Review 2006

Test 2
PHN 242

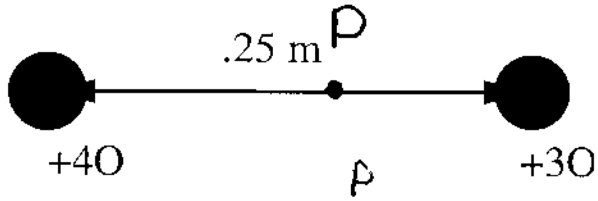
Dr. Weppner
April 2004

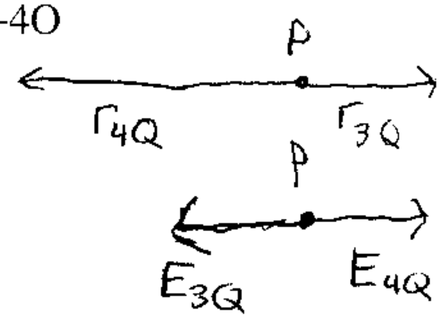
NAME ______________ SCORE____

Remember to get full credit label answers with correct units, show all work, draw diagrams.

Useful constants and formulas: $\epsilon_0 = 8.85\times10^{-12}F/m$, $\frac{1}{4\pi\epsilon_0} = 8.99\times10^{9}Nm^2/C^2$, $q_{electron} = 1.6\times10^{-19}C$, $A_{circle} = \pi r^2$, $A_{cylinder} = 2\pi rL$

1. [15 pts] For the figure to the right, determine the point (other than at infinity) where the total electric field is zero. Be explicit, solve mathematically and mark point on figure.





$$\vec{E}_{3Q} + \vec{E}_{4Q} = 0$$

$$\frac{K4Q}{r_{4Q}^2} = \frac{K3Q}{r_{3Q}^2}$$

$$.25m = r_{3Q} + r_{4Q}$$

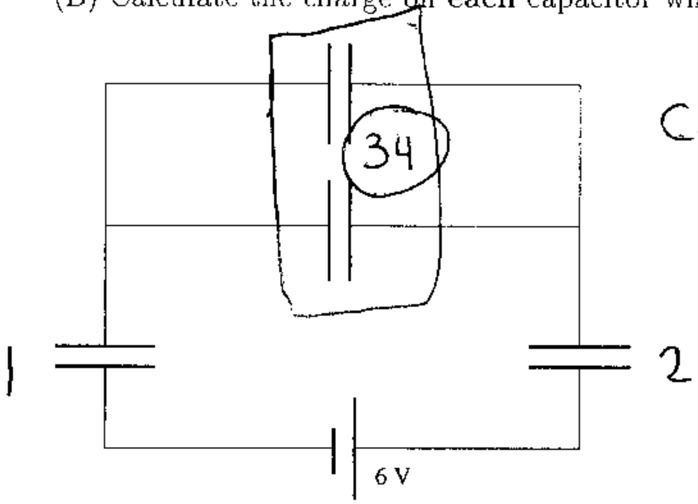
$$\frac{K4Q}{(.25 - r_{3Q})^2} = \frac{K3Q}{r_{3Q}^2} \Rightarrow 4r_{3Q}^2 = 3r_{3Q}^2 - 1.5r_{3Q} + .1875$$

$$r_{3Q}^2 + 1.5r_{3Q} - .1875 = 0$$

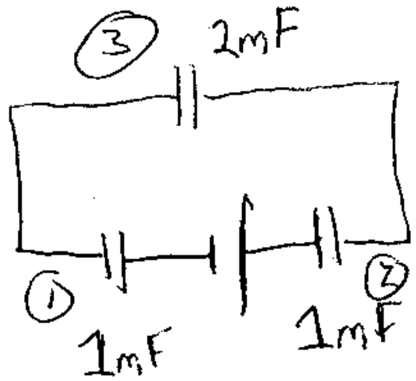
$$16r_{3Q}^2 + 24r_{3Q} - 3 = 0$$

$$r_{3Q} = \frac{-24 \pm\sqrt{576+192}}{32} = .116m$$

2. [15 pts] Assuming that each capacitor has $C = 1.0 mF$:
(A) Calculate the total capacitance of the circuit
(B) Calculate the charge on **each** capacitor when they are filled.



$C_{EQ} = 1mF + 1mF = 2mF$



$$\frac{1}{C_{EQ}} = \frac{1}{1mF} + \frac{1}{1mF} + \frac{1}{2mF} = \frac{5}{2mF}$$

SO $C_{EQ} = \frac{2}{5} mF$

Q on All these is same

SO $Q = CV$

$$1mF\,V_1 = 1mF\,V_2 = 2mF\,V_3$$

$$V_3 = V_1/2 = V_2/2$$

$V_1 + V_2 + V_3 = 6V$

$2.5\,V_1 = 6V$

$V_1 = 12/5\,V = V_2$

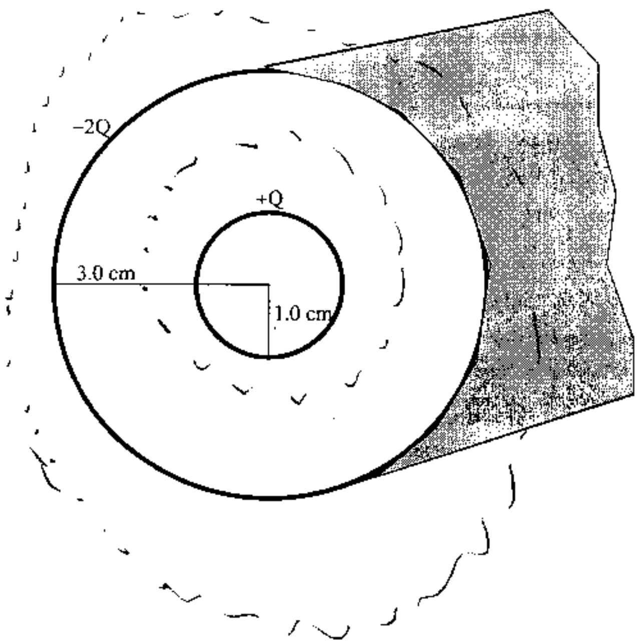
$V_3 = 6/5\,V$

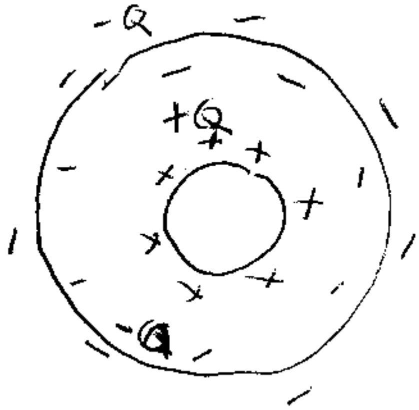
$Q = CV$

$Q_1 = 1mF\,(12/5\,V) = Q_2$

$= \frac{12}{5} mC$ $\quad Q_3 = Q_4 = 6/5\,mC$

3. [25 pts] A very long conducting cylindrical shell of length L with a total charge of $+Q$ is surrounded by a conducting cylindrical shell (also of length L) with total charge of $-2Q$, as shown in the figure below.
(a) Where is the excess charge located for the two shells (be careful the $-2Q$ is not all on the outside surface!). Draw on diagram
(b) using Gauss' Law derive the E field for the region outside both shells
(c) using Gauss' Law derive the E field for the region between both shells
(d) What is the E field on both conductors
(e) What is the voltage difference between the inner cylinder and outer cylinder if the inner radius is .01m and the outer radius is .03m? (Hint: $\int_{.01m}^{.03m} \frac{dr}{r} = \ln 3$)





+Q on outside of inner surface

-Q on inside of outer surface

-Q on outside of inner surface

b) $Q_{enc} = -Q$

$$E(2\pi r L) = \frac{-Q}{\varepsilon_0}$$

$$E = \frac{-Q}{2\pi\varepsilon_0 r L}$$

c) $Q_{enc} = +Q$

$$E 2\pi r L = +\frac{Q}{\varepsilon_0}$$

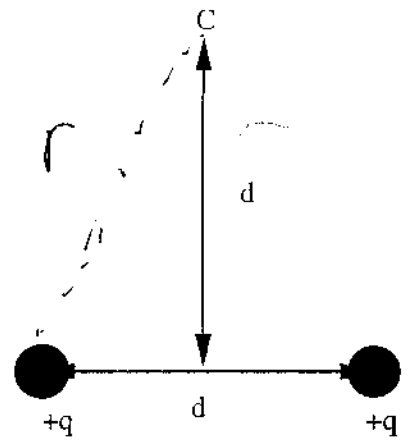
$$E = \frac{+Q}{2\pi\varepsilon_0 r L}$$

d) $E = 0$

e) $$\Delta V = -\int E \cdot dl = -\frac{Q}{2\pi\varepsilon_0 L}\int_{.01m}^{.03m} \frac{dr}{r}$$

$$= -\frac{Q}{2\pi\varepsilon_0 L} \ln 3$$

4. [15 pts] Assume the two charges below are fixed. How much work would it take to drag a point charge of $+q$ from infinity to point C?



$$W_{Human} = qV$$

$$V = \frac{Kq}{r} + \frac{Kq}{r}$$

$$r^2 = d^2 + \left(\frac{d}{2}\right)^2$$

$$\text{so } V = \frac{2Kq}{r} = \frac{2Kq}{\sqrt{d^2 + d^2/4}} = \frac{2Kq}{\sqrt{5/4\, d^2}} = \frac{4Kq}{\sqrt{5}\, d}$$

$$W = \frac{4Kq^2}{\sqrt{5}\, d}$$

5. [10 pts] A dielectric inserted into a capacitor increases the capacitance. For example, in a parallel plate capacitor, $C_{orig} = \frac{\epsilon_0 A}{d}$ becomes $C = \frac{K\epsilon_0 A}{d}$ where $K > 1$. In general what happens to the voltage and electric field of objects immersed in a dielectric?

$$E \sim \frac{1}{4\pi K\varepsilon_0} \frac{Q}{r^2}$$

$$V \sim \frac{1}{4\pi K\varepsilon_0} \frac{Q}{r}$$

so denomonator grows

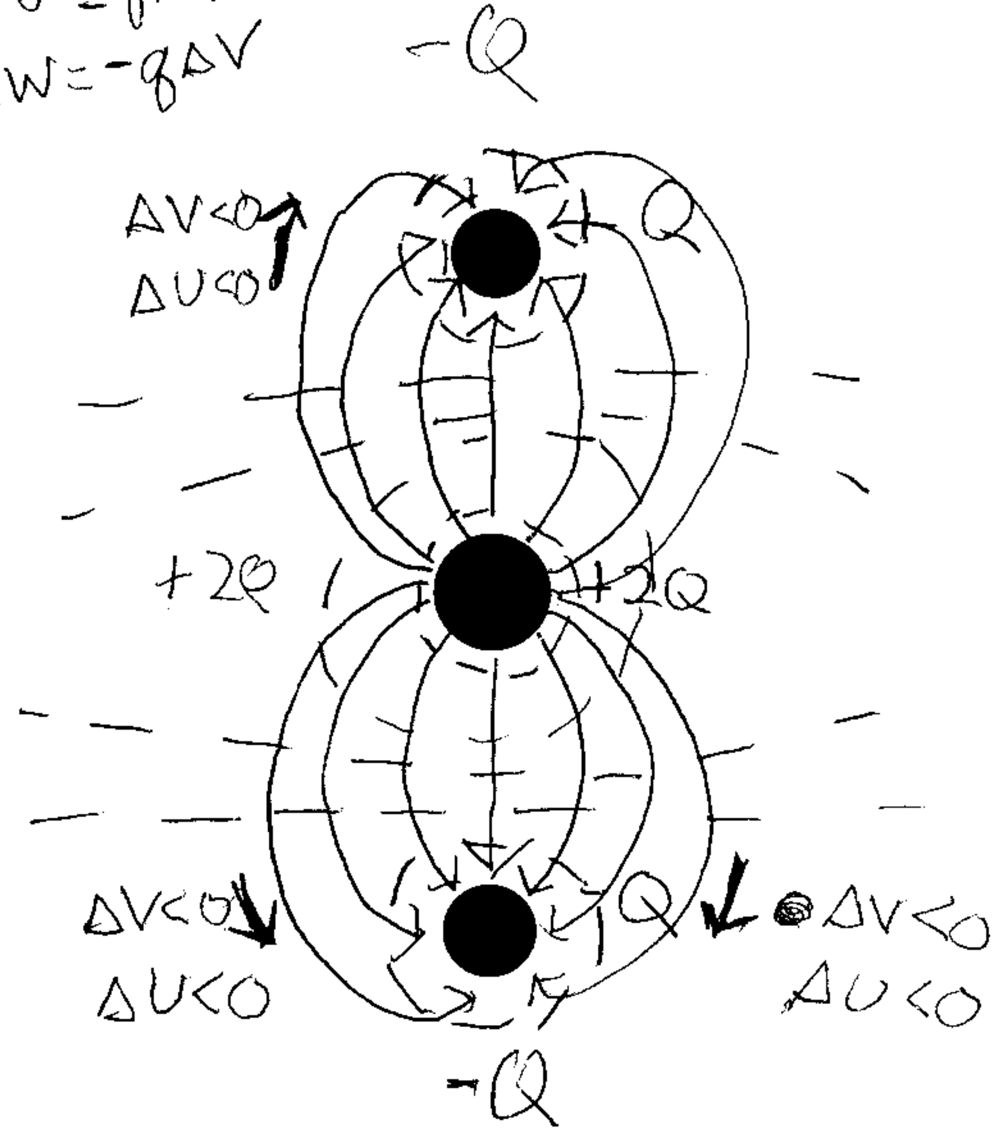
E and V weaken

6. [20 pts] Examine the charge set-up below:
(A) Draw electric field lines using solid lines (include direction!)
(B) Draw equipotential lines using dashed lines
(C) Show direction of decreasing voltage in three places on diagram
(D) Show direction of decreasing potential energy in three places

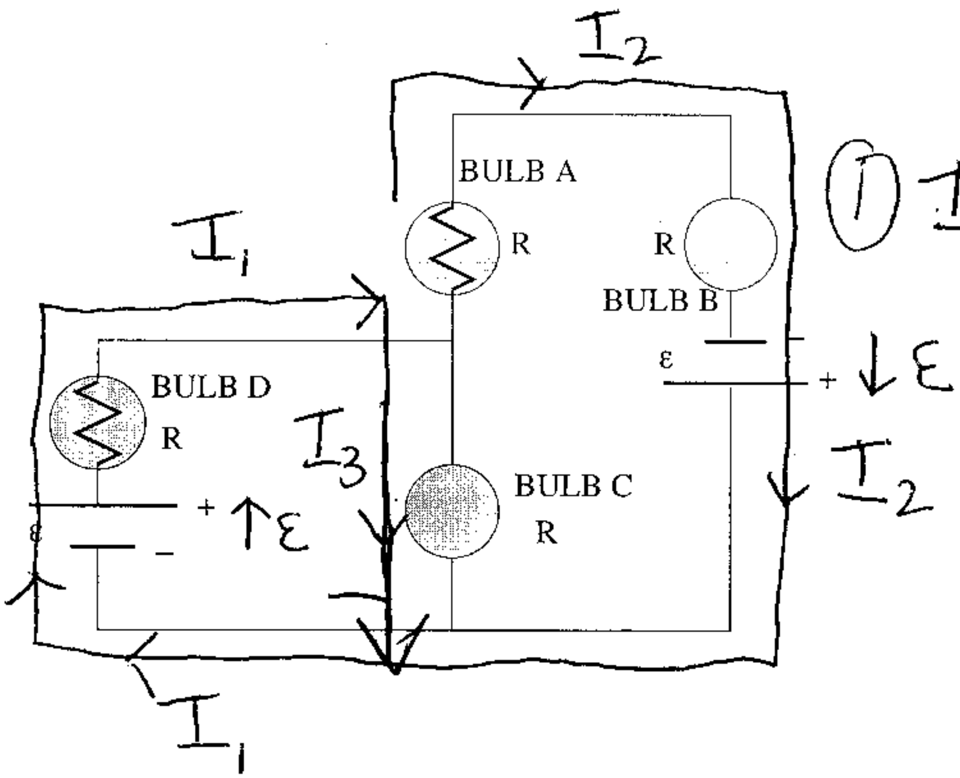
ΔV 0

$\Delta U = q\Delta V$

$\Delta W = -q\Delta V$



2. [20 pts] Analyze the circuit below. All bulbs have $R = 20\Omega$, and $\mathcal{E} = 1.5V$.
(A) What is the current going through each of the bulbs?
(B) What is the power going through each of the bulbs?



① $I_1 = I_2 + I_3$

② Left loop: $V - I_1R - I_3R = 0$

③ Right loop: $V + I_3R - I_2R - I_2R = 0$

② $V - (I_2 + I_3)R - I_3R = 0 = V - I_2R - 2I_3R$

2* ③ $V + I_3R - 2I_2R = 0$

$2③ + ②$

$3V - 5I_2R = 0$

so $\boxed{I_2 = \frac{3}{5}\frac{V}{R} = .045A}$

③ $V + I_3\cancel{8}R - \frac{6}{5}V = 0$

$\boxed{I_3 = \frac{1}{5}\frac{V}{R} = .015A}$

$\boxed{I_1 = I_2 + I_3 = \frac{4}{5}V/R = .06A}$

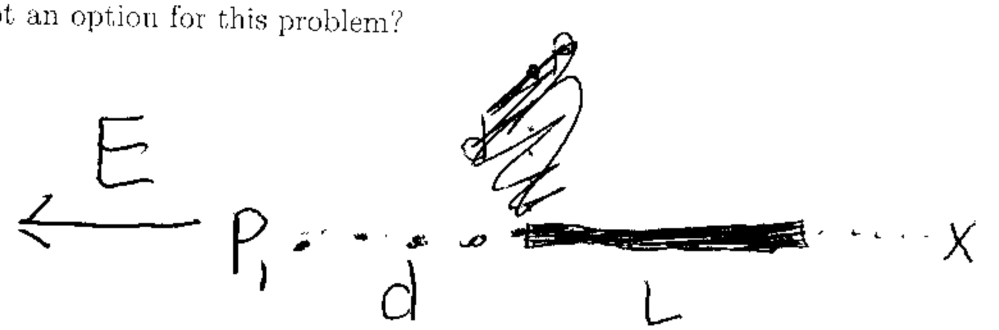
4. [20 pts] In the figure below there is a rod of length L with a uniform positive charge density of λ lying on the x-axis. Explain in as much detail as possible the steps to show that

(A) the Electric field at point P_1 is $|\vec{E}| = k_0 \int_0^L \frac{\lambda\, dx}{(d+x)^2}$

(B) What direction is the E-field in at point P_1?

(C) The answer for the integral is $|\vec{E}| = k_0\lambda(\frac{1}{d} - \frac{1}{d+L})$. Assume $L = d$, show that $|\vec{E}| = k_0\frac{Q}{2d^2}$ where Q is the charge on the rod.

(D) Why is Gauss' Law not an option for this problem?



In general

$E = K_0 \int \frac{dq}{r^2}$ $\quad q = \lambda L$ so $dq = \lambda dL = \lambda dx$

r is distane from P to "dx"

$r = d + x$

$r^2 = (d+x)^2$

$$E = K_0 \int_0^L \frac{\lambda dx}{(d+x)^2}$$

if $L = d$ $\quad |\vec{E}| = K_0\lambda\left(\frac{1}{d} - \frac{1}{d+L}\right) = K_0\lambda\left(\frac{1}{2d}\right)$

$\lambda = \frac{Q}{L} = \frac{Q}{d}$ so $|\vec{E}| = K_0\frac{Q}{d}\left(\frac{1}{2d}\right) = \frac{K_0 Q}{2d^2}$

Gauss' law cannot be used easily because rod is small, end has curved $\vec{E}$ fields

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Useful constants and formulas: $\epsilon_0 = 8.85 \times 10^{-12} C^2/Nm^2$, $\frac{1}{4\pi\epsilon_0} = k_0 = 8.99 \times 10^9 Nm^2/C^2, q_{electron} = -1.6 \times 10^{-19} C, A_{circle} = \pi r^2, A_{sphere} = 4\pi^2, A_{cylinder} = 2\pi r L$

1. [15 pts] Shown below is a charge configuration the distance between the plates is 1.0 cm.

(A) Using a **solid line** draw the $\vec{E}$ field lines.

(B) Using a **dashed line** draw the equal-potential lines.

(C) What is the capacitance of this set up if the area of one plate is $.05m^2$ and there is only air between the plates?

$C = \frac{A\epsilon_0}{d}$