125$

$Q = 0$

b. $\gamma$

From Step 2: $\Delta U = \frac{f}{2} N \Delta T$

$$-1513 = \frac{f}{2}(20)(-30.3)$$

$f = 5$ (degrees of freedom)

$\gamma = C_p/C_v$ $C_v = f/2$ $C_p = 1 + f/2$

$$\boxed{\gamma = \frac{7/2}{5/2} = 1.4}$$

OR:

From Step 3: $P_i V_i^{\gamma} = P_f V_f^{\gamma}$

$$P_i/P_f = (V_f/V_i)^{\gamma}$$

$$\ln(P_i/P_f) = \gamma \ln(V_f/V_i)$$

$$\boxed{\gamma = \frac{\ln(9.47/25)}{\ln(100/200)} = 1.4}$$

c. Step 1: $W = 1732$ Step 2: $W = 0$

Step 3: $Q = 0$ so $W = \Delta U = \frac{f}{2} N \Delta T$

$W = \frac{5}{2}(20)(-30.3) = -1513$

d. Net work= 219

## P.21.2 (Level 3)

*Topic(s):* engines, thermodynamic processes, engine efficiency

*Answer:*

a. **$\gamma$=1.4** Step 1: W=(1/1-$\gamma$)($p_fV_f$-$p_iV_i$) or use $pV^\gamma$=constant.

b. Step 1: **Q=0, W=1816**. Step 2: **W=0**. Q= **-3185.** Q= ΔU=(5/2)NΔT= -3185; Step 3: **W=-798 Q=0**; Step 4: **W=0; Q=4201**

c. **ε=.24.** ε= net work/$Q_{\text{hot reservoir}}$=(1816-798)/4201=.24

## P.21.3 (Level 3)

*Topic(s):* engines, thermodynamic processes, engine efficiency

*Answer:*

a. Step 1: isobaric; Step 2: adiabatic; Step 3: isobaric; Step 4: adiabatic

b. Step 1: **W=1500**. W=pΔV=1500; Step 2: **W=2001**. W= ΔU=(3/2)NΔT=2001; Step 3: **W= -967**; Step 4: **W= -1200**.

c. net **W=1334**.

d. **ε=0.36.** Step 1: Q=3750; ε=1334/3750=0.36..

## P.21.4 (Level 3)

*Topic(s):* engines, thermodynamic processes, engine efficiency

*Answer:*

a. **Adiabatic**: **Burning gas mixture** (step 1) & **Compressing gas** (step 3); **Isochoric:** (step 2) **beginning of expelling smoke & very end of animation** (beginning of burning gas mixture) (step 4).

b. **End up at same volume so net W=0. Heat is released in exhaust of smoke (and then intake of gas).**

c. Step 1: **W=183**. W= ΔU=(5/2)NΔT=183; Step 3: **W= -60.4**

d. Step 2: **Q= -150.** Q= ΔU=(5/2)NΔT=-150; Step 4: **Q=273**

e. **ε=0.55.** ε=(183-60)/273=.55.

## P.21.5 (Level 3)

Topic(s): engines, thermodynamic processes, engine efficiency

*Answer:*

a. **n=0.067 moles.** pV=nRT.

b. This is a Stirling cycle. Step 1: **W=0**; **Q=289J.** ΔU=(3/2)NΔT=291J. Step 2: **Q=W=200J**, W=nRTln($V_f/V_i$)=200J. ΔU=0; Step 3: **W=0; Q=-289J**; Step 4: **W=-67J=Q**.

d. **ε=0.27 OR ε=0.67.** ε=(200-67)/(289+200)=.27 **OR** ε=(200-67)/200=0.67. You can "store" the heat from the two isochoric expansions and release and then absorb it back to the gas (see for example Halliday, Resnick and Walker, Fundamentals of Physics 5th ed, p. 514-516 for an example of the high efficiency Stirling Engine).

a. Step 1: $p_i = 1 \text{ atm} = 1.01\times10^5 \text{ Pa}$ $T_i = 173 \text{ K}$

$p_f = 3 \text{ atm} = 3.03\times10^5 \text{ Pa}$ $T_f = 519 \text{ K}$

$V = .08 \times .04 \times .3 = 9.6\times10^{-4} \text{ m}^3$ $\underline{\text{Isochoric}}$

$pV = nRT$

$$n = \frac{pV}{RT} = \frac{(10^5)(9.6\times10^{-4})}{(8.32)(173)} = .067 \text{ moles}$$

$$\boxed{n = .067 \text{ moles}}$$

b. Step 1: $\Delta V = 0$ so $W = 0$

$\Delta U = \frac{3}{2} nR\Delta T = (\frac{3}{2})(.067)(8.32)(346) = 289 \text{ J}$

$Q = \Delta U + W = \underline{289 \text{ J}}$

Step 2:

$p_i = 3 \text{ atm}$ $V_i = 9.6\times10^{-4} \text{ m}^3$ $T = 519\text{K}$

$p_f = 1.5 \text{ atm}$ $V_f = 2V_i$

$\underline{\text{Isothermal}}$: $\Delta U = 0$

$Q = W = nRT \ln V_f/V_i = \underline{200 \text{ J}}$

Step 3:

$p_i = 1.5 \text{ atm}$ $V = 19.2\times10^{-4} \text{ m}^3$ $T_i = 519\text{K}$

$p_f = 0.5 \text{ atm}$ $T_f = 173\text{K}$

$\Delta V = \underline{0 = W}$

$\Delta U = \frac{3}{2} nR\Delta T = (\frac{3}{2})(.067)(8.32)(346) = -289 \text{ J}$

$\underline{Q = -289 \text{ J}}$

Step 4:

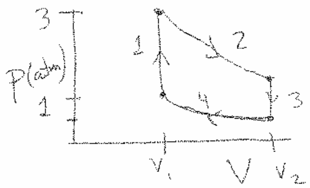
$p_i = 0.5 \text{ atm}$ $v_i = 19.2\times10^{-4}$ $T = 173 \text{ K}$

$p_f = 1 \text{ atm}$ $v_f = 9.6\times10^{-4}$

$\Delta U = 0$

$W = nRT \ln V_f/V_i = \underline{-67 \text{ J} = Q}$

c.



d.

Net work $= 200 - 67 = 133$ $Q_{absorbed} = 200 + 289 \text{ J}$
$= 489 \text{J}$

$\varepsilon = 133/(200+289) = .27$

OR $\varepsilon = 133/200 = .67$

if you store the heat from the two isochoric steps (see, for example, Halliday, Resnick, Walker, Fundamentals of Physics, 5th ed., p.514-516 for an example of the high efficiency Stirling engine.)

## P.21.6 (Level 1)

*Topic(s):* ideal gas law, entropy

*Answer:*

**Animation 1.** It is the only one with the correct change in T (if Vx2 then Tx2 for isobaric)—all have same change in S.

## P.21.7 (Level 2)

*Topic(s):* thermodynamic processes, entropy

*Answer:*

a**. isothermal**.

b. **$\Delta$S=27.** Graph is a horizontal line on T-S graph at T=80.

## P.21.8 (Level 3)

*Topic(s):* engines, thermodynamic processes, entropy

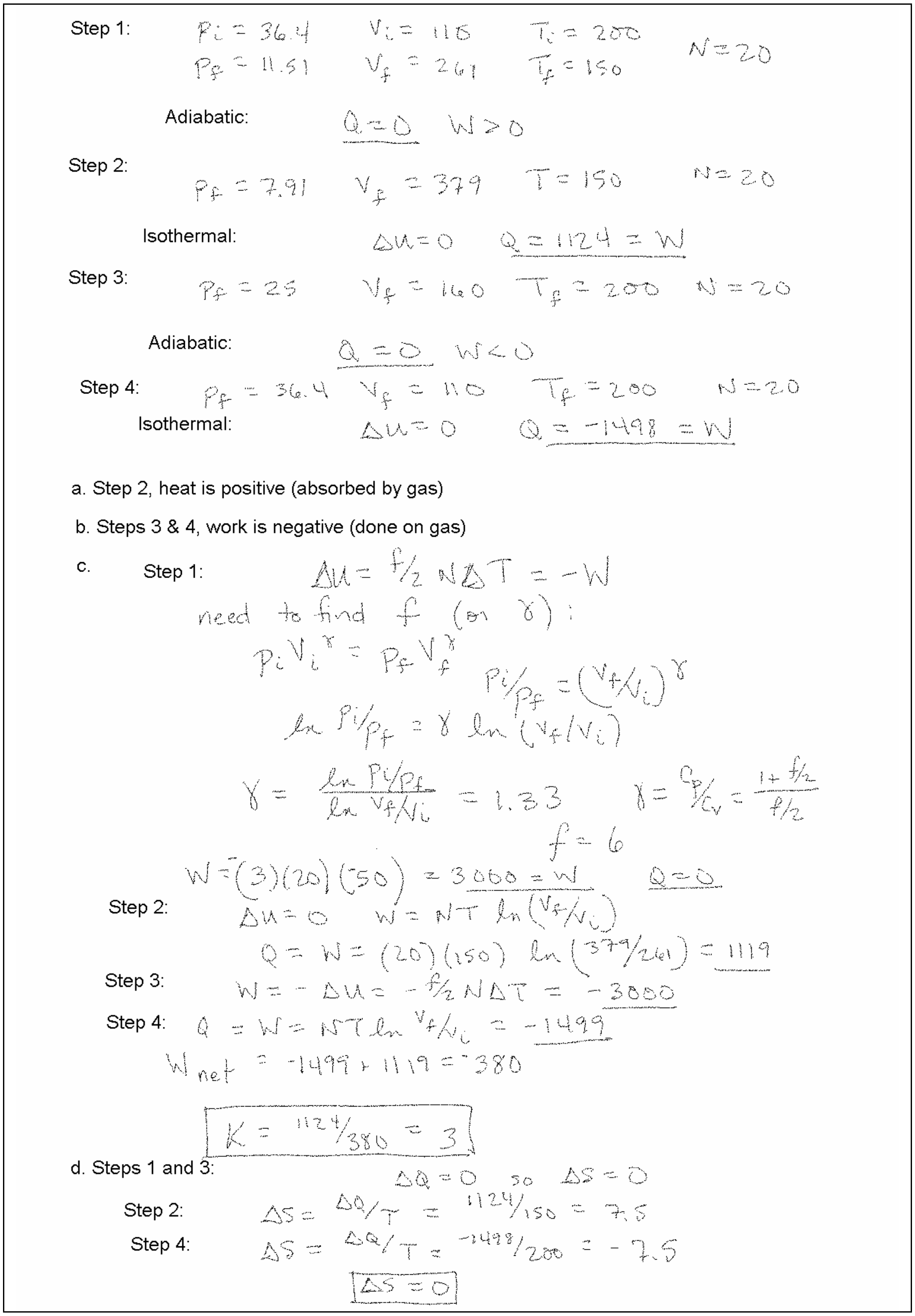
*Answer:*

a. step 2

b. steps 3 & 4

c. Step 1: **Q=0, W=3000**. adiabatic use $pV^{\gamma}$=constant and find **$\gamma$=1.33**, polyatomic. W=3000= -$\Delta$U; Step 2: **W=1119= $Q_c$**.isothermal so use W=NTln($V_f/V_i$); Step 3: **Q=0 W= -3000**. adiabatic Q=0 W=-$\Delta$U=-3000; Step 4: **W=-1499.** isothermal **K=3**. $W_{net}$= -380 (W<0 because work done on the gas) so K=1124/380=3.

d. **$\Delta$S=0**. Steps 1 and 3: $\Delta$S=0 and Step 2: $\Delta$S= $\Delta$Q/T=7.5 and Step 4: $\Delta$S= $\Delta$Q/T=-7.5.



Step 1: $P_i = 36.4$ $V_i = 115$ $T_i = 200$ $N = 20$

$P_f = 11.51$ $V_f = 261$ $T_f = 150$

Adiabatic: $\underline{Q = 0}$ $W > 0$

Step 2: $P_f = 7.91$ $V_f = 379$ $T = 150$ $N = 20$

Isothermal: $\Delta U = 0$ $\underline{Q = 1124 = W}$

Step 3: $P_f = 25$ $V_f = 160$ $T_f = 200$ $N = 20$

Adiabatic: $\underline{Q = 0}$ $W < 0$

Step 4: $P_f = 36.4$ $V_f = 110$ $T_f = 200$ $N = 20$

Isothermal: $\Delta U = 0$ $Q \underline{= -1498 = W}$

a. Step 2, heat is positive (absorbed by gas)

b. Steps 3 & 4, work is negative (done on gas)

c. Step 1: $\Delta U = f/2 \, N \Delta T = -W$

need to find $f$ (or $\gamma$):

$P_i V_i^{\gamma} = P_f V_f^{\gamma}$

$P_i/P_f = (V_f/V_i)^{\gamma}$

$\ln P_i/P_f = \gamma \ln (V_f/V_i)$

$\gamma = \frac{\ln P_i/P_f}{\ln V_f/V_i} = 1.33$ $\gamma = C_p/C_v = \frac{1 + f/2}{f/2}$

$f = 6$

$W = -(3)(20)(-50) = \underline{3000 = W}$ $\underline{Q = 0}$

Step 2: $\Delta U = 0$ $W = NT \ln (V_f/V_i)$

$Q = W = (20)(150) \ln (379/261) = \underline{1119}$

Step 3: $W = -\Delta U = -f/2 \, N \Delta T = \underline{-3000}$

Step 4: $Q = W = NT \ln V_f/V_i = \underline{-1499}$

$W_{net} = -1499 + 1119 = -380$

$\boxed{K = 1124/380 = 3}$

d. Steps 1 and 3: $\Delta Q = 0$ so $\Delta S = 0$

Step 2: $\Delta S = \Delta Q/T = 1124/150 = 7.5$

Step 4: $\Delta S = \Delta Q/T = -1498/200 = -7.5$

$\boxed{\Delta S = 0}$

# Chapter II.22: Coulomb's Law

## *Illustrations*

### Illustration 22.1: Charge and Coulomb's Law

*Topic(s):* Coulomb's law
*Purpose:* To demonstrate the forces between charged particles.
*Suggested Use:* As students build different charge configurations, they should notice the properties of Coulomb's Law. They should also notice that the force between two unequal charges is the same magnitude (points in the opposite direction)—a Newton's third law force pair. As an extension, students can develop a method to rank three charges (by magnitude and by sign).

### Illustration 22.2: Charge and Mass

*Topic(s):* Coulomb's law, charge/mass ratio, test charge
*Purpose:* To show the radial nature of the Coulomb force, the rapid decrease in the force as the radial distance increases, and the importance of the charge to mass ratio.
*Suggested Use:* Instructors should discuss the idea test charges although students have little difficulty with this concept as well as the limitations of measuring trajectories. Answers to initial questions: The sign of the fixed charge can be determined if the test charge is assumed to be positive. This is the accepted convention. Only the charge to mass ratio of the test charge can be determined from the trajectories. Like the gravitational force, the electrostatic interaction is radial and falls off as 1/(r^2). Unlike the gravitational force, the Coulombic force can be positive or negative.

### Illustration 22.3: Monopole, Dipole, and Quadrupole

*Topic(s):* Coulomb's law, charge distribution
*Purpose:* To demonstrate the difference in forces for charge distributions.
*Suggested Use*: Compare the graphs for monopoles, dipoles and quadrupoles and note that as you move further away, the charge distribution acts like the sum of the charges (zero for dipole and quadrupole).

### Illustration 22.4: Charging Objects and Static Cling

*Topic(s):* charging by induction, "static electricity"
*Purpose:* To introduce a microscopic model of what happens when objects are charged.
*Suggested Use:* Demonstrate the various methods of charging shown in the illustration (balloon on ceiling) and charging by induction. Can use "sticky tape" demonstrations as well.

## *Explorations*

### Exploration 22.1

*Topic(s):* Coulomb's law, superposition

### Exploration 22.2

*Topic(s):* Coulomb's law

### Exploration 22.3

*Topic(s):* Coulomb's law, test charge

### Exploration 22.4

*Topic(s):* Coulomb's law, dipole

### Exploration 22.5

*Topic(s):* Coulomb's law

### Exploration 22.6

*Topic(s):* Coulomb's law, test charge

## *Problems*

### P.22.1 (Level 1)

*Topic(s):* Coulomb's law

*Answer:*

a) **The observation that if two charges repel then another charge will either be attracted to both charges or repelled by both charges can be explained by postulating that there are only two types of charges**.

b) **Three charges are alike. Two are different**.

### P.22.2 (Level 2)

*Topic(s):* Coulomb's law

*Answer:*

**Charges do not have consistent forces: charge A is opposite polarity from B and C, but D, when it is near B and C repels one and attracts the other**.

### P.22.3 (Level 1)

*Topic(s):* Coulomb's law, charge distribution

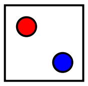
*Answer:*

**+3**

### P.22.4 (Level 1)

*Topic(s):* Coulomb's law, charge distribution

*Answer:*

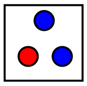


**red positive, blue negative**

## P.22.5 (Level 1)

Topic(s): Coulomb’s law, charge distribution

*Answer:*



**red positive, blue negative**

## P.22.6 (Level 2)

*Topic(s):* Coulomb’s law

*Answer:*

a) **Configuration 1: x=5; Configuration 2: x=7.5**

b) **no** (need value of one of the test charge or one of the charges)

c) **Configuration 1: x=5d/6; Configuration 2:x=5d/4**. Configuration 1: $25/r^2=1/(d-r)^2$. Configuration 2: $25/r^2=1/(r-d)^2$ so the two roots of the quadratic are the two solutions.

## P.22.7 (Level 2)

*Topic(s):* Coulomb’s law

*Answer:*

**#1 positive; #2 positive; #3 neutral; #4 negative**

## P.22.8 (Level 2)

*Topic(s):* Coulomb’s law

*Answer:*

a) **1.41 m/s**

b) $\mathbf{v_y=(10^4Q)^{1/2}}$**.** $F=kqQ/r^2=mv^2/r$. r=10, m=0.9, $q=10^{-5}$.

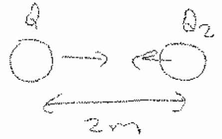
## P.22.9 (Level 3)

*Topic(s):* Coulomb’s law; charge distribution

*Answer:*

**3 e-5C; 1 e-5 C one of them positive and one negative**. Before they touch opposite sign, measure $force=kq_1q_2/r^2$ and after they touch, have equal charge of the same sign (now repel) and $force=kq_fq_f/r^2$ where $2q_f=|q_1|-|q_2|$ and solve quadratic equation.

Before contact:

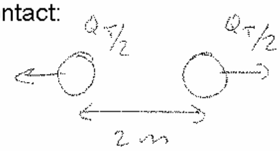


$F = 0.75N$

opposite sign

$$\big| |Q_1| - |Q_2| \big| = Q_T$$

After contact:



$F = 0.25N$

or

$$Q_1 + Q_2 = Q_T$$

(where either $Q_1 < 0$ or $Q_2 < 0$)

(after contact, net charge is distributed equally)

$$F = k\frac{Q_1 Q_2}{r^2}$$

$$k\frac{(Q_T/2)(Q_T/2)}{(2)^2} = 0{,}25 \longrightarrow Q_T^2 = 4/k$$

and

$$k\frac{|Q_1|\,|Q_2|}{2^2} = 0{,}75 \qquad |Q_2| = |Q_1| - |Q_T|$$

$$k\frac{Q_1(Q_1 - Q_T)}{4} = 0{,}75$$

$$k\frac{Q_1^2}{4} - k\frac{Q_1 Q_T}{4} = 0.75 \qquad Q_T = \sqrt{4/k}$$

$$k\frac{Q^2}{4} - k\frac{2}{\sqrt{k}}4\,Q_1 = 0.75$$

$$Q_1^2 - \frac{2}{\sqrt{k}}Q_1 = \frac{(4)(0.75)}{k} \qquad k = 9\times10^9$$

$$Q_1^2 - 2{,}11\times10^{-5}Q_1 - 3{,}33\times10^{-10} = 0$$

$$Q_1 = \frac{2{,}11\times10^{-5} \pm \sqrt{(2{,}11\times10^{-5})^2 + 4(3{,}33\times10^{-10})}}{2}$$

$$Q_1 = 1.05\times10^{-5} \pm 2.08\times10^{-5}$$
$$= 3.1\times10^{-5} \text{ or } -1.03\times10^{-5}\,C$$

$$\boxed{Q_1 = 3\times10^{-5}\,C \qquad Q_2 = -1\times10^{-5}\,C}$$

## P.22.10 (Level 3)

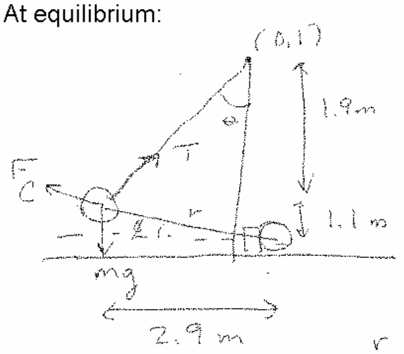
*Topic(s):* Coulomb's law

*Answer:*

**9.4e-8 C** (to 9.8e-8C). Equilibrium point with -3mC ball as close as possible is (-2.325,-0.9). r between charges=3.1; Fx=[k3e-3Q/9.62](2.9/3.1); Fy=[k3e-3Q/9.62](1.1/3.1). Components: T(.77)=Fx and T(.63)+Fy-mg=0.

*Note:* Re-work this problem (P.25.5) when get to potential to show how much easier this problem is to do (without vectors!).

At equilibrium:



Green charge at (-2.325, -0.9)

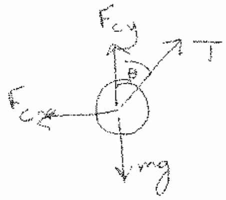
Black charge at (0.575, -2.0)

$$r = \sqrt{2.9^2 + 1.1^2} = \sqrt{9.62}$$

$$F_{Coulomb} = k\frac{Q_1 Q_2}{r^2} = k\frac{(3mC)Q}{9.62}$$

$$F_{Cy} = |F_C| \sin\phi \qquad \sin\phi = \frac{1.1}{\sqrt{9.62}}$$

$$F_{Cx} = |F_C| \cos\phi \qquad \cos\phi = \frac{2.9}{\sqrt{9.62}}$$



$$T\sin\theta = F_{Cx} \qquad T = \frac{F_{Cx}}{\sin\theta}$$

$$T\cos\theta + F_{Cy} = mg$$

$$F_{Cx}\frac{\cos\theta}{\sin\theta} + F_{Cy} = mg \qquad \tan\theta = \frac{2.325}{1.9}$$

$$9\times10^9\left(\frac{(3\times10^{-3})Q}{9.62}\right)\left(\frac{2.9}{\sqrt{9.62}}\right)\left(\frac{1.9}{2.325}\right) + 9\times10^9\left(\frac{3\times10^{-3}Q}{9.62}\right)\frac{1.1}{\sqrt{9.62}} = (0.03)(9.8)$$

$$2.14\times10^6 Q + .995\times10^6 Q = .294$$

$$Q = 9.4\times10^{-8} C$$

## P.22.11 (Level 2)

*Topic(s):* Coulomb's law, SHM

*Answer:*

a) **0.5 m**
b) **0.0044 N/m**. T=13.4s, ω=.47 rad/s $\omega^2$=k/m
c) **no**—spring remains the same
d) **x=-0.675m**
e) **4.5e-6C**. In equilibrium, kx=F=$kq_1q_2/r^2$.

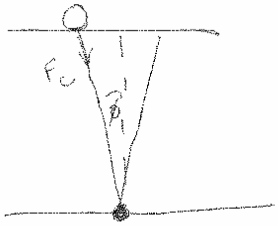
## P.22.12 (Level 3)

*Topic(s):* Coulomb's law, SHM

*Answer:*

**1.1e-4C**. Set it up so that the amplitude of motion is small so it will be SHM. Fx=$[kq_1q_2/r^2](x/r)$, but for small amplitude, r is essentially constant (=3m). Then force is a restoring force and from T=7.49, ω=0.86 rad/s so $kq_1q_2/r^3 = m\,\omega^2$.

Set velocity = 0 for small amplitude oscillations.



$$F_x = F_c \sin\phi$$

$$\sin\phi = \frac{x}{r}$$

$$F_x = k\frac{Q_1Q_2}{r^2}\left(\frac{x}{r}\right) \qquad r = \sqrt{x^2+y^2}$$

If $y >> x$, r is essentially constant so

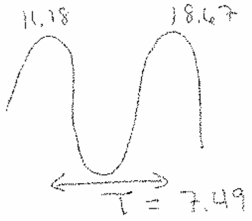
$$F_x = \underbrace{\left(k\frac{Q_1Q_2}{r^3}\right)}_{c} x$$ and this is a restoring force

so $ma = -cx$ (SHM)

$$x = x_0\cos\omega t \qquad \omega T = 2\pi$$

$$m\omega^2 = c$$

$$\omega = \sqrt{\frac{c}{m}} = 2\pi/T$$



$$\omega = 0.86$$

$$c = m\omega^2 = 0.074$$

$$c = k\frac{Q_1Q_2}{r^3} \qquad r \approx 3$$

$$.074 = \frac{9\times10^9\ Q_1Q_2}{27}$$

$$Q_1Q_2 = 2.2\times10^{-10}\ C$$

$Q_1 = 2\times10^{-6}$ C so $\boxed{Q_2 = 1.1\times10^{-4}C}$

# Chapter II.23: Electric Field

## *Illustrations*

### Illustration 23.1: What is a vector field?

*Topic(s):* vector field
*Purpose:* To introduce a representation of vector fields.
*Suggested Use:* Input different fields and note the field diagrams. Pick a point in a field and check that the vector points in the correct direction as suggested. Note that the find suggested field is for a point charge at the origin.

### Illustration 23.2: Force and E-Field due to Point Charges

*Topic(s):* vector field
*Purpose:* To demonstrate the electric field for point charges.
*Suggested Use:* Should discuss superposition. Students should be able to describe the charge distribution simply by looking at the vector fields so they need to develop a way to describe the connection between the charge distribution and the field.

### Illustration 23.3: Field Line Representation of Vector Fields

*Topic(s):* vector field, field lines
*Purpose:* To compare field line and vector field representations of e-fields.
*Suggested Use*: Have students carefully compare the vector field and field line representation. Note that the density of lines qualitatively represents the field strength. Also note that the vector field points in a direction tangent to the field vector at any point. This provides a quick connection between typical book diagrams (field lines) and physlets (field lines or vector fields). Configuration B has a net charge of zero (stack all the charges on top of each other).

### Illustration 23.4: Practical Uses of Charges and Electric Fields

*Topic(s):* e-field, force
*Purpose:* To illustrate uses of electric fields in modern electronics equipment: cathode ray tube (computer screen, TV, oscilloscope) and ink-jet printing.
*Suggested Use:* Demonstrate how different values of the speed, charge and electric field can control the point of impact on the right wall (screen). If available, show a cathode ray tube from an oscilloscope.

## *Explorations*

### Exploration 23.1

*Topic(s):* e-field

### Exploration 23.2

*Topic(s):* e-field, force

## Exploration 23.3

*Topic(s):* vector field, force

Answers

*0.5 C. For E=0.1N/C, equilibrium at (-1.367,-1.7) so mg=Fnsinφ and qE=Fncosφ and tanφ=(2.7/1.367).*

## *Problems*

### P.23.1 (Level 2)

*Topic(s):* e-field, vector field

*Answer:*

a) **$3.9\mathbf{x}10^{-10}$ C**. At (0,2), E=.875N/C and $E=kQ/r^2$.

b) **excess**. Vectors point into charge, so negative charge.

c) **$2.4\mathbf{x}10^9$ electrons**

### P.23.2 (Level 1)

*Topic(s):* e-field, field lines

*Answer:*

**negative**

### P.23.3 (Level 2)

*Topic(s):* vector field

*Answer:*

**I=C; II=B; III=D; IV=A; V=E**

### P.23.4 (Level 2)

*Topic(s):* e-field, force

*Answer:*

a) **Region I, E -y; Regions III & IV, E +y; Region II, E=0**. E is opposite direction than F for electron

b) **IV>III=I>II**—measure change in y-velocity through the regions

### P.23.5 (Level 2)

*Topic(s):* e-field, force

*Answer:*

a) **Region I, E -y; Region II &IV, E=+x; Region III, E +y**. E is opposite direction than F for electron

b) **II>III>IV=I**—measure change in velocity through the regions

### P.23.6 (Level 2)

*Topic(s):* e-field, force

*Answer:*

**E=0.55 N/C in +y**. In time of 1.7 us, Δy=0.14m so ay=9.7x$10^{10}$m/$s^2$.

| t | x | y |
|---|---|---|
| 1 | -10.5 | 0 |
| 1.1 | -4.5 | 0 |
| 1.2 | 1.5 | 0 |
| 1.7 | 31.5 | -1 |
| 1.9 | | -2.5 |
| 2.85 | | -14 |

$v_x = \frac{6}{.1\,\mu s} = 6 \times 10^5\ m/s$

$v_x = \frac{30}{.5} = 6 \times 10^5\ m/s$

so $F_x = 0$

$v_{oy} = 0$

$\Delta y = 14 \qquad t = 1.7\,\mu s$

$$y = \overset{0}{\cancel{v_{oy} t}} + \tfrac{1}{2} a_y t^2$$

$a_y = 9.7 \times 10^{10}\ m/s^2$

$m a_y = qE$

$m = 9.1 \times 10^{-31}\ kg \qquad q = 1.6 \times 10^{-19}\ C$

$$E = \frac{(9.1 \times 10^{-31})(9.7 \times 10^{10})}{1.6 \times 10^{-19}} = 0.55\ N/C$$

## P.23.7 (Level 2)

*Topic(s):* e-field, force

*Answer:*

a) **F**

b) **E=2.4 N/C** In time of 0.5 us, Δvy=15cm/us=150km/s and Δvy=15cm/us so |a|=4.2e11m/$s^2$.

## P.23.8 (Level 2)

*Topic(s):* e-field

*Answer:*

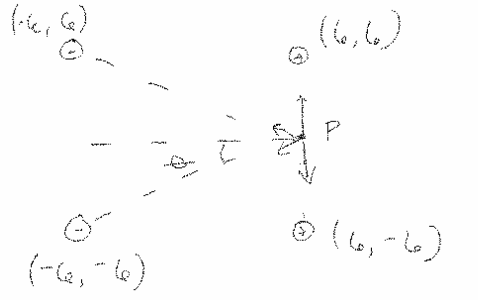
**2nC.**

## P.23.9 (Level 3)

*Topic(s):* e-field, charge distribution

*Answer:*

**E=Q*0.0895**. E from two right-hand charges cancels out as does the y-component of electric field from the two charges on the left. One charge: $|E|=kQ/(12^2+6^2)$ so $E_x=kQ/(180)*12/(180)^{1/2}$ for one of the left-hand charges.



Point P is (6, 0)

$E_y = 0$ (y-components cancel)

$E_x$ due to left-hand charges (at $x = -6$):

$$|E| = k\frac{Q}{r^2} = k\frac{Q}{12^2+6^2} = \frac{9\times10^9\ Q}{180}$$

Due to one negative charge

$$E_x = |E|\cos\theta = |E|\frac{12}{\sqrt{180}}$$

$$E_{Total} = \left(2|E|\frac{12}{\sqrt{180}}\right)(-\hat{\imath})$$

$$= 0.0895\ Q\ (-\hat{\imath})$$

## P.23.10 (Level 3, calculus)

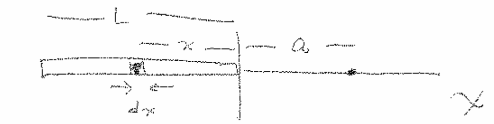
*Topic(s):* charge distribution

*Answer:*

a) **Q=9e-4C**; E=(kQ/8)[1/x-1/(8+x)]. $dE=(kQ/L)dx/(x+x1)^2$ where L is the length of the rod and x1 is the distance from the origin to the test charge and the limits of integration are 0 to L.

b) **Q=9e-4C**; $E=(kQ/y)1/(16+y^2)^{1/2}$. y-component of field is $dE_y=(kQ/L)[dx/(x^2+y^2)]y/(x^2+y^2)^{1/2}$ and integration is from –L/2 to L/2 (this integral is not trivial substitution—students should probably simply use an integral table or a mathematics package.)

a.



$dq = \left(\frac{Q}{L}\right) dx$

Constants: Q, L, a

length of rod

charge on rod

$dE = k\left(\frac{Q}{L}\right)\frac{dx}{(a+x)^2}$

$E = k\left(\frac{Q}{L}\right)\int_0^L \frac{dx}{(a+x)^2}$ $u = a+x$

$du = dx$

$E = k\left(Q/L\right)\int u^{-2}\,du = k\left(Q/L\right)\frac{u^{-1}}{-1}$

$E = -k\left(Q/L\right)\left.\frac{1}{a+x}\right|_0^L = -k\left(Q/L\right)\left(\frac{1}{L+a} - \frac{1}{a}\right)$

$E = k\left(Q/L\right)\left(\frac{1}{a} - \frac{1}{L+a}\right)$

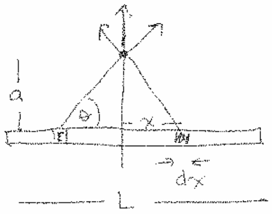
$F = qE$

At $a = 4$, $F = 1.7016$ N

$E(a=4) = \frac{1.7016}{10\times10^{-6}} = (9\times10^9)\left(\frac{Q}{8}\right)\left(\frac{1}{4} - \frac{1}{8+4}\right)$

$\boxed{Q = 9\times10^{-4}\,C}$

b.



Net field in y-direction.

$dE_y = k\frac{Q}{L}\frac{dx}{x^2+a^2}\sin\theta$

$\sin\theta = \frac{a}{\sqrt{x^2+a^2}}$ $E_y = k\frac{Q}{L}\int_{-L/2}^{L/2}\frac{a\,dx}{(x^2+a^2)^{3/2}}$

integral table

$E_y = k\frac{Qa}{L}\left.\frac{x}{a^2\sqrt{x^2+a^2}}\right|_{-L/2}^{L/2}$

$E_y = k\frac{Q}{La}\left[\frac{L/2}{\sqrt{L^2/4+a^2}} + \frac{L/2}{\sqrt{L^2/4+a^2}}\right]$

$E_y = k\frac{Q}{a}\left[\frac{1}{\sqrt{L^2/4+a^2}}\right]$

At $a = 4$, $F = 3.56$ N $F = qE$

$\frac{3.56}{10\times10^{-6}} = k\frac{Q}{4}\left[\frac{1}{\sqrt{16+4}}\right]$

$\boxed{Q = 9\times10^{-4}\,C}$

# Chapter II.24: Gauss's Law

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## *Illustrations*

### Illustration 24.1: Flux and Gaussian Surfaces

*Topic(s):* flux
*Purpose:* To make the connection between flux and charge enclosed by Gaussian surface.
*Suggested Use:* Explore the flux through the different size and shaped surfaces (remind students that Gaussian surfaces are 3-D and what they see are 2-D representations). Can begin to make the case that Gauss's law is always true, but not always useful in calculating the electric field.

### Illustration 24.2: Symmetry

*Topic(s):* symmetry
*Purpose:* To explore choices of Gaussian surfaces.
*Suggested Use:* Demonstrate that Gauss's law can be used to calculate the electric field in the "near view" and "far view" case, but not in the "intermediate view" because of the symmetry (relation of Gaussian surface to electric field). Can also demonstrate the connection between flux and charge enclosed by noting how the flux changes as the box is moved through the filament in the near view.

### Illustration 24.3: Cylinder of Charge

*Topic(s):* symmetry
*Purpose:* To compare cases of symmetry with ones with broken symmetry.
*Suggested Use*: Have students carefully compare the electric fields for one charged rod with half a cylinder and a complete cylinder. Have them propose Gaussian surfaces for those cases and discuss which ones would allow them to calculate the electric field.

## *Explorations*

### Exploration 24.1

*Topic(s):* flux, gauss's law
*Worksheets:*
Flux=352 Nm$^2$. q=3.1e-9C.

### Exploration 24.2

*Topic(s):* gauss's law, symmetry
*Worksheets:*

### Exploration 24.3

*Topic(s):* gauss's law, charge distribution
*Worksheets:*
Q=1nC

## Exploration 24.4

*Topic(s):* gauss's law

*Worksheets:*

Point charge: $E=kQ/r^2$ (configuration 1); Cylinder: $E=k2\lambda/r$ (configuration 2).

## *Problems*

### P.24.1 (Level 1)

*Topic(s):* flux

*Answer:*

1) a>d>e>b>c
2) a>e>d>c>b
3) a>d>c>e>b

### P.24.2 (Level 1)

*Topic(s):* flux

*Answer:*

e<c<a<b<d

### P.24.3 (Level 1)

*Topic(s):* flux

*Answer:*

Negative charge with charge density increasing radially out from origin.

### P.24.4 (Level 1)

*Topic(s):* flux, charge distribution

*Answer:*

Center region of negative charge (r about .1); surrounded by positive charge (decreasing readially) out to r about 1); small net positive charge (of 1 mC).

### P.24.5 (Level 1, calculus)

*Topic(s):* flux

*Answer:*

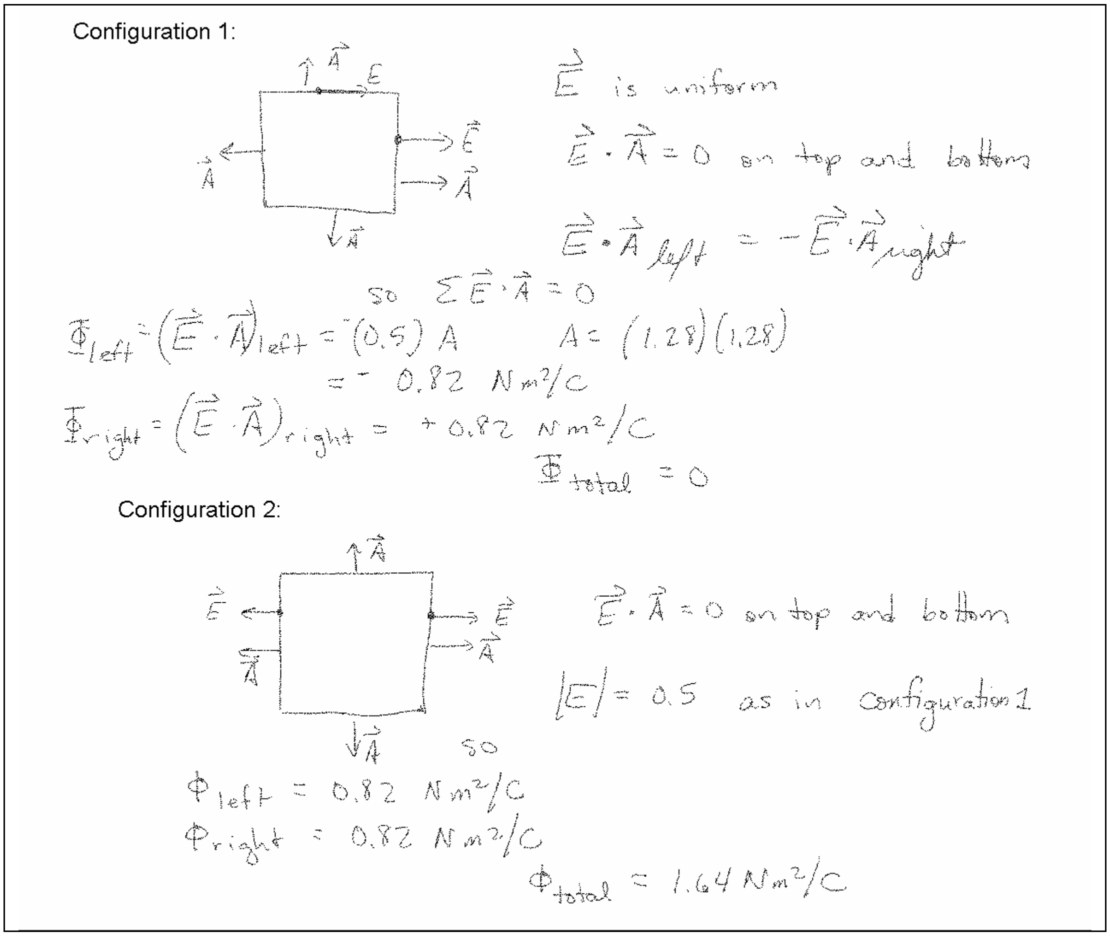
Flux=0 both configurations.

### P.24.6 (Level 2, calculus)

*Topic(s):* flux

*Answer:*

Configuration 1: total flux=0=top=bottom; right=.82 $Nm^2/C$=-left.
Configuration 2: 0=top=bottom; right=.82 $Nm^2/C$=left (net flux=1.6 $Nm^2/C$).



$\vec{E}$ is uniform

$\vec{E}\cdot\vec{A} = 0$ on top and bottom

$\vec{E}\cdot\vec{A}_{left} = -\vec{E}\cdot\vec{A}_{right}$

so $\Sigma \vec{E}\cdot\vec{A} = 0$

$\Phi_{left} = (\vec{E}\cdot\vec{A})_{left} = -(0.5)A$ $A = (1.28)(1.28)$

$= -0.82\ Nm^2/C$

$\Phi_{right} = (\vec{E}\cdot\vec{A})_{right} = +0.82\ Nm^2/C$

$\Phi_{total} = 0$

$\vec{E}\cdot\vec{A} = 0$ on top and bottom

$|E| = 0.5$ as in configuration 1

so

$\Phi_{left} = 0.82\ Nm^2/C$

$\Phi_{right} = 0.82\ Nm^2/C$

$\Phi_{total} = 1.64\ Nm^2/C$

## P.24.7 (Level 1)

*Topic(s):* symmetry

*Answer:*

Top is positive; bottom negative; cube (or cylinder) with one side between plates and one side above (or below plates).

## P.24.8 (Level 2, calculus)

*Topic(s):* symmetry, Gauss's law

*Answer:*

a) **point or line charge**=net charge at center (or at x=1,y=0 for Configuration 1).

b) **Configuration 1: E=0.8/r. Configuration 2: E=1.5/r$^2$. Configuration 3: E=0. Configuration 4: E=4/r**. In all cases, measure E, double radius and measure E to find out if line charge (~1/r) or point charge (~1/r$^2$) (line or point charge) and then find constant of proportionality.

## P.24.9 (Level 2, calculus)

*Topic(s):* symmetry, Gauss's law, charge distribution

*Answer:*

**Left: line of charge; right: charged plate**. Left: near charges, field drops off as 1/r and on right field does not drop much.

### P.24.10 (Level 2, calculus)

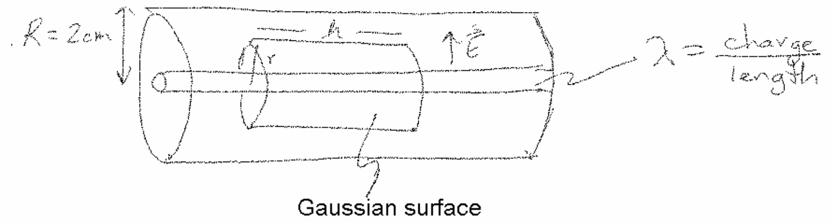
*Topic(s):* charge distribution, Gauss's law

*Answer:*

b) **λ=1.1e-11 C/m**. **E=-2k(charge/length)/r inside; E=0 outside**.

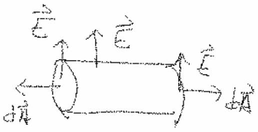
a. Since the field outside the charge distribution is zero, the net charge of the two must be zero so inner is negative (from direction of electric field vectors) and outer is positive.

b.



Gauss' Law: $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$

$\vec{E}\cdot d\vec{A} = 0$ on "caps" of cylinder:



At all points of the curved cylindrical surface, the electric field points in the same direction as $d\vec{A}$ so $\vec{E}\cdot d\vec{A} = E\,dA\cos 0 = E\,dA$

also $|\vec{E}|$ is the same at all points on the curved surface

so

$$\oint \vec{E}\cdot d\vec{A} = \int E\,dA = E\int dA = EA$$

surface area of curved surface

$A = 2\pi r h$

$$\oint \vec{E}\cdot d\vec{A} = E\,2\pi r h$$

Charge enclosed by Gaussian surface ($q_{enclosed}$):

$q_{enclosed} = \lambda h$

so

$\oint \vec{E}\cdot d\vec{A} = q_{enclosed}/\epsilon_0$ gives

$$E\,2\pi r\,\cancel{h} = \lambda \cancel{h}/\epsilon_0$$

$$E = \frac{\lambda}{2\pi r\epsilon_0}$$

$|E| = 40$ N/C

$\left(\vec{E} = -\frac{\lambda}{2\pi r}\hat{r}\right)$

$40 = \frac{\lambda}{2\pi\epsilon_0(.01)}$

$\lambda = 1.1\times10^{-11}$ C/m

$\vec{E} = -\frac{\lambda}{2\pi r}\hat{r}$

## P.24.11 (Level 1, calculus)

*Topic(s):* charge distribution, Gauss's law

*Answer:*

**$\sigma$=2.5e-11 C/m$^2$. E=5.6-5.8 N/C.**

## P.24.12 (Level 3, calculus)

*Topic(s):* charge distribution, Gauss's law

*Answer:*

a) **Animation 1: uniformly charged sphere. Animation 2: conducting sphere.**

b) **Animation 1: $E_{outside}$=300/4$\pi$r$^2$. $E_{inside}$=2.4r/4$\pi$.** Find that flux varies as t$^3$ (at t=2 flux=19.4 and at t=4, flux=155) so charge~r$^3$ inside.

**Animation 2: $E_{outside}$=300/4$\pi$r$^2$.E=0 inside.**

Animation 1:

| $t$ | $\phi$ |
|---|---|
| 2 | 19.41 |
| 4 | 155 |

$t \propto r$

$\phi \propto t^3$

so $\phi \propto r^3$ so $q \propto r^3$

$Q_T = \Phi \epsilon_0 = 300\,\epsilon_0$

$q = A r^3$ so that when $r=5$, $Q = 300\,\epsilon_0$

$300\,\epsilon_0 = A\,5^3$

$A = 2.4\,\epsilon_0$

$\underline{q = 2.4\epsilon_0 r^3}$ uniform spherical charge distribution

$\vec{E} = k\, 300\epsilon_0 / r^2\, \hat{r} = \frac{300}{4\pi r^2}\hat{r}$ (outside)

Inside: $\int \vec{E}\cdot d\vec{A} = q_{enclosed}/\epsilon_0$

$E\, 4\pi r^2 = 2.4\,\epsilon_0\, r^3/\epsilon_0$

$\vec{E} = 2.4\, r/4\pi\ \hat{r}$ (inside)

Animation 2: $\phi = 0$ inside $\rightarrow$ $q = 0$ inside

Q on surface $\rightarrow$ conducting sphere

$\Phi = 300$ $Q = \Phi\epsilon_0 = 300\,\epsilon_0$

$\vec{E} = \frac{300}{4\pi r^2}\ \hat{r}$ outside

$\vec{E} = 0$ inside

# Chapter II.25: Electric Potential

## *Illustrations*

### Illustration 25.1: Energy and Voltage

*Topic(s):* work, energy, voltage
*Purpose:* To help students make connections between work, potential energy, kinetic energy and electrostatic potential (voltage).
*Suggested Use:* Demonstrate the ways you can increase and decrease the potential and kinetic energy of the particle. Have students find the (arbitrary) point of PE=0 (middle in this case). Students should recognize that the total energy is arbitrary and so what we are interested in is the change in energy. Extensions: Have students find the voltage difference between the two plates is (10V) and/or calculate work done.

### Illustration 25.2: Work and Equipotential Surfaces

*Topic(s):* equipotential surfaces, work, voltage
*Purpose:* To explore equipotential surfaces.
*Suggested Use:* Move the test charge and note the work required to move along an equipotential contour (0) and between contours. Students should be able to identify the charge distribution (3 positive; 1negative). Note the direction of electric field (from force vector) in relation to contours. Extension: Have students calculate the charge of the test charge (given the work and voltage).

### Illustration 25.3: Potential of Point Charges

*Topic(s):* voltage, equipotential surfaces, charge distributions
*Purpose:* To compare equipotential surfaces for different charge distributions.
*Suggested Use:* Change the value of the charge and note the differences in the contours. Add the third (dragable) charge and note the contours. Have students develop a technique for determining relative size of charges from contour maps.

### Illustration 25.4: Conservative Forces

*Topic(s):* work, potential
*Purpose:* To compare conservative and non-conservative forces.
*Suggested Use:* Compare the work done to go along different paths from one point to another to determine which one is a conservative force (Force 1). Using the questions in the illustration, have students develop an explanation for why a conservative force is required in order to develop a potential associated with the force.

## *Explorations*

### Exploration 25.1

*Topic(s):* equipotential surfaces, field vectors

## Exploration 25.2

*Topic(s):* equipotential surfaces, field vectors

## Exploration 25.3

*Topic(s):* equipotential surfaces, conductors

## Exploration 25.4

*Topic(s):* voltage, energy
*Note:* Suggested calculation ignores the time required to accelerate particles into the field free region (but the time is short enough that this is negligible).

## Exploration 25.5

*Topic(s):* voltage, charge distribution, conductors
*Note:* calculus required for parts (c) & (d).

# *Problems*

## P.25.1 (Level 2)

*Topic(s):* voltage, energy
*Answer:*

Animation 1) **PE increases; higher V at beginning; could be the field**
Animation 2) **PE increases; higher V at end; external force to left=qE** (after initial start)
Animation 3) **PE increases; higher V at beginning; external force=qE to right** (after initial start)
Animation 4) **PE decreases; higher V at end; could be the field**
Animation 5) **PE same; V same; F=qE to right to keep along path** (initial F to start motion)

## P.25.2 (Level 1)

*Topic(s):* work, equipotential surfaces
*Answer:*

**0 J** (on same equipotential)

## P.25.3 (Level 1)

*Topic(s):* work, equipotential surfaces
*Answer:*

**4=5>1>2=3**.

## P.25.4 (Level 2)

*Topic(s):* voltage
*Answer:*

a) **$10 \times 10^{-10}$ C**
b) **$-5 \times 10^{-10}$ C**
c) **$5 \times 10^{-10}$ C**

## P.25.5 (Level 2)

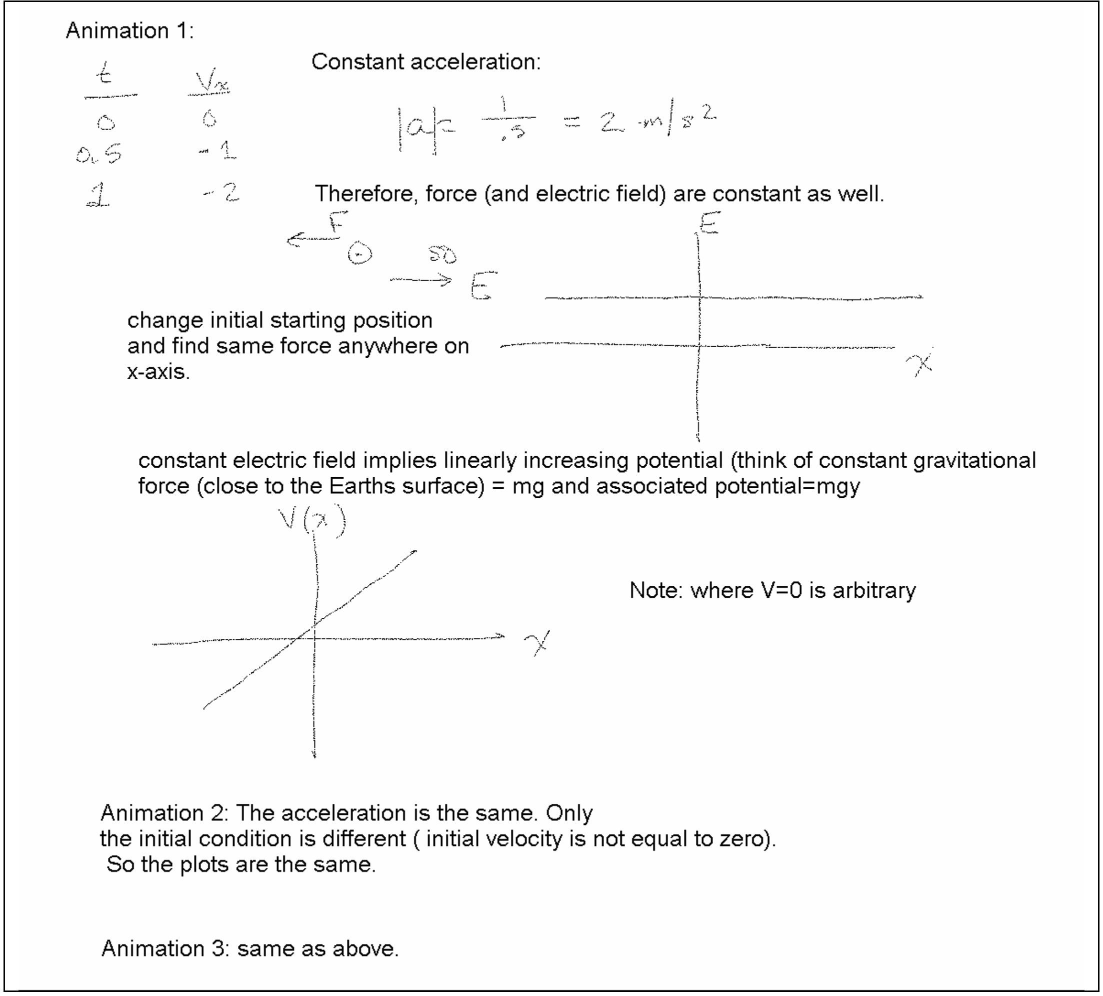
*Topic(s):* voltage, energy
*Answer:*

**$10^{-6}$ kg**. Start at rest at left of field and then record velocity as it leaves the field. ½mv^2=qV.

## P.25.6 (Level 1)

*Topic(s):* voltage, electric field, energy
*Answer:*

**Animations 1-3: voltage linearly increases in +x direction; electric field is constant and points in +x direction**.



Animation 1:

| t | Vx |
|---|---|
| 0 | 0 |
| 0.5 | -1 |
| 1 | -2 |

Constant acceleration:

$$|a| = \frac{1}{.5} = 2\ m/s^2$$

Therefore, force (and electric field) are constant as well.

change initial starting position and find same force anywhere on x-axis.

constant electric field implies linearly increasing potential (think of constant gravitational force (close to the Earths surface) = mg and associated potential=mgy

Note: where V=0 is arbitrary

Animation 2: The acceleration is the same. Only the initial condition is different ( initial velocity is not equal to zero). So the plots are the same.

Animation 3: same as above.

## P.25.7 (Level 1)

*Topic(s):* voltage, energy, equipotential surfaces
*Answer:*

**Animation 4**

## P.25.8 (Level 2)

*Topic(s):* voltage, electric field, equipotential surfaces, conductors
*Answer:*
a) **3>2>1**
b) **3>1>2**
c) **2>1>3>4**

## P.25.9 (Level 2)

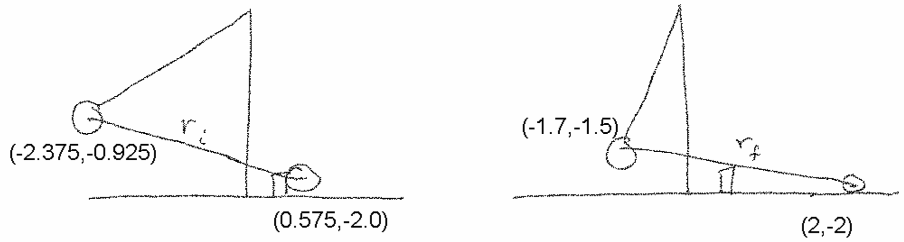
*Topic(s):* voltage, electric field, charge distribution
*Answer:*
a) **V=kQ/2**.
b) **E=0 at midpoint**.
c) **V is not zero** (V=0 is infinity or some other arbitrary reference point).

## P.25.10 (Level 3)

*Topic(s):* voltage, energy
*Answer:*

**Q=1x10$^{-7}$ C**. At equilibrium (set vel=0 until stops moving) change in gravitational PE (mgh) equals change in electrostatic PE (qQk/r). When black charge is at x=2, equilibrium is (-1.7, -1.5) so r=3.7 between the two. When black charge is at x=.575, equilibrium is (-2.325,-.925) so r=3.1.
*Note:* Same problem as Problem 22.10, but easier to do from energy point of view.

Equilibrium positions:



$$E_i = E_f$$

V = 0 in both cases :

Set $PE_{mgh} = 0$ at $y = -2$.

$$PE_{mg} + PE_{electrostatic} = \text{constant}$$

$$PE_{mg} = mgh$$

$$PE_{electrostatic} = k\,q_1 q_2/r$$

$$r_i^2 = (2.375 + 0.575)^2 + (2.0 - .925)^2 \qquad r_f^2 = (2 + 1.7)^2 + (2 - 1.5)^2$$

$$r_i = 3.14 \qquad r_f = 3.73$$

$$E_i = E_f$$

$$mg(1.075) + kq_1q_2/3.14 = mg(0.5) + kq_1q_2/3.73$$

$m = 0.03$ kg

$$(.575)(0.03)(9.8) = k\,q\,(3 \times 10^{-3})\left(\frac{1}{3.33} - \frac{1}{3.14}\right)$$

$$q = -1 \times 10^{-7}\ \text{C}$$

## P.25.11 (Level 2)

*Topic(s):* voltage, conductors, charge distribution

*Answer:*

**$Q=6.7\times10^{-12}$ C**.

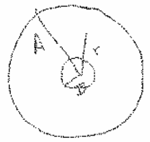
## P.25.12 (Level 3, calculus)

*Topic(s):* charge distribution, voltage

*Answer:*

**Cylinder**. For sphere: V=kQ(1/r-1/R) where R is the distance to outer ring; Q=1.4e-11C. Cylinder: V=2kλln(R/r); λ=3.7e-10C/m. Checking voltage values (x=0, y=2, V=5.3 and x=0, y=4, V=0.9, matches co-centric cylinder equation.)

a.



Sphere:

$$\vec{E} = k\, Q/r^2\, \hat{r}$$

$$V = -\int \vec{E}\cdot d\vec{r}$$

$$= -\int_A^r k\, Q/r^2\, dr = -kQ\left(\frac{1}{-r}\right)\Big|_A^r$$

$$\boxed{V(r) = kQ\left(\frac{1}{r} - \frac{1}{A}\right)}$$

when $r = B$, $V = 10$ so

$$10 = kQ\left(\frac{1}{B} - \frac{1}{A}\right)$$

$A = .045\ m$, $B = 0.01\ m$

$$10 = kQ\left(\frac{1}{.01} - \frac{1}{0.045}\right)$$

$$\underline{Q = 1.4\times10^{-11}\ C}$$

b. Cylinder: $\vec{E} = 2k\,\lambda/r\,\hat{r}$

$$V = -\int \vec{E}\cdot d\vec{r} = -\int_A^r 2k\,\lambda/r\, dr = -2k\lambda \ln r\Big|_A^r$$

$$V = -2k\lambda \ln r/A = \boxed{2k\lambda \ln A/r}$$

when $r = B$, $V = 10$ so

$$10 = 2k\lambda \ln A/B = 2k\lambda \ln\left(\frac{.045}{.01}\right)$$

$$\underline{\lambda = 3.7\times10^{-10}\ C}$$

c. Cylinders or Spheres?

matches this better

Cylinder: $r = 2$ $V = 2k\lambda \ln A/2$

$V = 5.4$

$r = 4$ $V = 2k\lambda \ln A/4 = 0.8$

Sphere: $r = 2$ $V = kQ\left(\frac{1}{.02} - \frac{1}{.045}\right) = 3.5$

$r = 4$ $V = kQ\left(\frac{1}{.04} - \frac{1}{.045}\right) = 0.35$

Measured values:

$x = 0$, $y = 2$, $V = 5.3$

$x = 0$, $y = 4$, $V = 0.9$

# Chapter II.26: Capacitance and Dielectrics

## *Illustrations*

### Illustration 26.1: Microscopic View of Capacitor

*Topic(s):* capacitance, dielectrics, electric field
*Purpose:* To provide a model microscopic view of what happens in a capacitor.
*Suggested Use:* Demonstrate the microscopic model. From looking at the model, students should be able to develop the idea that addition of a dielectric (bound charges here in comparison with the conductor where the charges can separate) decreases the net electric field so battery with dielectric is a smaller battery (smaller V).

### Illustration 26.2: Capacitor Connected to a Battery

*Topic(s):* capacitance, equipotential surfaces, voltage, electric field
*Purpose:* To connect electric fields with capacitor parameters.
*Suggested Use:* With increasing plate separation (as students drag the red plate) and/or reducted voltage, students should quickly see that the electric field is reduced. Students should be able to see this on both the vector field plot and the equipotential contours. Students should begin to understand what happens to charges (and why it happens) on the plates as these parameters change.

### Illustration 26.3: Capacitor with Dielectric

*Topic(s):* capacitance, dielectrics, equipotential surfaces, voltage, electric field
*Purpose:* To illustrate the effect of dielectrics within capacitors.
*Suggested Use:* Students should observe (and describe) what happens to both the electric field and the equipotential contours when the dielectric moves in and out of the region between the plates. Students should be able to predict what will happen if the dielectric constant is changed. Note the increase in the charges on the plates near the dielectric. Have students identify the location of the largest electric fields (between dielectric and plate) and explain why the field is so large at that point.

### Illustration 26.4: Capacitors in Parallel and in Series

*Topic(s):* capacitance, voltage
*Purpose:* To provide a microscopic view of parallel and series capacitors.
*Suggested Use:* Use the different configurations to illustrate that the potential across parallel capacitor is the same while the charge on series capacitors is the same. This can lead naturally into the series and parallel equivalent capacitor circuits equations.

## *Explorations*

### Exploration 26.1

*Topic(s):* equipotential surfaces, capacitance, energy
a) V=20V; A=0.18m^2; b) W>0 to push together; c) U~1/d.

## Exploration 26.2

*Topic(s):* capacitance, voltage

## Exploration 26.3

*Topic(s):* capacitance, dielectrics, conductors, equipotential surfaces

## Exploration 26.4

*Topic(s):* capacitance, equivalent capacitance

## Exploration 26.5

*Topic(s):* voltage, charge distribution, capacitance, gauss's law
*Note:* calculus required.

## *Problems*

### P.26.1 (Level 1)

*Topic(s):* capacitance
*Answer:*
**Animations 1, 3 & 4**

### P.26.2 (Level 1)

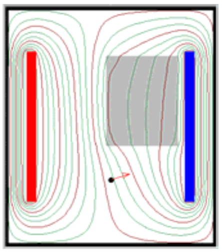
*Topic(s):* capacitance, field vectors
*Answer:*
a) **positive**
b) **negative**
c) **charges only on surfaces**—most on surface facing other plate (a few charges on surface facing outside).
d) **9-10V.** V=Ed, d=3 and E=3.13

### P.26.3 (Level 2)

*Topic(s):* capacitance, energy
*Answer:*
**Not connected to battery**. $U=CV^2/2=Q^2/(2C)$; here U~1/A so Q is constant, not V.

### P.26.4 (Level 1)

*Topic(s):* capacitance, dielectric, equipotential surfaces
*Answer:*



## P.26.5 (Level 2)

*Topic(s):* dielectric
*Answer:*
**C>A>B**. Smallest electric field, largest dielectric.

## P.26.6 (Level 2)

*Topic(s):* dielectric, equipotential surfaces
*Answer:*
**A>C>B**.

## P.26.7 (Level 1)

*Topic(s):* capacitance, dielectric
*Answer:*
**Animation 1: connected to battery**
**Animation 2: not connected to a battery (constant Q)**

## P.26.8 (Level 2)

*Topic(s):* dielectric, capacitance, electric field
*Answer:*
**Dielectric constant=8**. Ratio of electric fields outside and within dielectric.

## P.26.9 (Level 1)

*Topic(s):* capacitance, equivalent capacitance
*Answer:*
a) **Capacitor on right has bigger capacitance.**
b) **Both have same V**.
c) **One on right has bigger E.**

## P.26.10 (Level 2)

*Topic(s):* capacitance, equivalent capacitance
*Answer:*
**Battery of series circuit is larger**. Use V=Q/C and Q single=10; Q series=6-7 (for each).

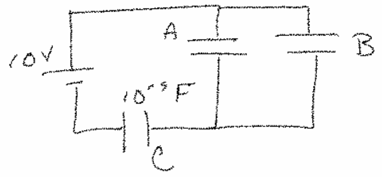
## P.26.11 (Level 3)

*Topic(s):* capacitance, equivalent capacitance

*Answer:*

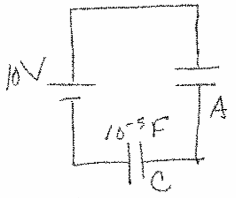
**The voltage across C should be biggest when both switches are closed.**

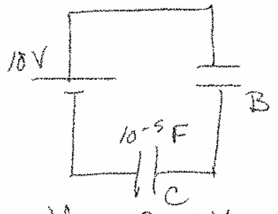
Both switches closed:



$V_C = 9.14$ V $\quad V_A = V_B = 0.86$

One switch open, one closed:





$V_c = 5.71$ V $\quad V_A = 4.29$ V $\quad V_C = 9.52$ V $\quad V_B = 0.48$ V

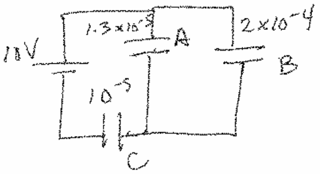
With one switch open and one closed, the charge on capacitor C and the other capacitor must

$Q = CV \quad Q_c = Q_A \qquad Q_C = Q_B$

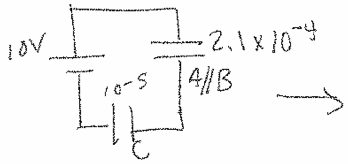
$Q_C = (5.71)(10^{-5}) = 5.71\times10^{-5}C = Q_A \qquad Q_c = (9.52)(10^{-5}) = 9.5\times10^{-5}C = Q_B$

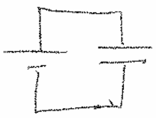
$C_A = Q_A/V = 1.3\times10^{-5}F \qquad C_B = Q_B/V_A = 1.98\times10^{-4}$ F

Further, when capacitors A and B are in series, their total capacitance adds. This means that when both switches are closed, the equivalent capacitance is greater so that the charge stored should be greater as well and the voltage across capacitor C should be the biggest of all the configurations with the switches (and it is NOT).



A and B in parallel $C_{A//B} = C_A + C_B$





$C_{eq} = \left(\frac{1}{C_c} + \frac{1}{C_{A//B}}\right)^{-1} = 9.5\times10^{-6}$

$Q_T = CV = 9.5\times10^{-5}$

$Q_c = Q_{A//B}$ so $V_c = Q_c/10^{-5} = 9.55$ V

and $V_{A//B} = 0.45$ V

which does not agree with the animation. So the animation does not correctly show the voltages across the capacitors.

## P.26.12 (Level 3, calculus)

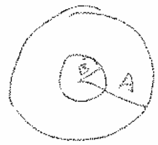
*Topic(s):* charge distribution, voltage

*Answer:*

a) V=kQ(1/a-1/b) so Q=V/(k(1/a-1/b)) where b is the outer radius and a is the inner sphere radius.

b) C=Q/V=1/(k(1/a-1/b)) so as b increases, C decreases (as expected) until C=a/k.

a. $\vec{E} = k\, Q/r^2\, \hat{r}$



$$V = -\int \vec{E}\cdot d\vec{r} = -\int_A^B k\, Q/r^2\, dr = + kQ/r \Big|_A^B$$

$$= kQ\left(\frac{1}{B} - \frac{1}{A}\right)$$

$B = 2\text{ cm} = 0.02\text{ m}$

$A = 9.6\text{ cm} = 0.096\text{ m}$

$$50 = kQ\left(\frac{1}{.02} - \frac{1}{.096}\right)$$

(which agrees with animation)

$$Q = 1.4\times10^{-10} = 140\,nC$$

$$Q = \frac{V}{k}\,\frac{1}{\left(1/B - 1/A\right)} = \left(\frac{V}{k}\right)\frac{BA}{(A-B)} = 1.1\times10^{-10}\,\frac{A}{(A-0.02)}$$

$$Q(R) = 1.1\times10^{-10}\,\frac{R}{(R-0.02)} \quad (R \text{ in meters})$$

b. $C = Q/V$

$$C = Q/V = \frac{\cancel{Q}}{k\cancel{Q}\left(\frac{1}{B} - \frac{1}{A}\right)} = \frac{1}{k}\left(\frac{AB}{A-B}\right)$$

$$\boxed{C = 2.2\times10^{-12}\,\frac{R}{(R-.02)}} \quad (R \text{ in meters})$$

c. $C = 2.2\times10^{-12}\,\dfrac{1}{(1-.02/R)}$

as the radius increases, the capacitance decreases which agrees with the animation since the voltage is constant but the charge stored decreases with increasing radius.

# Chapter II.27: Magnetic Fields and Forces

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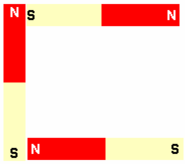
## *Illustrations*

### Illustration 27.1: Magnets and Compass Needles

*Topic(s):* magnets, magnetic field
*Purpose:* To connect permanent magnets with magnetic field vectors and compasses.
*Suggested Use:* Demonstrate how the compass maps out the magnetic field. Try a variety of permanent magnet configurations: have students predict the field first and then build it. Try, for example, making a "horseshoe" magnet:



### Illustration 27.2: Earth's Magnetic Field

*Topic(s):* magnets, magnetic field
*Purpose:* To demonstrate the Earth's magnetic field.
*Suggested Use:* Students should use the "compass" to map the magnetic field and find that the geographic "north pole" is really a magnetic "south pole."

### Illustration 27.3: The Mass Spectrometer

*Topic(s):* Lorentz force
*Purpose:* To demonstrate an application of magnetic forces on charged particles.
*Suggested Use:* After students predict what should happen when B=0 and when E=0, set E=vB so that the particles are "selected." Students should be able to predict what will happen as the charge and mass are changed. Pick a charge and mass and have students predict different values of charge and mass that end in collision at the same spot (q/m the same).

### Illustration 27.4: Magnetic Forces on Currents

*Topic(s):* Lorentz force
*Purpose:* To show the magnetic force on the current carriers in a wire.
*Suggested Use:* Have students use q**vxB** to predict the forces on the moving electrons and check their predictions with the animation. From here it is fairly easy for students to make predictions about force on wires in magnetic fields (for example, try a demonstration of a bar magnet next to a freely hanging wire carrying a couple amps of current (short-circuit a rechargeable D-cell battery).

### Illustration 27.5: Permanent Magnets, Ferromagnetism

*Topic(s):* magnets
*Purpose:* To demonstrate a simplified model of ferromagnetism.

*Suggested Use:* Discuss magnetization and how ferromagnetic materials retain their magnetization even when the external field is zero. Also discuss the impact of increasing the temperature of ferromagnetic materials (when ferromagnetic materials reach their Curie point, they are no longer magnetized).

## Explorations

### Exploration 27.1

*Topic(s):* magnetic field, Lorentz force

### Exploration 27.2

*Topic(s):* Lorentz force
20x100m/s

### Exploration 27.3

*Topic(s):* Lorentz force
a) E=25N/C; d) $m=qB^2R/E$

## Problems

### P.27.1 (Level 1)

*Topic(s):* magnets
*Answer:*
**Green: south pole; Yellow: north pole**

### P.27.2 (Level 1)

*Topic(s):* magnetic field
*Answer:*
**Configuration 3.**

### P.27.3 (Level 2)

*Topic(s):* Lorentz force
*Answer:*
**Animation 2**. F=0 (B-field in –x and velocity in +x directions)

### P.27.4 (Level 1)

*Topic(s):* Lorentz force, current
*Answer:*
**Configuration 1**: current to left B out of screen
**Configuration 2**: current to left, B out of screen

### P.27.5 (Level 2)

*Topic(s):* magnet, Lorentz force
*Answer:*
a) **Green: south pole; Blue: north pole**.

b) **Animation 1**

## P.27.6 (Level 3)

*Topic(s):* Lorentz force

*Answer:*

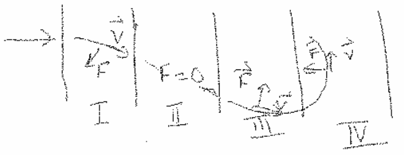
a) **I into screen; II 0; III=IV out of screen**

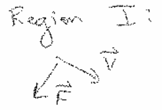
b) **III=IV>I>II**

c) **same shape, just curves up first (would look like running the animation in reverse)**

d) **decrease speed: R=mv/qB and for smaller R, decrease v**.

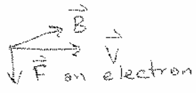
a.



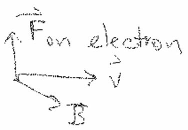


$\vec{B}$ into or out of screen?

Into screen:



Out of screen:



Region I: into screen; Region II: $\vec{B} = 0$

Region III and IV: out of screen

b. $qvB = ma = m\, v^2/R$

$R = \frac{mv}{q} \frac{1}{B}$ so smaller B field, larger R

radius of Region I > radius of III and IV

Guess: $|B_{III}| = |B_{IV}| > |B_{I}| > |B_{II} = 0|$

Check: Region I:

| $t$ | $v_x$ | $v_y$ |
|---|---|---|
| 1 | 83.28 | -34.12 |
| 1.2 | 74.84 | -50 |

$|v| = 90$ cm/s

$a_x = 42.2$

$a_y = 79.4$

$|a| = 90$ cm/s$^2$

Region III:

| $t$ | $v_x$ | $v_y$ |
|---|---|---|
| 2 | 85.27 | -28.79 |
| 2.2 | 89.75 | 6.69 |

$|v| = 90$ cm/s

$a_x = 22.4$

$a_y = 177.4$

$|a| = 180$ cm/s$^2$

Region IV

| $t$ | $v_x$ | $v_y$ |
|---|---|---|
| 2.5 | 70.30 | 56.20 |
| 2.7 | 42.87 | 79.14 |

$|v| = 90$ cm/s

$a_x = 137.15$

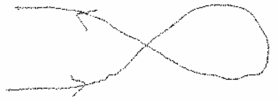
$a_y = 114.7$

$|a| = 179$ cm/s$^2$

Since $F = q|v||B|\underbrace{\sin\phi}_{1} = ma$

Ranked by magnitude of magnetic III = IV > I > II

c.



d. Want smaller radius in I so:

$R = \frac{mv}{qB}$ decrease speed

## P.27.7 (Level 3)

*Topic(s):* Lorentz force

*Answer:*

a) **magenta=red (120 cm/s)>yellow=blue=green(60 cm/s)>cyan(30 cm/s)**

b) **|v| constant**

c) **cyan=yellow (-6 mC)<magenta (0)<blue=red(6 mC)<green(12 mC)**
d) **B=0.83 T**

a. magenta and red arrive first so have highest speeds; then blue, yellow and green arrive with the next highest speeds; finally, the slowest one-cyan arrives.

Speed: $\underline{\text{magenta} = \text{red}} > \underbrace{\text{blue} = \text{yellow} = \text{green}} > \text{cyan}$

$v_i = \frac{\Delta x}{\Delta t} = \frac{54}{.45} \approx 120 \text{ cm/s}$ $v_i = 60 \text{ cm/s}$ $v_i = \frac{50}{1.65} = 30 \text{ cm/s}$

b. | v | stays the same (uniform circular motion)

c. $\vec{F} = q\vec{v}\times\vec{B} = m\vec{a}$

$qvB = m\, v^2/R$

$q = \left(\frac{m}{B}\right) v/R$

Blue: $R = 60$ cm $q = 6$ mC

Green: $R = 30$ $v$ same so $q > 6$ mC; $q = 12$ mC

Yellow: $R = 60$ $q = -6$ mC

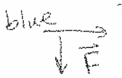
Magenta: $q = 0$

Red: $R = 120$, speed = 2× (speed of blue) ≈ 120 cm/s
so in comparison with blue, 2v and 2R so
$q_{red} = 6$ mC

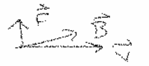
Cyan: $R = 30$ speed ≈ ½ (speed of blue) = 30 cm/s
so in comparison with blue, ½v and ½R so
$q = -6$ mC

Charge (lowest → highest):
cyan = yellow < magenta < blue = red < green

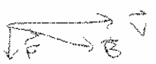
d.



$\vec{B}$ into screen:



$\boxed{\vec{B} \text{ out of screen}}$:



$qvB = m\, v^2/R$

$B = \frac{m}{q}\, v/R$ blue: $m = 0.005$ $q = 6\times10^{-3}$ C
$v = 0.6$ m/s $R = 0.6$ m

$B = \left(\frac{(.005)}{6\times10^{-3}}\right)\left(\frac{0.6 \text{ m/s}}{0.6 \text{ m}}\right) = \underline{0.83 \text{ T}}$

## P.27.8 (Level 2)

*Topic(s):* Lorentz force
*Answer:*
a) **negative charge** (curves in B-field)

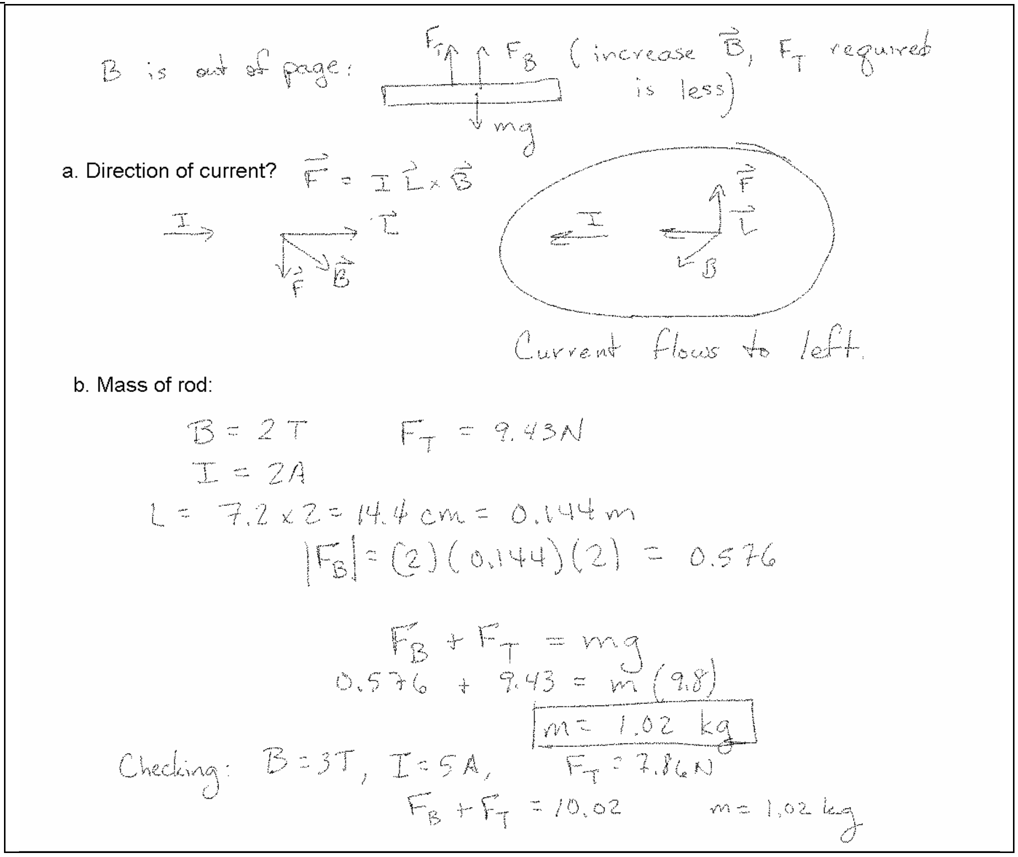
b) **2 T**. E=vB, v=40m/s
c) **2 C/kg**. q/m=v/RB, R=10m.

## P.27.9 (Level 2)

*Topic(s):* Lorentz force
*Answer:*
a) **positive**
b) **37.5 V/m**. E-field only: $a_y$=5000m/s2. ma=qE.
c) **0.75 T**. vB=E or B-field only: a=4970m/$s^2$ (measure changes in vx and vy) and use ma=qvB.
d) **2.25 V**. V=Ed and d=.06m.

## P.27.10 (Level 2)

*Topic(s):* Lorentz force
*Answer:*
a) **left**.
b) **m=1 kg**. $F_{mag}$=ILB as $F_{mag}$ increases, F_measured decreases: $F_{mag}$ up. Mg- $F_{mag}$ =F_measured.



B is out of page:

( increase $\vec{B}$, $F_T$ required is less)

a. Direction of current? $\vec{F} = I\vec{L}\times\vec{B}$

Current flows to left.

b. Mass of rod:

$B = 2T$ $F_T = 9.43N$
$I = 2A$
$L = 7.2 \times 2 = 14.4 \text{ cm} = 0.144 \text{ m}$
$|F_B| = (2)(0.144)(2) = 0.576$

$F_B + F_T = mg$
$0.576 + 9.43 = m(9.8)$
$\boxed{m = 1.02 \text{ kg}}$

Checking: $B = 3T$, $I = 5A$, $F_T = 7.86N$
$F_B + F_T = 10.02$ $m = 1.02 \text{ kg}$

# Chapter II.28: Ampere's Law

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## *Illustrations*

### Illustration 28.1: Fields from Wires and Loops

*Topic(s):* current, magnetic field
*Purpose:* To show the magnetic fields from a variety of current carrying wires.
*Suggested Use:* After looking at the magnetic field from one wire, have students make predictions about the field from the different configurations. Students should be able to use a right-hand rule to find the direction of the magnetic field near a wire. Note that the convention is that red means current out of the screen and blue means current into the screen.

### Illustration 28.2: Forces between Wires

*Topic(s):* Lorentz force, magnetic field
*Purpose:* To demonstrate the forces between wires.
*Suggested Use:* Have students use I**LxB** to predict the forces on the wires and check their predictions with the animation. Students should also be able to determine which direction current is flowing in the center wire and recognize that the forces shown do constitute a Newton's 3rd law force pair.

### Illustration 28.3: Ampere's Law and Symmetry

*Topic(s):* symmetry, magnetic field, Ampere's law
*Purpose:* To demonstrate symmetries useful for Ampere's law.
*Suggested Use:* Compare the symmetry present in the single wire and two wire cases. At far enough distances away, the two wire case can be approximated by a single wire carrying the same total current. Have students explain what "far away" is for different spacing between the wires.

### Illustration 28.4: Path Integral

*Topic(s):* Ampere's law
*Purpose:* To demonstrate Ampere's law.
*Suggested Use:* Students should trace different closed loops around the current carrying wires (and around regions where there is no current) to verify Ampere's law. This Illustration can really help students "visualize" a path integral as it calculates the path integral for the dragging mouse. Extension: Have students calculate the current in the wires (magnitude=1A).

## *Explorations*

### Exploration 28.1

*Topic(s):* Ampere's law

## Exploration 28.2

*Topic(s):* Ampere's law, symmetry

## Exploration 28.3

*Topic(s):* Lorentz force, magnetic fields

## *Problems*

## P.28.1 (Level 1)

*Topic(s):* Lorentz force

*Answer:*

a) **2**

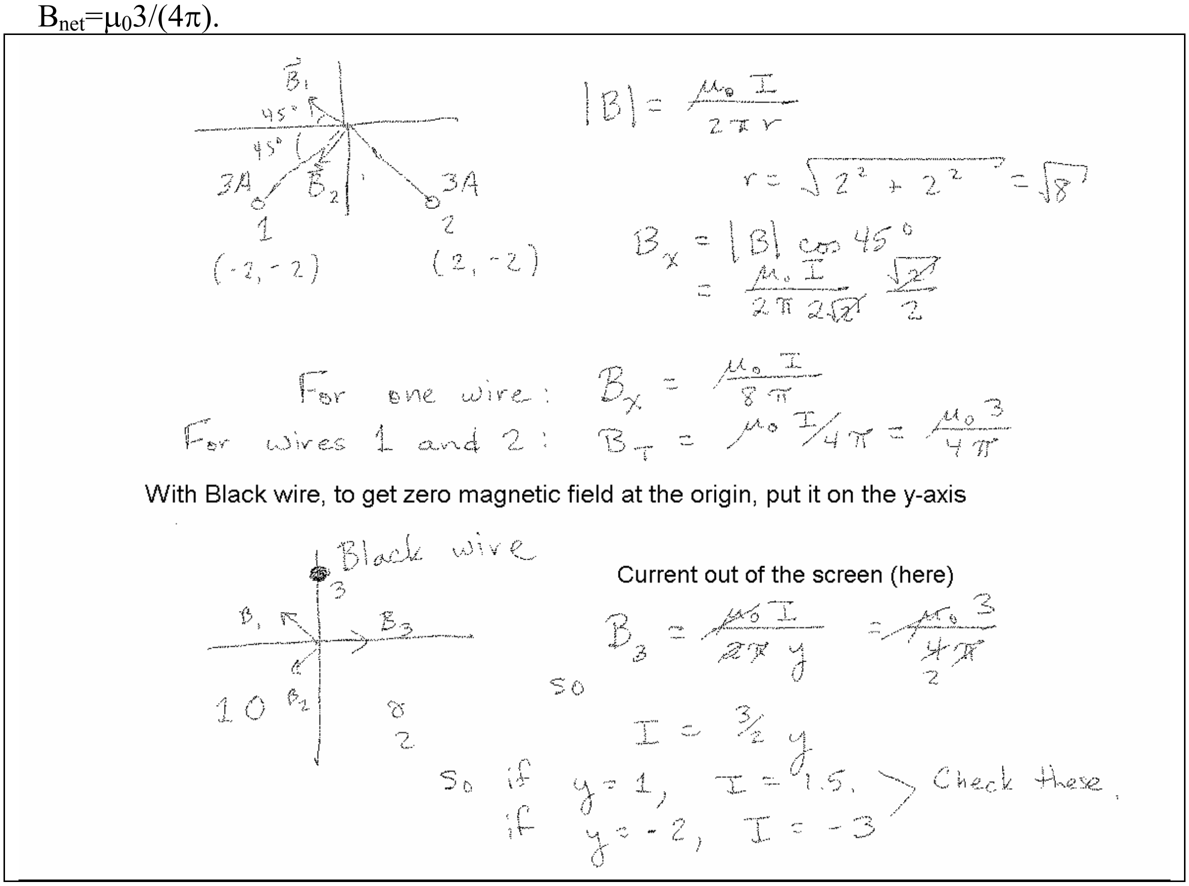
b) **1**

## P.28.2 (Level 2)

*Topic(s):* magnetic field

*Answer:*

**y=(2/3)I**. One red wire: |B|=$\mu_0$I/(2π2√2) so Bx= |=$\mu_0$I/(2π2√2)cos45; both wires $B_{net}$=$\mu_0$3/(4π).



$$|B| = \frac{\mu_0 I}{2\pi r}$$

$$r = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$B_x = |B| \cos 45^\circ = \frac{\mu_0 I}{2\pi\, 2\cancel{\sqrt{2}}} \frac{\cancel{\sqrt{2}}}{2}$$

For one wire: $B_x = \frac{\mu_0 I}{8\pi}$

For wires 1 and 2: $B_T = \mu_0 I/4\pi = \frac{\mu_0 3}{4\pi}$

With Black wire, to get zero magnetic field at the origin, put it on the y-axis

Current out of the screen (here)

$$B_3 = \frac{\cancel{\mu_0} I}{\cancel{2\pi} y} = \frac{\cancel{\mu_0} 3}{\cancel{4\pi}_{2}}$$

so $I = \frac{3}{2} y$

So if $y = 1$, $I = 1.5$, if $y = -2$, $I = -3$ > Check these.

## P.28.3 (Level 2, calculus)

*Topic(s):* Ampere's law
*Answer:*
**Pink=10A into screen; Yellow=10A out of screen; Green=5A out of screen; Cyan=15A into screen**.

## P.28.4 (Level 1, calculus)

*Topic(s):* Ampere's law
*Answer:*
a) **top=20x$10^{-6}$ Tm, bottom=-20x$10^{-6}$ Tm, sides=0**
b) **0**, no current inside box

## P.28.5 (Level 2)

*Topic(s):* Ampere's law
*Answer:*
a) **no external magnetic fields**
b) **symmetry and current enclosed equals zero**
c) **I=0.1 A**

## P.28.6 (Level 2)

*Topic(s):* Lorentz force
*Answer:*
**0.7A.** current off: at y=2cm, x=0, B=4.97x$10^{-5}$ T. Current on: F=3.5x$10^{-5}$ at same position. Use F=ILxB so F/L=IB.

## P.28.7 (Level 2, calculus)

*Topic(s):* Ampere's law
*Answer:*
a) **out of the screen.**
b) **25A.** Outside the wire, use $B=\mu_0 I/2\pi r$.

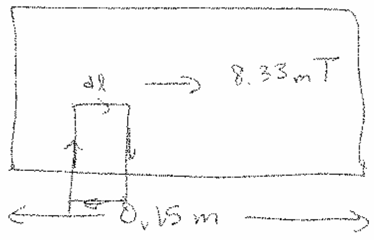
## P.28.8 (Level 3, calculus)

*Topic(s):* Ampere's law
*Answer:*
a) **0.**
b) **6 A/cm.** Plate is in uniform field: current from plate creates 0.38 above (adds to external field) and 0.38 beneath (subtracts from external field) so find current/length to create B=0.38mT: $2BL=\mu_0 KL$ where K=current/meter.

## P.28.9 (Level 3, calculus)

*Topic(s):* Ampere's law
*Answer:*
a) **Integral is zero for all sides except top.** For I=2.526, current enclosed=83.8A; I/cm=33.5 A/cm.
b) **Depends on the length.**

c) **199 turns**.
d) **B=25I/L** (for B in mT and L in cm). B=$\mu_0$NI/L.



$I = 5A$

a. $\int \vec{B} \cdot d\vec{l} = 0$ for all sides except middle because $\vec{B}$ and $d\vec{l}$ are perpendicular to each other on the left + right sides and $\vec{B} = 0$ outside the solenoid.

Middle: $\int \vec{B} \cdot d\vec{l} = \int B dl = B \int dl = BL$

$\vec{B}$ + $d\vec{l}$ are parallel

B is constant along path

$\oint \vec{B} \cdot d\vec{l} = BL = (8.33\,mT)(0.025\,m) = 2.08 \times 10^{-4}$

$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ $\quad I_{enc} = \frac{2.08 \times 10^{-4}}{4\pi \times 10^{-7}} = 165\,A$

b. $165A / 0.025\,m = 6600\,A/m$

Total length = 0.015 m so total current = 990 A

c. Each loop has 5A so $\underline{198 \text{ turns}} \approx 200$ turns

d. $BL = \mu_0 I_{enc}$ $\quad I_{enc} = LnI$ $\quad n = \#$ of turns/length

$I$ = current in 1 loop

$B\cancel{L} = \mu_0 \cancel{L} n I$ $\quad B = \mu_0 n I$

$n = 200/\text{length}$

$B = \frac{(\mu_0)\, 200\, I}{\text{length}} = 2.5 \times 10^{-4}\, I/\text{length}$

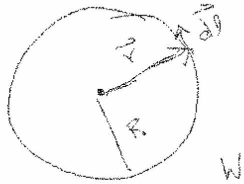
$\underline{B(mT) = 25\, I/\text{length(cm)}}$ ← B in mT, length in cm

## P.28.10 (Level 3, calculus)

*Topic(s):* Biot-Savart law
*Answer:*
a) **30A**.

b) **B=18.5/R** where B in mT and R in mm.

a.



$\vec B$ at center points out of page.
(to the right in Physlet)

When $R = 2$ mm, $\vec B$ (center) = 9.32 mT

$$d\vec B = \frac{\mu_0 I}{4\pi} d\vec l \times \vec r / r^3 \qquad \vec r = R\hat r$$

$$|d\vec B| = \frac{\mu_0\, dl\, RI}{4\pi R^3} \qquad d\vec l \times \vec r = (dl) R$$ in direction out of screen

$$|\vec B| = \int \frac{\mu_0 I}{4\pi R^2} dl = \frac{\mu_0 I}{4\pi R^2}\int dl = \frac{\mu_0 I}{R^2} L$$

$$L = 2\pi R$$

$$|\vec B| = \frac{\mu_0 I}{\cancel{4\pi}_{2} \cancel{R^2}} \cancel{2\pi R} = \frac{\mu_0 I}{2R}$$

$$|\vec B(R=0.002)| = 9.32 \text{ mT}, \quad I = \frac{(9.32\times10^{-3})2(0.002)}{4\pi\times10^{-7}}$$

$$I = 29.7 \text{ A} \approx \underline{30A}$$

b.

$$|\vec B| = \left(\frac{\mu_0 I}{2}\right)\frac{1}{R} = \underline{(1.86\times10^{-5})\frac{1}{R}}$$

$|B| = \underline{18.5/R}$ for B in mT and R in mm

Check: R=1 mm $|B| = 18.46$ mT ✓

# Chapter II.29: Faraday's Law

## *Illustrations*

### Illustration 29.1: Varying Field and Varying Area

*Topic(s):* flux, emf, Faraday's law
*Purpose:* To demonstrate the connection between flux and induced emf.
*Suggested Use:* Have students examine the graphs of flux and emf, describing the differences. Students should be able to predict one given the other one. Students should also be able to explain (have them explain to a peer) how what is happening to the magnetic field or the loop relates to the graphs.

### Illustration 29.2: Loop in a Changing Vector Field

*Topic(s):* flux, emf, Faraday's law
*Purpose:* To illustrate the different ways to have a changing flux.
*Suggested Use:* Students should be able to explain that the rate of change of the flux is the same for both animations (given the same slider values) and that the difference between whether the field is increasing and decreasing or rotating simply changes the way you calculate the flux. Exploration 29.4 asks students to do more quantitative work with this same animation.

### Illustration 29.3: Electric Generator

*Topic(s):* flux, emf, Faraday's law
*Purpose:* To demonstrate an application of Faraday's law: generating electricity.
*Suggested Use:* Have students describe different ways to change the magnetic flux through a wire loop. Students should be able to explain what happens in this illustration and connect it to what happens in a demonstration Genecon (hand generator) or bicycle wheel generator. Extension: Have students calculate the magnetic field (can use flux view or normal view. Answer: 0.2T)

## *Explorations*

### Exploration 29.1

*Topic(s):* Lenz's law

### Exploration 29.2

*Topic(s):* Faraday's law, Lorentz force, power

### Exploration 29.3

*Topic(s):* flux, emf, Faraday's law

### Exploration 29.4

*Topic(s):* Faradays' law, emf

## Exploration 29.5

*Topic(s):* inductance, Faraday's law, emf

## *Problems*

### P.29.1 (Level 2)

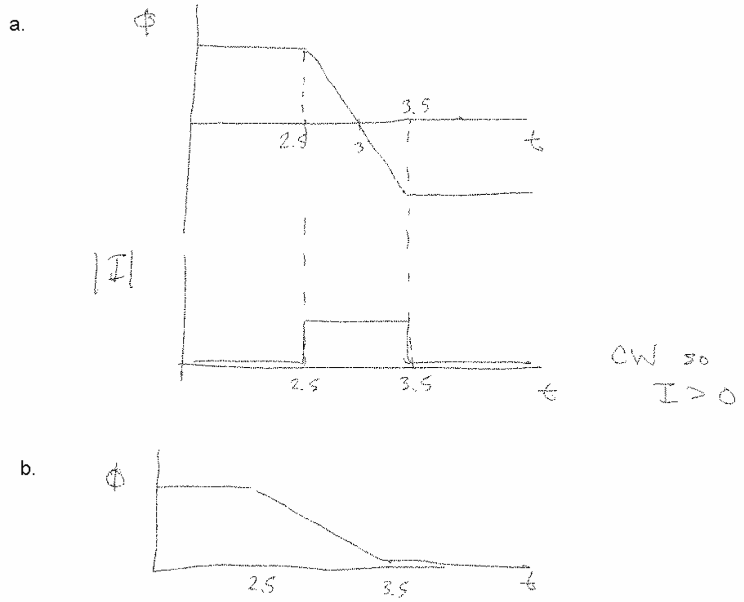
*Topic(s):* Lenz's law
*Answer:*

a) **A=CCW (counter clock-wise); B=0; C=CW(clock-wise)**
b) **A=CW; B=0; C=CCW**
c) **A=CW; B=0; C=CCW**

### P.29.2 (Level 2)

*Topic(s):* Faraday's law
*Answer:*

a) **I=0 for t<2.5s and t>3.5s, I>0 (constant value) for 2.5<t<3.5**.
b) **I would be smaller** because smaller change in flux for the same time.



so current would be less because

. Smaller ΔΦ in same Δt.

### P.29.3 (Level 2)

*Topic(s):* Faraday's law, Lorentz force
*Answer:*

a) **0.15A**. I=BLv/R, v=3m/s, L=5m.
b) **F=ILxB in left wire is opposite direction of pulling force**
c) **F=ILB=1.5N**
d) **accelerate to right**

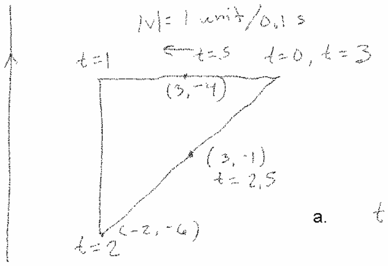
## P.29.4 (Level 3)

*Topic(s):* Lenz's law, Faraday's law

*Answer:*

a) **I=0 for t=1.5 and t=3.5; I=CW for t=2.5; I=CCW for t=0.5 and t=4.5.**

b) **I(t=2.5)=I(t=0.5)>I(t=1.5)=I(t=3.5)=0.** Note that the solution below includes the time t=4.5 s in the answer for part (b). We deemed this to be too difficult for students but left in the solution for instructors' purposes.



From t = 4 to 4.5

r increased 1 unit

a. t = 1.5 no change in flux (same distance from long wire)

t = 3.5 I = 0

t = 0.5, increasing flux into screen so induced current to create flux out of screen: CCW ↺

t = 2.5 decreasing flux into screen so ↻ CW

t = 4.5 radius increasing so increasing flux into screen so ↺ CCW

b. t = 0.5 10 m/s in x-direction

t = 2.5 s $v_x = 5/.5 = 10\,m/s$ so moves away from long wire at same rate as moved toward it (and same distance away)

so $\left|I(t=0.5)\right| = \left|I(t=2.5s)\right|$

t = 4.5 s R changes 1 unit in 0.5 s but B is lower

$$t = 0.5/2.5:\quad \frac{d\phi}{dt} = A\frac{dB}{dt} = A\frac{\mu_0 I}{2\pi}\frac{d}{dt}\left(\frac{1}{r}\right) = A\frac{\mu_0 I}{2\pi}\left(\frac{-1}{r^2}\right)v$$

$$\left|\frac{d\phi}{dt}\right| = AB\,v/r \qquad r = 13 \quad v = 10\,m/s$$

$$\approx (\cancel{\pi})(2)^2\frac{\mu_0 I}{\cancel{2\pi}(13)}\frac{10}{13} \overset{x}{\approx} \frac{\mu_0 I\,20}{169}$$

$$t = 4.5:\quad \frac{d\phi}{dt} = B\frac{dA}{dt} = B\pi 2R\frac{dR}{dt} \qquad dR/dt = 1/.5 = 2$$

$$= B\pi 2(3)(2) \approx \frac{\mu_0 I}{\cancel{2\pi}(\cancel{18})_3}\cancel{\pi}\,\cancel{2}\cdot\cancel{6} = \frac{\mu_0 I}{3}$$

So $\frac{d\phi}{dt}$ at t = 4.5 s is greater.

(Note: this assumes $\Phi = BA$ when actually $\Phi = \int B\,dA$ and B is $\underline{\text{not}}$ constant)

$$\text{So}\quad \left.\frac{d\Phi}{dt}\right|_{t=4.5} > \left.\frac{d\Phi}{dt}\right|_{t=0.5} = \left.\frac{d\Phi}{dt}\right|_{t=2.5} > \left.\frac{d\Phi}{dt}\right|_{\substack{t=1.5\\t=3.5}}$$

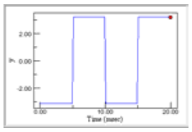
$$\left|I(t=4.5s)\right| > \left|I(0.5) = I(2.5)\right| > \left|I(1.5) = I(3.5)\right|$$

## P.29.5 (Level 2)

*Topic(s):* Faraday's law, emf

*Answer:*

a)



b) **3.7V**. dB/dt=1.2/.005; A=$\pi(.07)^2$.

c) **current vs time~emf vs time**

## P.29.6 (Level 2)

*Topic(s):* Faraday's law, emf

*Answer:*

**up**

## P.29.7 (Level 3)

*Topic(s):* Faraday's law, emf

*Answer:*

**Animation 1.** As N pole enters, induced current CW (negative emf) in center of magnet, field is constant so emf=0, as magnet exits, induced current CCW (flux dropping off).

## P.29.8 (Level 2)

*Topic(s):* Faraday's law, emf

*Answer:*

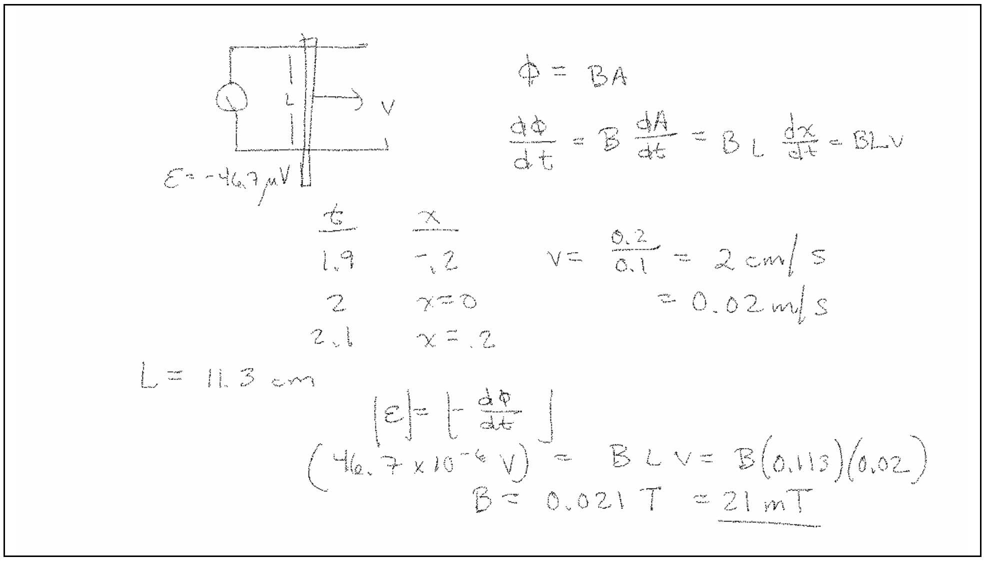
**Out of screen.** As you reduce the area, CCW current induced (to keep a higher flux out of screen so B must be out of screen).

## P.29.9 (Level 3)

*Topic(s):* Faraday's law, Lorentz force, emf

*Answer:*

**21 mT**.



$$\Phi = BA$$

$$\frac{d\Phi}{dt} = B\frac{dA}{dt} = BL\frac{dx}{dt} = BLv$$

| $t$ | $x$ |
|---|---|
| 1.9 | -.2 |
| 2 | $x=0$ |
| 2.1 | $x=.2$ |

$$v = \frac{0.2}{0.1} = 2\ \text{cm/s} = 0.02\ \text{m/s}$$

$L = 11.3$ cm

$$|\mathcal{E}| = \left|-\frac{d\Phi}{dt}\right|$$

$$(46.7\times10^{-6}\ V) = BLv = B(0.113)(0.02)$$

$$B = 0.021\ T = \underline{21\ mT}$$

## P.29.10 (Level 3)

*Topic(s):* Faraday's law, emf

*Answer:*

a) **0.07T**. T=3.14, ω=2 and $\text{emf}_{max}=\omega B_{max}A$ and $A=0.012\text{m}^2$.

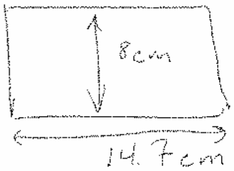
b) **CCW**

a.

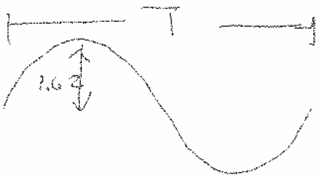
$$\mathcal{E} = -\frac{d(B\cdot A)}{dt} = -BA\frac{d}{dt}\cos\theta \qquad \theta = \omega t$$

$$= -BA\frac{d}{dt}(\cos\omega t)$$

$$\mathcal{E} = BA\omega\sin\omega t$$



$$A = (0{,}08)(.147) = 0{,}01176\ m^2$$



$$T = 3{,}14\,s$$

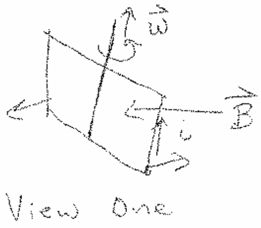
$$\omega = \frac{2\pi}{T} = 2\ rad/s$$

$$\mathcal{E} = (1.62\ \sin 2t)\,mV \leftarrow \text{from plot}$$

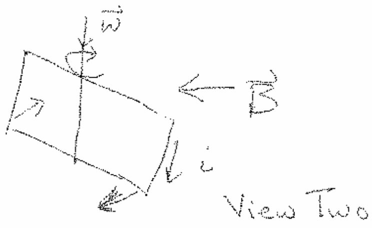
$$0{,}00162\ (\sin 2t) = B\,(0{,}01176)(2)\ \sin 2t$$

$$B = 0{,}07\ T$$

b.



OR



For view one, as it rotates, the flux decreases so the current must be induced to increase the flux, (opposing change) resulting in the current as labeled (down in righthand wire. The converse is true of view two so you would expect the current drawn in view two. If you look at the animation, you notice that the right wire has upward current; thus, view one is correct. Looking down from above, you would see a loop rotating counter clockwise.

## P.29.11 (Level 3)

*Topic(s):* Faraday's law, inductance

*Answer:*

a) **0.9 mH.** emf=L(dI/dt)=ΔL/Δt=L10/4 to solve for L

b) **250 loops/m.** $B=\mu_0 nI$ to solve for n.

c) **1-m.** Use $L=\mu_0 N^2 A/\text{length}==\mu_0 n^2 A\text{length}$ to find length.

# Chapter II.30: DC Circuits

## *Illustrations*

### Illustration 30.1: Complete Circuits

*Topics:* voltage, current
*Purpose*: To introduce the idea of a complete circuit and how to determine the voltages and currents open circuits.
*Suggested Use*: Have students predict the voltages for the possible broken bulbs. Students do not find it surprising that if you have no current and if the wire is not broken, then the voltage across the wire is zero, but they do find it surprising that the voltage difference between one broken wire and another does not have to be zero. Here, it may be useful for students to think of two wires not connected to anything and simply attached to two ends of a battery. Students having difficulty with Problem 30.1 should make sure they understand this Illustration completely.

### Illustration 30.2: Circuits, Voltage and Current

*Topics:* voltage, current, switches
*Purpose*: To help students understand voltages in complete and open circuits.
*Suggested Use*: This is an extension of Illustration 30.1. Again, students should try to predict the voltage readings.

### Illustration 30.3: Current and Voltage Dividers

*Topics:* voltage, current, resistance
*Purpose*: To introduce voltage dividers and current dividers.
*Suggested Use*: Provides a practical application of using resistor networks to make voltage and current dividers. Students should develop a feel for how the current and voltages change in these "dividers" as the resistance changes. Students find it surprising that increasing R can either increase or decrease current from the source depending on the configuration.

### Illustration 30.4: Batteries and Switches

*Topics:* switches, batteries, voltages
*Purpose*: To distinguish between batteries added in parallel and in series.
*Suggested Use:* Students are very surprised that when **S3 closes nothing happens** (often they will need a real demonstration of this because many are convinced that the animation is wrong because they believe more batteries necessarily means brighter lights). Students should use this to explain what happens when batteries are added in series in comparison with adding batteries in parallel.

### Illustration 30.5: Ohm's "Law"

*Topics:* Ohm's law

*Purpose*: To show that there are electronic (non-linear) devices that do not obey Ohm's law so it is not a "law" in the same sense as Newton's Laws.
*Suggested Use*: Observe differences between the curves. Note that students can find resistance of bulb: 150 Ω as well as Bulb—low current/voltage part of curve and higher current/voltage. Diode allows current in one direction (I=0 when reverse biased) and has a voltage drop (in this case) of about 0.5 V across it.

### Illustration 30.6: RC Circuit

*Topics:* switched RC
*Purpose*: To introduce switched RC circuits and the voltage across different elements when the capacitor is charging and discharging.
*Suggested Use*: Students should explain why current flows (why the bulb lights) when the capacitor is charging and discharging (and why the bulb goes out once the capacitor is fully charged). Students are often surprised that the bulb lights in both cases—the graphs can help, but the resistor voltage going from negative1V to 0 while discharging is often surprising until they realize it is simply an expression of the current going in the other direction.

### Illustration 30.7: The Loop Rule

*Topics:* Kirchhoff's loop rule
*Purpose*: To demonstrate the voltage drops across circuit elements and the use of Kirchhoff's laws.
*Suggested Use*: Students should first predict the voltage as a function of time (as the charge goes around the loop) and then compare their predictions to the plot.

## *Explorations*

### Exploration 30.1

*Topics:* voltage, current, resistance

### Exploration 30.2

Topics: switches, resistance, voltage, current

### Exploration 30.3

Topics: voltage, resistance, power

### Exploration 30.4

*Topics:* current, meters, internal resistance

### Exploration 30.5

***Topics: voltage, meters, internal resistance***

### Exploration 30.6

*Topics:* switched RC

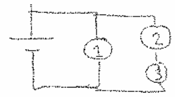
## *Problems*

### P.30.1 (Level 2)

*Topic(s):* switches, voltage, current

Answer:

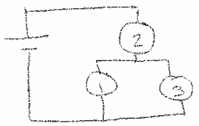
a. Circuit 1: Three Bulbs



when Bulb 1 is unscrewed,it doesn't change Bulbs 2&3 so it must
be in parallel with them.

when Bulb 2 or 3 is unscrewed, the other one goes out (no current) so they are in series with each other.

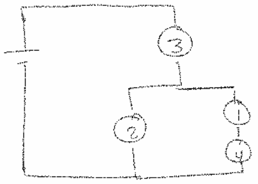
b. Circuit 2: Three Bulbs



Bulb 2 is brighter than 1 and 3 and when it is removed, all bulbs go out.

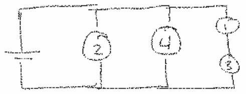
When either 1 or 3 is removed, the remaining bulb and 2 are the same brightness.

c. Circuit 3: Four Bulbs



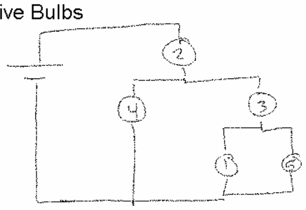
Strategy:
Look for the Bulb with the most current and remove it from the circuit. The Bulbs that go out at the same time are in series with it while the Bulbs that stay on are in a parallel branch.

d: Circuit 4: Four Bulbs

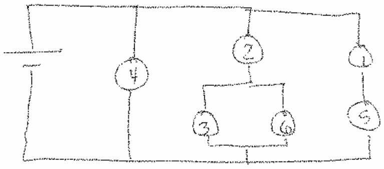


Systematically test the rest of the Bulbs to determine parallel and series branchs.

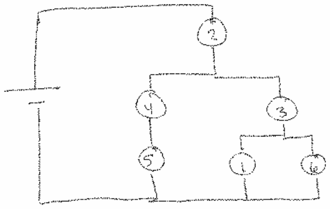
e. Circuit 5: Five Bulbs



f. Circuit 6: Six Bulbs



g. Circuit 7: Six Bulbs



## P.30.2 (Level 1)

*Topic(s):* current, voltage, resistance

Answer:

a) **as R increases, C stays the same and A & B are dimmer**.

b) **in parallel**

c) as **R increases, A & B have less current (more total resistance in the branch through A & B) so less current from the battery**

d) **No**

e) **as R increases, the voltages across A & B are less because there is less current and more voltage drops across R (total resistance of the branch increases).**

f) **constant V source, but current changes to meet the resistive load.**

## P.30.3 (Level 2)

*Topic(s):* voltage, current, power, resistance

Answer:

a) **600Ω. 15V. Pmax=321mW; Pmin=141mW**. 9V/15mA=600Ω=R. Vbattery=(400+600) Ω *.015A=15V. P=IV.

b) $V_{battery}$=**7V. R=400Ω; Pmax=653mW Pmin=212mW.** $I_{unknown}=I_{battery}-I_{variable}=23.33$ for $R_{variable}$=400 Ω.

c) $V_{battery}$=**12V R=700Ω. Pmax=180mW; Pmin=85mW.** $V_{battery}$=12V when $R_{variable}$=400 Ω, V=6V across each resistor when $R_{variable}$= 700 Ω.

Note: 3 very different circuit configurations in the parts of this problem.

## P.30.4 (Level 2)

Topic(s): current, voltage, resistance

Answer:

a) **Rb<Ra<Rc.** Switches open I=0.25 through A, S2 closed, I=0.75 so 0.50 through B, Rb<Ra. S1 closed, I=0.37 so 0.12 through C, Rc>Ra so Rb<Ra<Rc.

b) **Rc<Rb<Ra.** S1 open and S2 closed, I through Rb is 0.25. S2 open, I=0.08 so Ra>Rb (if Ra=Rb, I=0.125). S2 open and S1 closed, I=0.75 so current through Rc is 0.5 so Rc<Rb.

Note: 2 different circuit configurations in parts of this problem.

## P.30.5 (Level 2)

Topic(s):voltage, Ohm's law, power, current

Answer:

**Graphs A, B and D.** V is constant and $P=V^2/R$, I=V/R so both P vs. R and I vs. R have same shape. Graphs A, B and D are correct—others show a linear relationship between I and R and P and R.

## P.30.6 (Level 2)

Topic(s): current, voltage

Answer:

a) **10V in parallel with 20V** (two 10V in series) so short circuit batteries.
b) **short circuit battery** (through branch with switch).
c) **too much current through 100Ω** resistor (P=1W so power rating on resistor must be less than 1W).

## P.30.7 (Level 2)

Topic(s): voltage, current, resistance
Answer:

A. **Rx=162 Ω**.
B: **82 Ω**
C: **330Ω**. when I=0, voltage across 390 is the same as is voltage across variable and voltage across 561 is the same as across Rx: Voltage divider: 561/(390+561)=Rx/(Rx+Rvariable) so Rx=Rvariable(561/390). Circuit A: Rvariable=113 Ω so Rx=162 Ω. Circuit B, Rvariable=57 Ω so Rx=82 Ω. Circuit C: Rvariable=230 Ω so Rx=330Ω.

## P.30.8 (Level 3)

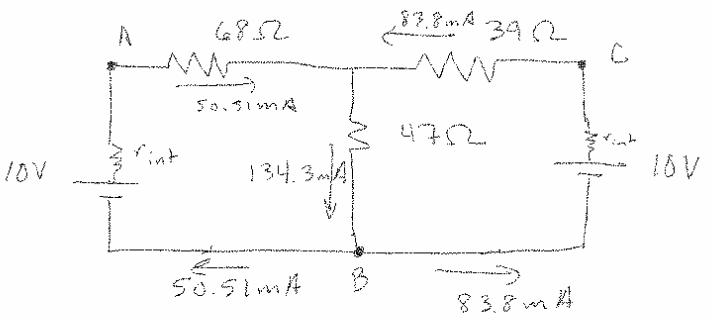
Topic(s): voltage, current, internal resistance, meters
Answer:

a) **$R_{ammeter}$=.82V/0.409A=2Ω**. Use Circuit A to find $R_{ammeter}$. Keep variable resistor small (similar to ammeter)—with $R_{variable}$=20, ammeter reads 0.409A so voltage drop across $R_{variable}$ is 8.18 so voltage drop across ammeter is 0.82V. Thus, $R_{ammeter}$=.82V/0.409A=2Ω.
b) **$R_{voltmeter}$=9V/9x10$^{-5}$A=100kΩ**. Use circuit B to find $R_{voltmeter}$—need big $R_{variable}$ (in the range of $R_{voltmeter}$). $R_{variable}$=200kΩ, $I_{variable}$=9/200kW=4.5x10$^{-5}$A which leaves 1.35x10$^{-4}$A-4.5x10$^{-5}$A through voltmeter so $R_{voltmeter}$=9V/9x10$^{-5}$A=100kΩ.
c) **Use Circuit A for unknown big resistors** (if use Circuit B, a value of R close to $R_{voltmeter}$ will give current reading for an effective resistance and result in an underestimate of resistance)
d) **Use circuit B for unknown small resistors** (with Circuit A, the voltage drop through the ammeter could be of the same size as the voltage drop through the resistor).

## P.30.9 (Level 3)

Topic(s): internal resistance, voltage, current
Answer:

**R = 5Ω**. With V=10V for both (can adjust values): V drop across 68Ω resistor is 3.43V and across 47Ω resistor is 6.31 (current through 47Ω resistor=sum of currents: 50.51mA+83.8mA) so voltage drop through left battery is 0.26V so R=.26V/.0515A=5Ω. Same type of analysis for right battery.



V across 68Ω:

$V_{68\Omega} = IR = 3.43V$

V across 39Ω:

$V_{39\Omega} = IR = 3.27V$

$V_{47\Omega} = IR = 6.31V$

$\Delta V$ from a to b $= V_{68\Omega} + V_{47\Omega} = 9.74V$

$\Delta V$ from a to b $= 10V - I r_{int} = 9.74V$

Left: $r_{int} = .26V / .05051A = \boxed{5\Omega}$

$\Delta V$ from c to b $= V_{39\Omega} + V_{47\Omega} = 9.58V$

Right: $r_{int} = \frac{(10 - 9.58)V}{.0838A} = \boxed{5\Omega}$

## P.30.10 (Level 2)

Topic(s): power, resistance, current, voltage
Answer:

Circuit A: **118Ω**. R=100 P=0.84=I2R=V2/R so $I_{variable}$=.0917A and $V_{variable}$=9.165V so $V_{unknown}$=20-9.165=10.83V. When $R_{variable}$=$R_{unknown}$, peak on power curve so simply read R for peak power:

Circuit B: **180Ω**

Circuit C: **270Ω**.

## P.30.11 (Level 1, calculus required)

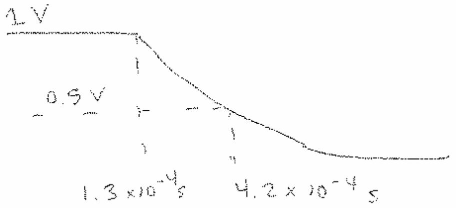
Topic(s): switched RC
Answer:

**Rb>Ra>Rc.** Circuit A: time for 1V to 0V is about .84ms. Circuit B: time is about 1.9ms and Circuit C: time is .52ms and longer time means bigger RC time constant.

## P.30.12 (Level 3, calculus required)

Topic(s): switched RC
Answer:

**C = 4.2 μF**. R=100, time for 1V to 0.5V is about .29ms. ½ =1e-(t/RC) so ln(½)=-t/RC.



$$\Delta t = 2.9 \times 10^{-4}\ \text{s}$$

$$V(t) = V_0\, e^{-t/RC}$$

$$\tfrac{1}{2}\text{V} = 1\,\text{V}\, e^{-t/RC}$$

$$\ln \tfrac{1}{2} = -t/RC$$

$$\ln 2 = t/RC$$

$$C = \frac{t}{R \ln 2}$$

$$C = \frac{2.9 \times 10^{-4}}{100 \ln 2} = 4.2 \times 10^{-6}\ \text{F}$$

$$\boxed{C = 4.2\ \mu\text{F}}$$

## P.30.13 (Level 2, calculus required)

Topic(s): switched RC
Answer:

**Graph C.** With two capacitors in parallel, total capacitance is doubled so time to discharge to same point doubled (but voltage still goes from 1 to 0 when discharging).

# Chapter II.31: AC Circuits

## *Illustrations*

### Illustration 31.1: Circuit Builder

*Topics:* RC, RL, and RLC; voltage, current
*Purpose*: To introduce a circuit simulator.
*Suggested Use*: This is a very flexible method of simulating circuits: both ac and dc. In class, you can build any circuit and plot the voltage and current of any circuit element.

### Illustration 31.2: AC Voltage and Current

*Topics:* voltage, current
*Purpose*: To introduce the idea of alternating current.
*Suggested Use*: Change the frequency and point out the change in the rate the lights blink, but also that the lights go and off. If you compare the plot of voltage versus time with a carefully wired (be careful with outlet current) oscilloscope measurement of voltage versus time for a light bulb connected to a wall outlet, you can show that even though we can not see it, the light bulb must be going on and off. It is also designed to provide a quick introduction to rms voltage and current in an independent exercise for the students (they can verify the power of bulb).

### Illustration 31.3: Transformers

*Topics:* transformers
*Purpose*: To introduce the idea of transformers and the reason we use AC (not DC).
*Suggested Use*: By changing the windings in the transformer, step the voltage up and down and find that the ratio of the voltages is equal to the ratio of windings. Students should verify the power calculations to show that delivering power at low current and high voltages over transmission lines is best since transformers can easily step the voltage down at a house.

### Illustration 31.4: Phase Shifts

*Topics:* reactance, capacitor and inductor loads
*Purpose*: To introduce the phase shift and reactance associated with capacitive and inductive loads.
*Suggested Use*: Vary the frequency to demonstrate what happens to the current as a function of frequency. Can use this to develop the idea of reactance as well as noting the phase shift between current and voltage for inductors and capacitors (leading and lagging of current).

### Illustration 31.5: Power and Reactance

*Topics:* power, capacitor and inductor loads
*Purpose*: To develop the connection between power and the phase shift between the current and voltage.

*Suggested Use*: Have students explain why the phase shift matters in calculating the power dissipated.

### Illustration 31.6: Voltage and Current Phasors

*Topics:* phasors, impedance, RC
*Purpose*: To connect a phasor representation to voltage and currents in circuit elements.
*Suggested Use*: Change the frequency and note the change in the phasor as well as the voltage and current plotted (more on phasors in Illustration 31.7 and Explorations 31.5 and 31.6).

### Illustration 31.7: RC Circuits Voltage Phasors

*Topics:* phasors, RC
*Purpose*: To show the frequency response of impedance as R,L and C are varied.
*Suggested Use*: Students should begin to be able to explain how the phasor representation connects to the voltages: comparing the plot of the voltage and the projection of the vector (more on phasors in Illustration 31.6 and Explorations 31.5 and 31.6).

### Illustration 31.8: Impedance and Resonance, RLC Circuit

*Topics:* impedance, resonance, RLC
*Purpose*: To show the frequency response of impedance as R, L and C are varied.
*Suggested Use*: Change L and C to demonstrate how the peak (resonance) changes in frequency, but changing R simply changes the sharpness of the peak. Note that a decrease in impedance means more current is going through the circuit so that when $\omega L=1/\omega C$, $Z=R$ (its minimum value) there is maximum current.

## *Explorations*

### Exploration 31.1

*Topics:* voltage

### Exploration 31.2

Topics: reactance

### Exploration 31.3

Topics: reactance, filters

### Exploration 31.4

*Topics:* impedance, power

### Exploration 31.5

*Topics:* impedance, phasors, RL

## Exploration 31.6

*Topics:* impedance, phasors, RLC

## Exploration 31.7

*Topics:* impedance, resonance

## Exploration 31.8

*Topics:* impedance, resonance, switched RLC

## *Problems*

## P.31.1 (Level 2)

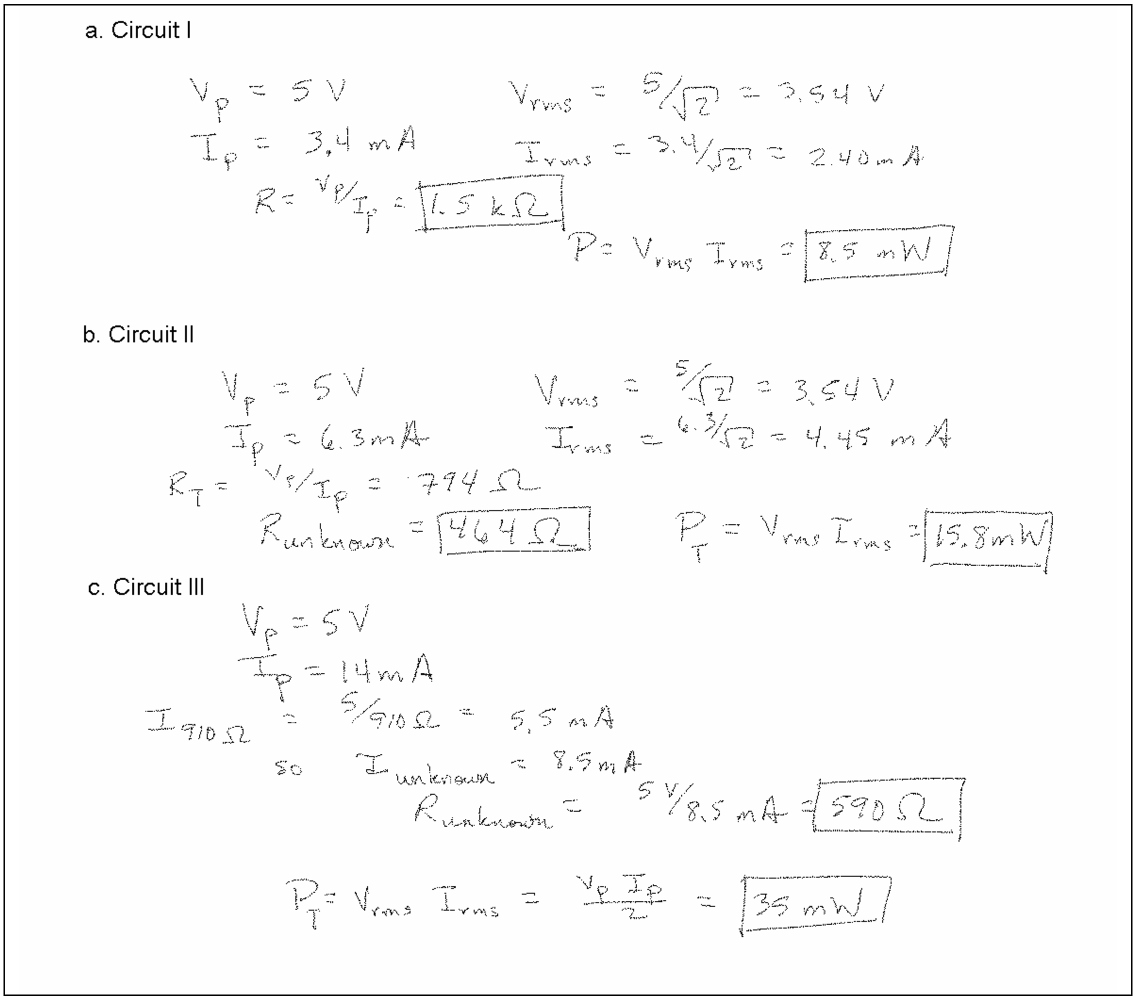
*Topic(s):* switches, power, voltage
Answer:
A: **P = 45 W.** close S2,S4,S5
B: **P = 60 W.** close S1,S3,S5
C: **P = 60 W**. close S1,S2,S5
D: **P = 100 W**. close S1,S2,S4:

## P.31.2 (Level 2)

*Topic(s):* transformers, power
Answer:
a) **104 windings**; Vp=171; Vs=44.5 so Ns=(400)Vp/Vs=104
b) **Irms=.041 A**.; Vrms=31.5V so Irms=.16A, P=5W for both secondary and primary so primary Irms=.041A.
c) **5 W**

## P.31.3 (Level 2)

Topic(s): power, resistors
Answer:
a) $R_{unknown}$= **1.5 kΩ** ;Pavg= **8.5 mW**
b) $R_{unknown}$= **470 Ω**; Pavg= **16 mW**
c) $R_{unknown}$= **590 Ω**; Pavg= **35 mW**



a. Circuit I

$V_p = 5\ V$ $\quad V_{rms} = 5/\sqrt{2} = 3.54\ V$

$I_p = 3.4\ mA$ $\quad I_{rms} = 3.4/\sqrt{2} = 2.40\ mA$

$R = V_p/I_p = \boxed{1.5\ k\Omega}$

$P = V_{rms} I_{rms} = \boxed{8.5\ mW}$

b. Circuit II

$V_p = 5\ V$ $\quad V_{rms} = 5/\sqrt{2} = 3.54\ V$

$I_p = 6.3\ mA$ $\quad I_{rms} = 6.3/\sqrt{2} = 4.45\ mA$

$R_T = V_p/I_p = 794\ \Omega$

$R_{unknown} = \boxed{464\ \Omega}$ $\quad P_T = V_{rms} I_{rms} = \boxed{15.8\ mW}$

c. Circuit III

$V_p = 5\ V$

$I_p = 14\ mA$

$I_{910\Omega} = 5/910\Omega = 5.5\ mA$

so $I_{unknown} = 8.5\ mA$

$R_{unknown} = 5\ V/8.5\ mA = \boxed{590\ \Omega}$

$P_T = V_{rms} I_{rms} = \frac{V_p I_p}{2} = \boxed{35\ mW}$

## P.31.4 (Level 1)

Topic(s):power, reactance

Answer:

a) **B, resistor**

b) **A, inductor**

c) **C, capacitor**

## P.31.5 (Level 2)

Topic(s):reactance

Answer:

a) **6.5e-5 F**. At f=2550, Peak V=17.45; Peak I=18.25 so X=.956=1/ωC.

b) **5.6e-4 H**. At f=800, Peak V=28.15; peak I=10.0 so X=2.81=ωL.

## P.31.6 (Level 2)

Topic(s): voltage

Answer:

**Graph B.** Since there are no values given, simply need to check Kirchoff's laws: the voltage across the load elements must equal the source voltage at all points and it is easiest to look at points where the source voltage crosses (is equal to) one of the load values to see if the other voltage is 0 at that same time.

## P.31.7 (Level 2)

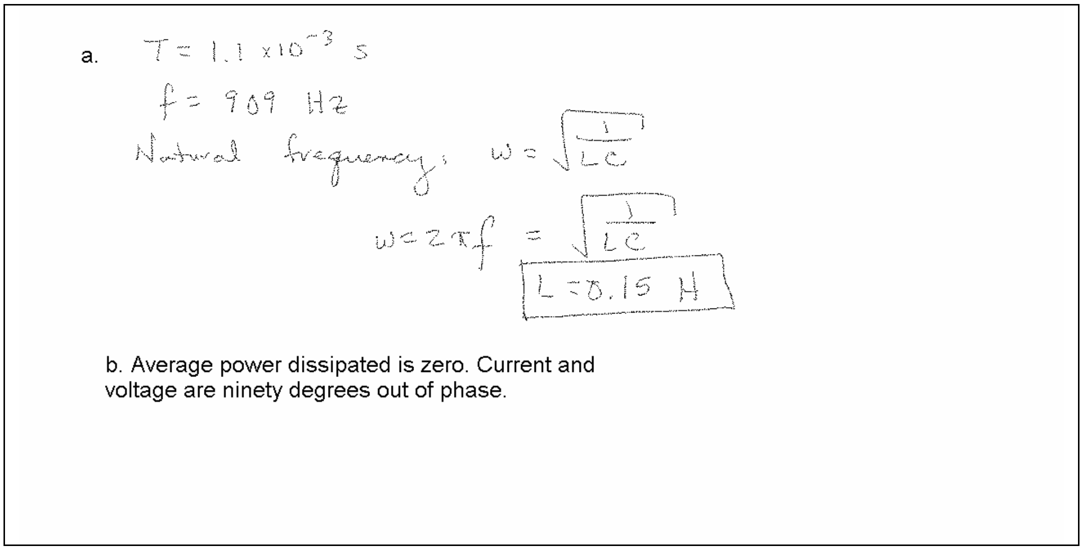
Topic(s): voltage
Answer:

**Graph C.** Since there are no values given, simply need to check Kirchoff's laws: the voltage across the load elements must equal the source voltage at all points and it is easiest to look at points where the source voltage crosses (is equal to) one of the load values to see if the other voltage is 0 at that same time.

## P.31.8 (Level 2)

Topic(s): impedance, LC
Answer:

**L=0.15 H**.



a. $T = 1.1 \times 10^{-3}$ s

$f = 909$ Hz

Natural frequency: $\omega = \sqrt{\frac{1}{LC}}$

$\omega = 2\pi f = \sqrt{\frac{1}{LC}}$

$\boxed{L = 0.15 \text{ H}}$

b. Average power dissipated is zero. Current and voltage are ninety degrees out of phase.

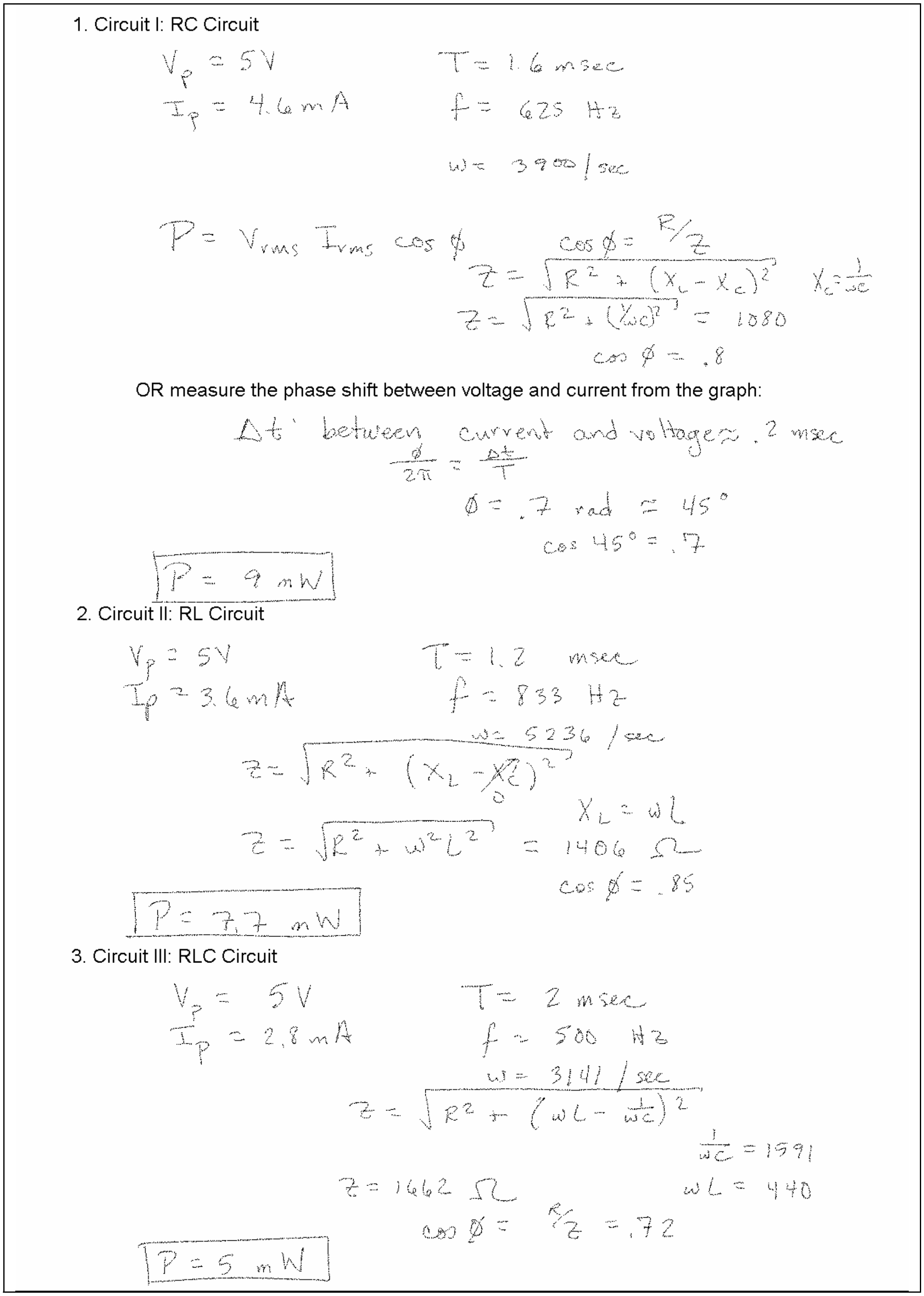
## P.31.9 (Level 3)

Topic(s): power, impedance
Answer:

1) **9 mW.** T=1.7e-3; f=588Hz; Vrms=3.5; Irms=3.25 and phase shift=42° (measured from graph) or R/Z=.79

2) **8 mW.** T=1.25e-3; f=140 Hz; Vpeak=5, Ipeak=3.6mA and phase shift=30° (measured from graph) or R/Z=.85

3) **4-5mW.** T=2e-3; f=500 Hz; Vpeak=5, Ipeak=2.9mA and phase shift=50° (measured from graph) or R/Z=.72



1. Circuit I: RC Circuit

$V_p = 5V$ $T = 1.6$ msec

$I_p = 4.6$ mA $f = 625$ Hz

$\omega = 3900/\text{sec}$

$P = V_{rms} I_{rms} \cos\phi$ $\cos\phi = R/Z$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ $X_C = \frac{1}{\omega C}$

$Z = \sqrt{R^2 + (1/\omega C)^2} = 1080$

$\cos\phi = .8$

OR measure the phase shift between voltage and current from the graph:

$\Delta t$ between current and voltage $\approx .2$ msec

$\frac{\phi}{2\pi} = \frac{\Delta t}{T}$

$\phi = .7$ rad $\approx 45°$

$\cos 45° = .7$

$\boxed{P = 9 \text{ mW}}$

2. Circuit II: RL Circuit

$V_p = 5V$ $T = 1.2$ msec

$I_p = 3.6$ mA $f = 833$ Hz

$\omega = 5236$ /sec

$Z = \sqrt{R^2 + (X_L - \underset{0}{\cancel{X_C}})^2}$

$X_L = \omega L$

$Z = \sqrt{R^2 + \omega^2 L^2} = 1406\ \Omega$

$\cos\phi = .85$

$\boxed{P = 7.7 \text{ mW}}$

3. Circuit III: RLC Circuit

$V_p = 5V$ $T = 2$ msec

$I_p = 2.8$ mA $f = 500$ Hz

$\omega = 3141$ /sec

$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

$\frac{1}{\omega C} = 1591$

$Z = 1662\ \Omega$ $\omega L = 440$

$\cos\phi = R/Z = .72$

$\boxed{P = 5 \text{ mW}}$

## P.31.10 (Level 2)

Topic(s): reactance, filters
Answer:

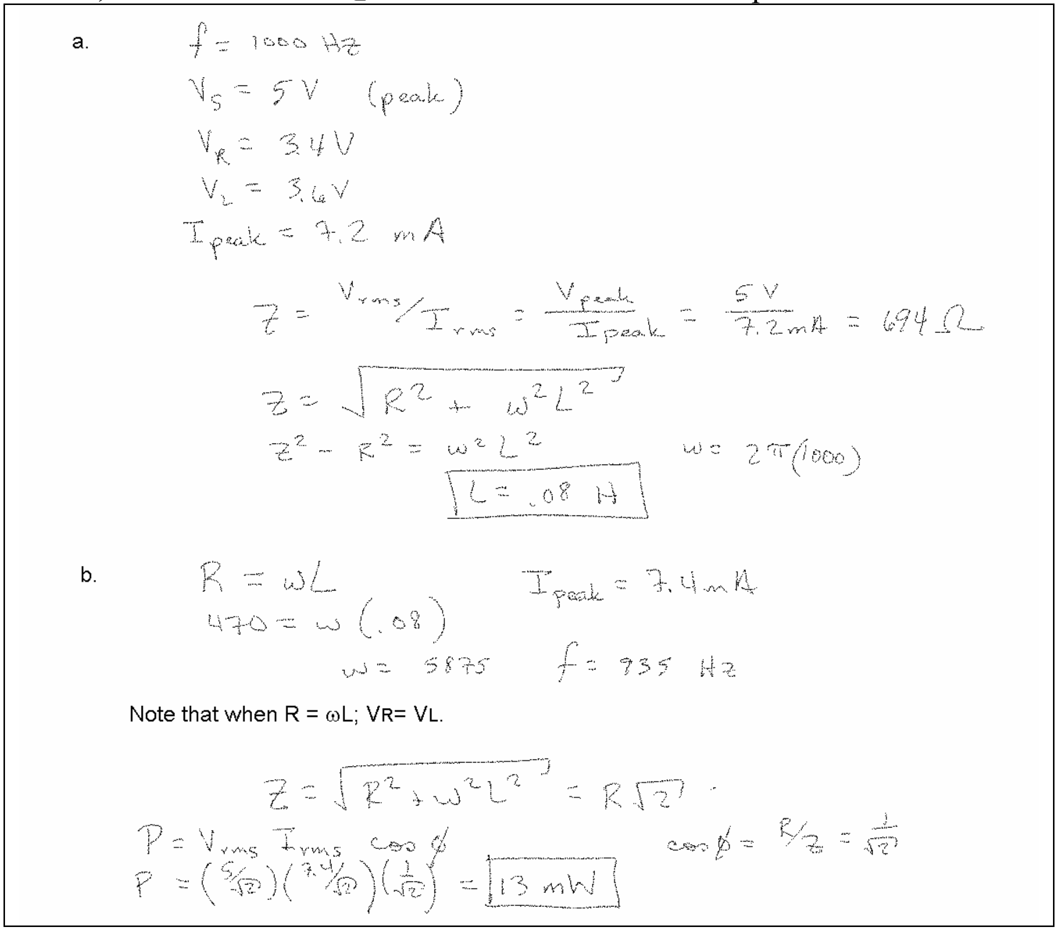
a) **Green**
b) **Blue**
c) **A-tweeter and B-woofer.** Blue is high freq part (filters out low freq) and is capacitor since capacitors act like open circuit for DC and low frequencies (and short circuits for high frequencies)

## P.31.11 (Level 2)

Topic(s): reactance, power
Answer:
a) **C=3e-7 F**
b) **P=11 mW**. Change f until voltage across R=voltage across C then Xc=R. Or measure f, V and I and find Xc. f=947Hz I=6.3mA and find power.

## P.31.12 (Level 2)

Topic(s): reactance, power
Answer:
a) **L=0.08 H**
b) **P=13 mW**. Change f until voltage across R=voltage across L then $X_L$=R. Or measure f, V and I and find $X_L$. f=935Hz I=7.5mA and find power.



a. $f = 1000$ Hz

$V_S = 5V$ (peak)

$V_R = 3.4V$

$V_L = 3.6V$

$I_{peak} = 7.2$ mA

$$Z = V_{rms}/I_{rms} = \frac{V_{peak}}{I_{peak}} = \frac{5V}{7.2mA} = 694\,\Omega$$

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$Z^2 - R^2 = \omega^2 L^2 \qquad \omega = 2\pi(1000)$$

$$\boxed{L = .08\ H}$$

b. $R = \omega L$   $I_{peak} = 7.4$ mA

$470 = \omega(.08)$

$\omega = 5875$   $f = 935$ Hz

Note that when R = ωL; VR= VL.

$$Z = \sqrt{R^2 + \omega^2 L^2} = R\sqrt{2}.$$

$$P = V_{rms} I_{rms} \cos\phi \qquad \cos\phi = R/Z = \frac{1}{\sqrt{2}}$$

$$P = \left(5/\sqrt{2}\right)\left(7.4/\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \boxed{13\ mW}$$

## P.31.13 (Level 1)

Topic(s): switched RLC

Answer:

**Rc<Ra<Rb**. Time to damp longest, largest resistance.

## P.31.14 (Level 1)

Topic(s): resonance

Answer:

a) **210 Hz.** When have maximum current (max voltage across resistor), at resonance.

b) **Graph A.**

# Chapter II.32: EM Waves

## *Illustrations*

### Illustration 32.1: Creation of Electromagnetic Waves

*Topic(s):* em waves
*Purpose:* To demonstrate the creation of EM waves by accelerating charges.
*Suggested Use:* As an in-class demonstration, this can be used to quickly make the connection between a charge (and its electric field) and waves.

### Illustration 32.2: Wave Crests

*Topic(s):* em waves
*Purpose:* To make the connection between EM waves and sound (pressure) waves.
*Suggested Use:* Discuss differences and similarities between em waves, sound waves, and transverse waves on a string.

### Illustration 32.3: Electromagnetic Plane Waves

*Topic(s):* em waves, wavelength, frequency
*Purpose:* To illustrate the vector nature of EM waves.
*Suggested Use:* Standard physics texts present overviews of the electromagnetic spectrum and its relationship to wavelength and frequency. This illustration reinforces these concepts and points out the vector nature of E&M waves. Students must move the transparent square along the z axis to measure wavelength and must observe the time it takes for the field to repeat itself to measure period. The vector field representation of a plane wave is unfamiliar and ought to be discussed in class. This discussion should focus on the misconception that the electric field only exists along the z axis.
Answers: no; wave equation; $\lambda$=20m (radio); f=15 MHz; v=3x$10^8$ m/s.

### Illustration 32.4: Electromagnetic Waves, E x B

*Topic(s):* em waves
*Purpose:* To demonstrate the connections between E & B fields and wave properties.
*Suggested Use:* Discuss with students the orientation between E and B, that **ExB** determines the direction of propagation of the wave (can check for the 2 waves), and the connections to the equations that describe E & B.

## *Explorations*

### Exploration 32.1 - Representation of Plane Waves

*Topic(s):* em waves, frequency, wavelength
*Answer:* b) V< IV = 0 < I=II=III, c) I=V<II=IV=0<III, d) 2m, e) 6.7Hz, f) 13.4 m/s

### Exploration 32.2 - Plane Waves and the Electric Field Equation

*Topic(s):* em waves

Answer: a) The field varies with both position along the z-axis and time. b) The direction of the field is the x-direction. c) B(z,t) = Emax/c cos(kz-wt) **j**

## Problems

### P.32.1 (Level 1)

*Topic(s):* em waves
*Answer:* **red=electric; green=magnetic**

### P.32.2 (Level 1)

*Topic(s):* em waves, wavelength
*Answer:* **UV**
wavelength of B is ½ A so frequency is twice. Green: $\lambda$=532nm so wavelength of is around 266nm (UV). Light has same intensity (same power).

### P.32.3 (Level 1)

*Topic(s):* em waves, wavelength
*Answer:* **A=red; B=violet; C=green**

### P.32.4 (Level 2)

*Topic(s):* em waves, wavelength, frequency
*Answer:* **a) B-field along y-axis, b) $\lambda$=.78 microns; f=3.8x$10^{14}$ Hz; v=3x$10^{8}$ m/s**

### P.32.5 (Level 2)

*Topic(s):* em waves, wavelength, frequency
*Answer:* **a) B-field perpendicular to xy-plane, containing the line y=-x., b) $\lambda$=1.11 mm; f=0.27x$10^{12}$ Hz; v=3x$10^{8}$ m/s;**

### P.32.6 (Level 1)

*Topic(s):* em waves
*Answer:* **a) Into and out of the screen (perpendicular and in phase), b) along horizontal axis through center of oscillation (where e-field wave amplitude is largest).**

# Chapter II.33: Mirrors

## *Illustrations*

### Illustration 33.1: Mirrors and the Small-Angle Approximation

*Topic(s):* mirrors
*Purpose:* To provide a flexible set-up for testing optical systems.
*Suggested Use:* Have students explore the difference between beam sources (parallel rays), point sources and objects (which show only the principal rays) as well as the difference between converging and diverging mirrors (positive and negative focal lengths). Students should move the sources around to get a sense of what happens light reflects off of mirrors.

### Illustration 33.2: Flat Mirrors

*Topic(s):* mirrors
*Purpose:* To illustrate the formation of images in mirrors.
*Suggested Use:* Students can work individually on this to adjust the object size and notice the “reflection in the mirror” as well as changing the angle. Introduces the idea of a virtual image. Students should be able to explain why we see the images we do in mirrors and where the light comes from (and why it is useful to extend the ray behind the mirror to determine image properties).

## *Explorations*

### Exploration 33.1 - Image in a Flat Mirror

*Topic(s):* mirrors

### Exploration 33.2 - Looking at Curved Mirrors

*Topic(s):* mirrors

### Exploration 33.3 - Ray Diagrams

*Topic(s):* mirrors, focal point

### Exploration 33.4: Focal Point and Image Point

*Topic(s):* mirrors, focal point

### Exploration 33.5: Convex Mirrors, Focal Point, and Radius of Curvature

*Topic(s):* mirrors, focal point

## *Problems*

### P.33.1 (Level 1)

*Topic(s):* mirrors, focal point
*Answer*: **1.3m**

### P.33.2 (Level 1)

*Topic(s):* mirrors, focal point
*Answer:* **1 m**

### P.33.3 (Level 1)

*Topic(s):* mirrors, focal point
*Answer:* **C**

### P.33.4 (Level 1)

*Topic(s):* mirrors, focal point
*Answer:* **0.75-m**

### P.33.5 (Level 2)

*Topic(s):* mirrors, focal point
*Answer*: **-1m**

### P.33.6 (Level 2)

*Topic(s):* mirrors, focal point
*Answer*: **green**

### P.33.7 (Level 2)

*Topic(s):* mirrors
*Answer:* **a) Can't be Concave, b) Can't be Plane, c) Convex.**

### P.33.8 (Level 2)

*Topic(s):* mirrors
*Answer:* **a) A-Convex Mirror, B-Concave Mirror, C-Plane Mirror, D-Concave Mirror, b) D>B (both concave mirrors)>A(convex); C=plane mirror (f→±infinity)**

# Chapter II.34: Refraction

## *Illustrations*

### Illustration 34.1: Huygens' Principle and Refraction

*Topic(s):* refraction, Huygens' principle
*Purpose:* To illustrate Huygens' principle.
*Suggested Use:* Students need to pay particular attention to what happens at the interface. The bending of the light is subtle the connection to the wavefronts can be missed if students aren't paying attention. Have students describe to each other (or write down) what happens at the interface, explaining the wavefront.

### Illustration 34.2: Fiber Optics

*Topic(s):* total internal reflection
*Purpose:* To illustrate how fiber optics work.
*Suggested Use:* Students should bring the source into the blue region and then change the angle to see the total internal reflection. Demonstration: laser beam internally reflected in a water flow (punch a small hole in the side of a clear plastic tennis ball "can," align the laser beam so it will point in the same direction as the water leaving the hole and fill "can" with water).

### Illustration 34.3: Prisms and Dispersion

*Topic(s):* refraction, dispersion
*Purpose:* To illustrate how a prism works.
*Suggested Use:* Students should be able to explain the refraction of light through a prism: predicting the approximate exit angle of the beam. As the index of refraction changes for each wavelength of light, students should also be able to explain why the exit angle changes the way that it does.

## *Explorations*

### Exploration 34.1 - Lens and a Changing Index of Refraction

*Topic(s):* refraction, lenses

### Exploration 34.2 - Snell's law and Total Internal Reflection

*Topic(s):* refraction, total internal reflection, Snell's law

### Exploration 34.3 - Towards Building a Lens

*Topic(s):* refraction, lenses

### Exploration 34.4 - Fermat's Principle and Snell's Law

*Topic(s):* refraction

## Exploration 34.5 - Index of Refraction and Wavelength

*Topic(s):* refraction, dispersion

## *Problems*

### P.34.1 (Level 1)

*Topic(s):* refraction, total internal reflection
*Answer*: **a) n=2.1-2.3, b) ~28°**

### P.34.2 (Level 1)

*Topic(s):* refraction
*Answer*: **water, n=1.3**

### P.34.3 (Level 2)

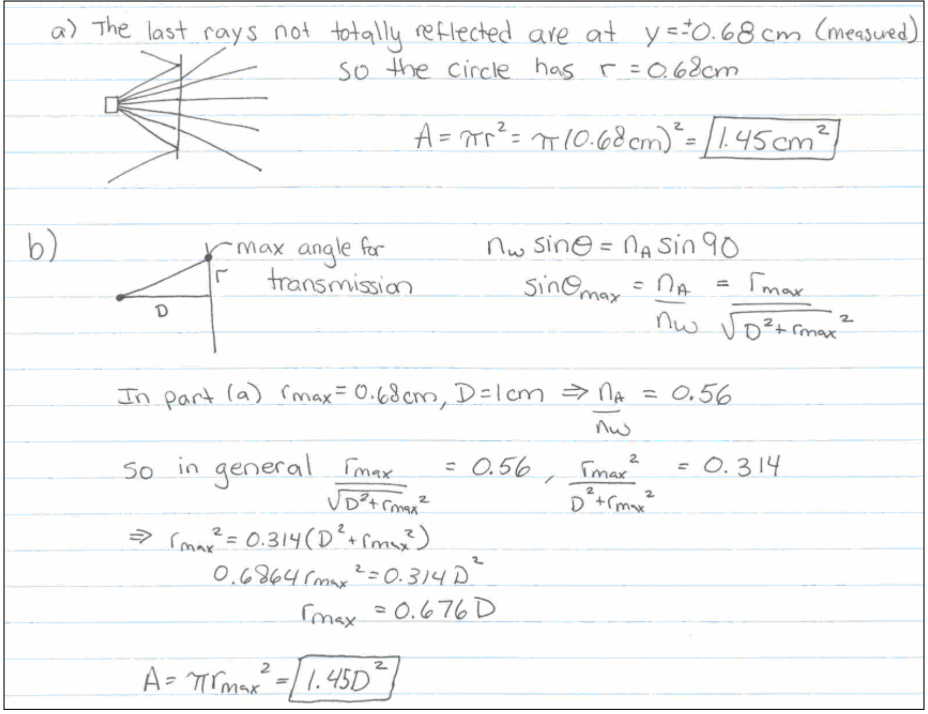
*Topic(s):* refraction
*Answer:* **x=1.2m**

### P.34.4 (Level 1)

*Topic(s):* refraction
*Answer:* **1.7-1.8**

### P.34.5 (Level 3)

*Topic(s):* refraction, internal reflection
*Answer*: **a) area=1.45 cm$^2$, b) area=1.45 D^2**



a) The last rays not totally reflected are at $y = \pm 0.68\,cm$ (measured)

so the circle has $r = 0.68cm$

$A = \pi r^2 = \pi (0.68cm)^2 = \boxed{1.45cm^2}$

b) max angle for transmission

r

D

$n_w \sin\theta = n_A \sin 90$

$\sin\theta_{max} = \frac{n_A}{n_w} = \frac{r_{max}}{\sqrt{D^2 + r_{max}^2}}$

In part (a) $r_{max} = 0.68cm, D = 1cm \Rightarrow \frac{n_A}{n_w} = 0.56$

so in general $\frac{r_{max}}{\sqrt{D^2+r_{max}^2}} = 0.56$, $\frac{r_{max}^2}{D^2+r_{max}^2} = 0.314$

$\Rightarrow r_{max}^2 = 0.314(D^2 + r_{max}^2)$

$0.6864\, r_{max}^2 = 0.314\, D^2$

$r_{max} = 0.676D$

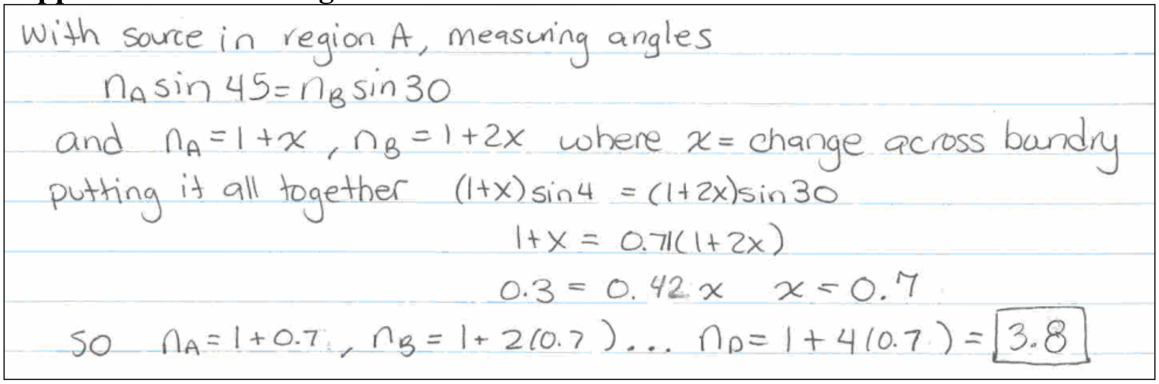
$A = \pi r_{max}^2 = \boxed{1.45D^2}$

## P.34.6 (Level 2)

*Topic(s):* refraction
*Answer:* **C>D>A>B**

## P.34.7 (Level 3)

*Topic(s):* refraction, internal reflection
*Answer:* **3.8. The critical angle from A to air (n=1) interface is 36° while B to A is 45.1° (so beam starting in B barely makes it through), but C to B, the critical angle is 50.7° so beam starting in C gets through B, but is refracted enough to be greater than the critical angle for the B/A interface. Similar argument to explain what happens if the beam begins in D.**



With source in region A, measuring angles

$n_A \sin 45 = n_B \sin 30$

and $n_A = 1 + x$, $n_B = 1 + 2x$ where $x$ = change across bandry

putting it all together $(1+x)\sin 4 = (1+2x)\sin 30$

$1 + x = 0.71(1+2x)$

$0.3 = 0.42x \quad x = 0.7$

so $n_A = 1 + 0.7$, $n_B = 1 + 2(0.7)$ ... $n_D = 1 + 4(0.7) = \boxed{3.8}$

## P.34.8 (Level 2)

*Topic(s):* refraction
*Answer:* **a) v=fλ, v smaller, λ smaller, b) n=c/v=λair/λmedium=2, c) v=.5c**

## P.34.9 (Level 1)

*Topic(s):* refraction
*Answer:* **a) smaller, b) same, c) bigger, d) same**

## P.34.10 (Level 1)

*Topic(s):* refraction, dispersion
*Answer:* **Animation A**

# Chapter II.35: Lenses

## *Illustrations*

### Illustration 35.1: Lenses and the Thin-Lens Approximation

*Topic(s):* lenses
*Purpose:* To provide a flexible set-up for testing optical systems.
*Suggested Use:* Have students explore the difference between beam sources (parallel rays), point sources and objects (which show only the principal rays) as well as the difference between converging and diverging lenses (positive and negative focal lengths). Students should move the sources around to get a sense of what happens light enters the lens.

### Illustration 35.2: Image from a Diverging Lens

*Topic(s):* lenses
*Purpose:* To demonstrate images from diverging lenses.
*Suggested Use:* Have students make a ray diagram to provide a basis for their prediction or explanation of their observations.

## *Explorations*

### Exploration 35.1 - Image Formation

*Topic(s):* lenses, ray diagrams

### Exploration 35.2 - Ray Diagrams

Topic(s): lenses

### Exploration 35.3 – Moving a Lens

Topic(s): lenses

### Exploration 35.4 – What is Behind the Curtain

Topic(s): lenses

### Exploration 35.5 – Lens Maker's Equation

*Topic(s):* lenses, lensmaker's equation

## *Problems*

### P.35.1 (Level 1)

*Topic(s):* lenses, focal point
*Answer:* **a) Converging Lenses for D and A, Diverging Lenses for B and C**

**b) B<C (both negative)<D<A [Magnitude: |C|<|D|<|A|<|B|. Comparing magnitudes requires some measurement for the divergent lenses.]**

## P.35.2 (Level 1)

*Topic(s):* lenses, focal point
*Answer:* **a) A: diverging lens; B: nothing; C&D: converging lenses, b) A(negative)<D<C**

## P.35.3 (Level 1)

*Topic(s):* lenses, focal point
*Answer:* **Same focal length**.

## P.35.4 (Level 1)

*Topic(s):* lenses, focal point
*Answer:* **0.7-m**

## P.35.5 (Level 2)

*Topic(s):* lenses, focal point
*Answer:* **a) 1m; b) 1.42m & 2m**

## P.35.6 (Level 2)

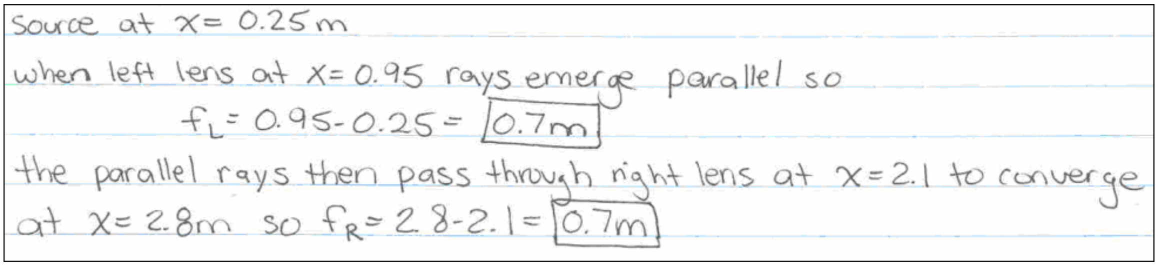
*Topic(s):* lenses
*Answer:* **a) image at position 3.3m (1.3m from lens); b) inverted**
f=0.8m

## P.35.7 (Level 2)

*Topic(s):* lenses
*Answer*: **a) image at position 5m (3m from lens); b) inverted**
f=1.2m

## P.35.8 (Level 2)

*Topic(s):* lenses, focal point
*Answer:* **0.7 m for both**



Source at $x = 0.25$ m

when left lens at $x = 0.95$ rays emerge parallel so

$f_L = 0.95 - 0.25 = \boxed{0.7\text{m}}$

the parallel rays then pass through right lens at $x = 2.1$ to converge at $x = 2.8$m so $f_R = 2.8 - 2.1 = \boxed{0.7\text{m}}$

## P.35.9 (Level 2)

*Topic(s):* lenses, focal point
*Answer:* **left is 0.7 m and right is 1.4 m**

When source at $x=0.25$, left lens at $x=0.96$, Right Lens at $x=1.46$ rays emerge parallel from left and converge at $x=2.84$

$$f_L = 0.96-0.25 = \boxed{0.71m}$$

$$f_R = 2.84-1.46 = \boxed{1.38m}$$

## P.35.10 (Level 3)

*Topic(s):* lenses

*Answer:* **A=1.2m; B=0.8m; C=2m.**

When source at $x=0$, lens A at $x=1.22$ rays emerge from lens A parallel. When lens B at $x=2.1$ these rays converge at $x=2.88$.

So $\boxed{F_A = 1.22m}$ $F_B = 2.88-2.1 = \boxed{0.78m}$

The rays then emerge parallel from C when C is at $x=4.83$

$$F_C = 4.83-2.88 = \boxed{1.95m}$$

## P.35.11 (Level 3)

*Topic(s):* lenses, lensmaker's equation

*Answer:* **f=0.86cm; R = 1.14 cm**

$$\frac{1}{f} = \frac{1}{d_O} + \frac{1}{d_I} \quad \text{and} \quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

So $$1/d_O + 1/d_I = (n-1)\left(1/R_1 + 1/R_2\right)$$

$$\frac{1}{2-0.5} + \frac{1}{4-2} = (2.5-1)\left(\frac{1}{R_1} + \frac{1}{-10cm}\right)$$

$$\boxed{R_1 = 1.14cm}$$

$$1/f = 1/1.5 + 1/2 \quad \boxed{f = 0.86cm}$$

# Chapter II.36: Applications

## *Illustrations*

### Illustration 36.1: The Human Eye

*Topic(s):* lenses
*Purpose:* To demonstrate how the eye accommodates and the purpose of glasses.
*Suggested Use:* Have students explore how the eye focuses light from far sources and near sources for the healthy eye in contrast with the near-sighted and far-sighted eye. Students can add glasses to the near-sighted and far-sighted eyes to correct vision and this provides a concrete application of the geometric optics they have been studying.

### Illustration 36.2: Camera

*Topic(s):* lenses
*Purpose:* To demonstrate camera focusing and different lenses.
*Suggested Use:* Students can "focus" the camera so that there is an image on the film using the different lenses. They can compare the different lenses (see also Exploration 36.1).

### Illustration 36.3: Laser Cavity

*Topic(s):* mirrors
*Purpose:* To demonstrate an application of spherical mirrors to make a laser cavity.
*Suggested Use:* Students can explore when the cavity is stable and unstable as they change the mirror separation, focal length of the mirrors, and position of the source. Provides another example of applications of optics. Can have students think of other examples of uses of curved mirrors (in convenience stores that allow the clerk to see customers, car side mirrors, fun houses, telescopes, Direct TV satellite dishes (not visible light mirror, but radio frequency "mirrors").

## *Explorations*

### Exploration 36.1 - Camera

*Topic(s):* lenses

### Exploration 36.2 - Telescope

*Topic(s):* lenses

## *Problems*

### P.36.1 (Level 2)

*Topic(s):* lenses
*Answer:* **a) near-sighted (can not focus a far source at the back of the retina)**
**b) -4.5**

## P.36.2 (Level 2)

*Topic(s):* lenses
*Answer:* **a) far-sighted, c) +2; no can't focus on a far-away point so use bifocals to look over the glasses to see far-away.**

## P.36.3 (Level 2)

*Topic(s):* lenses
*Answer:* **a) normal**

## P.36.4 (Level 2)

*Topic(s):* lenses
*Answer:* **x=2 (2.1 units in front of front of eye)**

## P.36.5 (Level 2)

*Topic(s):* lenses
*Answer:* **x=1 (1/2 cm from the objective).**

# Chapter II.37: Interference

## *Illustrations*

### Illustration 37.1: Ripple Tank

*Topic(s):* interference
*Purpose:* To demonstrate interference using a ripple tank.
*Suggested Use:* Have students explore the constructive and destructive interference as they move the sources around and change the wavelength. Since many books focus on contrasting constructive and destructive interference, should also discuss interference that is neither completely destructive nor completely constructive interference.

### Illustration 37.2: Dielectric Mirrors

*Topic(s):* interference, thin film
*Purpose:* To show how layering materials of different indices of refraction can create a mirror.
*Suggested Use:* Show a camera lens (or laser mirror) with a dielectric coating. Problem 37.11 asks students to explain the physics behind this illustration.

## *Explorations*

### Exploration 37.1 – Varying Numbers and Orientations of Sources

*Topic(s):* interference

### Exploration 37.2 – Changing the Separation Between Sources

*Topic(s):* interference, double-slit

## *Problems*

### P.37.1 (Level 1)

*Topic(s):* interference
*Answer:* **a) Bright central spot—fading to dark ring at a radius of about 7. b) Screen does not change over time.**

### P.37.2 (Level 1)

*Topic(s):* interference
*Answer*: **A: completely destructive; C&D: completely constructive; B somewhere in between**

### P.37.3 (Level 1)

*Topic(s):* interference
*Answer:* **In phase**.

## P.37.4 (Level 1)

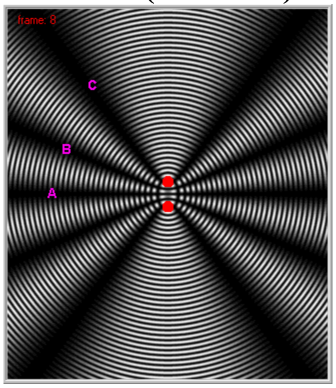
*Topic(s):* interference
*Answer:* **½ wavelength**.

## P.37.5 (Level 1)

*Topic(s):* interference
*Answer:* **a) regions of constructive and destructive interference would be switched., b) no difference**

## P.37.6 (Level 2)

*Topic(s):* interference
*Answer*: **a) out of phase, b) 1.25, c) A stay the same; C same; B completely destructive (see below)**



## P.37.7 (Level 2)

*Topic(s):* interference, double-slit
*Answer:* **2-nm.**
Wavelength≈0.8-0.9 nm.

## P.37.8 (Level 2)

*Topic(s):* interference, thin films
*Answer*: **a) 2<3<4<1 (smaller wavelength means bigger index of refraction), b) Interference of reflected wave. Medium 2 is 2- wavelengths thick and Medium 3 is 3-wavelengths of light thick, thus light that reflects from the 2-3 interface interferes destructively (phase shift with reflection) with incident light and same occurs for the 3-4 interface. Thus, the overall reflection amplitude is less with a multi-layer situation.**

## P.37.9 (Level 2)

*Topic(s):* interference, thin films
*Answer:* **a) As the thickness of the film is the same as an integer divided by 2 of the wavelength of light in the medium, the reflected wave goes to zero (destructive interference between the incoming light and the reflected light from the back of the medium-air interface)., b) 2. Wavelength in the material is 8 and $\lambda_{material}=\lambda_{air}/n$.**

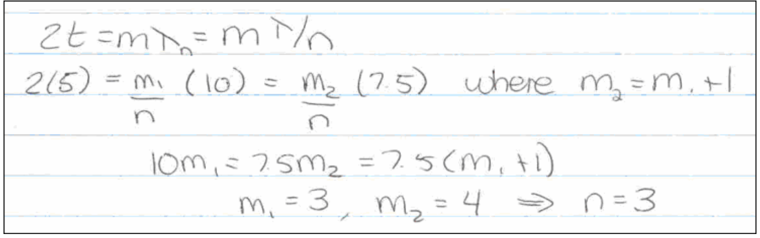
## P.37.10 (Level 3)

*Topic(s):* interference, thin films

*Answer:*

**a) Destructive interference will occur for the reflected light when mλ=2*thickness of medium. As the wavelength is varied the reflected light will cycle through destructive and constructive interference, leading to peaks in the transmission.**

**b) 3**



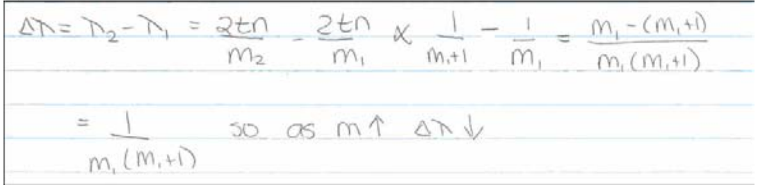
$2t = m\lambda_n = m\lambda/n$

$2(5) = \frac{m_1}{n}(10) = \frac{m_2}{n}(7.5)$ where $m_2 = m_1 + 1$

$10m_1 = 7.5m_2 = 7.5(m_1 + 1)$

$m_1 = 3, \ m_2 = 4 \Rightarrow n = 3$

**c) See Below**



$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{2tn}{m_2} - \frac{2tn}{m_1} \propto \frac{1}{m_1+1} - \frac{1}{m_1} = \frac{m_1 - (m_1+1)}{m_1(m_1+1)}$

$= \frac{1}{m_1(m_1+1)}$ so as $m \uparrow$ $\Delta\lambda \downarrow$

## P.37.11 (Level 2)

*Topic(s):* interference, thin films

*Answer:* **Constructive interference. For example, with one film: constructive interference of the wave reflected at the interface between the two films with the reflected wave from the front surface. Continue this process with multiple layers.**

# Chapter II.38: Diffraction

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## *Illustrations*

### Illustration 38.1: Single Slit Diffraction

*Topic(s):* diffraction, single slit
*Purpose:* To demonstrate the effect of slit width and wavelength on diffraction patterns.
*Suggested Use:* Have students explore the differences between the diffraction patterns produced by different sources (slit widths and wavelengths). Building a model of single slit diffraction in this animation involves arranging point sources together to the size of a slit (students are asked to do this in Exploration 38.1) which can lead into a discussion of Huygen's principle (see Illustration 34.3).

### Illustration 38.2: Application of Diffraction Gratings.

*Topic(s):* diffraction, grating
*Purpose:* To show how diffraction gratings are used to identify the wavelength of light and look at spectra.
*Suggested Use:* Discuss applications of gratings. Bring in diffraction gratings for students to look at filament bulbs and sodium, neon, etc lights. Demonstrate how CDs can be used as diffraction gratings. As a quick hands-on exercise, students can use laser pointers and make appropriate measurements to determine the spacing between grooves on a CD.

## *Explorations*

### Exploration 38.1 – Modeling Diffraction from a Slit

*Topic(s):* diffraction, single slit

### Exploration 38.2 – Diffraction Grating

*Topic(s):* diffraction, grating

## *Problems*

### P.38.1 (Level 1)

*Topic(s):* diffraction, single slit
*Answer:* **A<B<C**
Wavelength same for all.

### P.38.2 (Level 1)

*Topic(s):* diffraction, single slit
*Answer:* **A<C<B**
(since spreading of the beam is essentially the same, smallest wavelength must be entering smallest slit)

## P.38.3 (Level 2)

*Topic(s):* diffraction, single slit
*Answer:* **2.0-2.4μm**.
Wavelength=0.9μm.

## P.38.4 (Level 2)

*Topic(s):* diffraction, single slit
*Answer:* **600-640nm**

## P.38.5 (Level 2)

*Topic(s):* diffraction, grating
*Answer:* **510-540nm**

## P.38.6 (Level 2)

*Topic(s):* diffraction, grating
*Answer:* ***150 lines/mm***

# Chapter II.39: Polarization

## *Illustrations*

### Illustration 39.1: Polarization

*Topic(s):* polarization
*Purpose:* To connect the x and y components of a wave to the polarization of a wave.
*Suggested Use:* Start with a phase difference of 0 and then change the values of the amplitudes of the electric field in the x and y direction and have students predict the corresponding view of the traveling wave. Then students can try changing the phase difference to see what that does. This is a good opportunity to discuss linear, circular and elliptical polarization or have students work through Exploration 39.1 which is a tutorial on this animation and the different types of polarization.

### Illustration 39.2: Polarized Electromagnetic Waves

*Topic(s):* polarization, polarizer
*Purpose:* To demonstrate different polarizations and the effect of polarizers.
*Suggested Use:* Students can compare linearly polarized light with circularly polarized light as well what happens with a polarizing filter. Students should compare this representation of the waves with the representation in Illustration 39.1 (or Exploration 39.1).

## *Explorations*

### Exploration 39.1 – Polarization Tutorial

*Topic(s):* polarization

### Exploration 39.2 - Polarizers

*Topic(s):* polarization, polarizer

## *Problems*

### P.39.1 (Level 2)

*Topic(s):* polarization
*Answer:*

**a) left circularly polarized (Ex=8; Ey=8; PD=1.5*pi=-0.5*pi)**
**b) right elliptically polarized (Ex=4, Ey=8; PD=0.5*pi)**
**c) linearly polarized (Ex=4, Ey=8, PD=0) along Exis at an angle of 63° above x-Exis (angle=arctan(8/4)).**
**d) right circularly polarized (Ex=8, Ey=8, PD=0.5*pi)**

### P.39.2 (Level 2)

*Topic(s):* polarization

*Answer:*
Ex=Exsin(t+PD) Ey=Eysint
**a) Ex=3, Ey=8, PD=1*pi**
**b) Ex=3, Ey=8, PD=0**
**c) Ex=5, Ey=8, PD=0.25*pi**
**d) Ex=5, Ey=8, PD=1.25*pi**
**e) Ex=8, Ey=5, PD=0.28*pi**

## P.39.3 (Level 1)

*Topic(s):* polarization
*Answer:* **B**

## P.39.4 (Level 1)

*Topic(s):* polarization
*Answer*: **D**

## P.39.5 (Level 1)

*Topic(s):* polarization
*Answer:*
**a) +z**
**b) linearly polarized in x direction**

## P.39.6 (Level 1)

*Topic(s):* polarization
*Answer:*
**a) +z**
**b) right-circularly polarized**

## P.39.7 (Level 1)

*Topic(s):* polarization
*Answer:*
**a) +z**
**b) linearly polarized in plane 45$^o$ above x-Exis**

## P.39.8 (Level 1)

*Topic(s):* polarization
*Answer:*
**a) +z**
**b) left-circularly polarized**

## P.39.9 (Level 1)

*Topic(s):* polarization
*Answer:* **Animation 4**

## P.39.10 (Level 3)

*Topic(s):* polarization

*Answer:*

**a) $|\text{Amplitude}_{\text{exit}}|=|\text{Amplitude}_{\text{in}}|\cos\alpha$ where $\alpha$ is the polarization angle.**

**b) $\text{energy}_{\text{exit}}=\text{energy}_{\text{in}}\cos^2\alpha$ and energy~intensity of beam~$E^2$.**

# Chapter II.40: Optics Appendix

## *Problems*

### P.Appx.1 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) convex mirror**
**b) -1**

### P.Appx.2 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) converging lens**
**b) 0.7**

### P.Appx.3 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) diverging lens**
**b) -1.3**

### P.Appx.4 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) concave mirror**
**b) 0.7**

### P.Appx.5 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) plane mirror**
**b) infinity**

### P.Appx.6 (Level 1)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) concave mirror**
**b) 0.7**

### P.Appx.7 (Level 1)

*Topic(s):* lenses, mirrors, focal point
*Answer:*

**a) plane mirror**
**b) infinite**

## P.Appx.8 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) convex mirror**
**b) ≈ -1.3 to -1.7**

## P.Appx.9 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) diverging lens**
**b) ≈ -0.5 to -0.7**

## P.Appx.10 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) converging lens**
**b) 1.4**

## P.Appx.11 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) plane mirror**
**b) infinite**

## P.Appx.12 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) diverging lens**
**b) ≈ -0.9 to -1.1**

## P.Appx.13 (Level 1)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) concave mirror**
**b) 0.7**

## P.Appx.14 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*
**a) converging lens**
**b) 0.3**

## P.Appx.15 (Level 2)

*Topic(s):* lenses, mirrors, focal point
*Answer:*

**a) convex mirror**
**b) ≈ -0.7 to -0.9**