

NAME _____ Pledged SCORE ____ / 100 pts

Remember to get full credit label answers with correct units, show all work, draw diagrams.

Useful constants and formulas: $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$, $\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$, $q_{\text{electron}} = -1.6 \times 10^{-19} \text{ C}$, $mass_{\text{electron}} = 9.11 \times 10^{-31} \text{ kg}$, $q_{\text{proton}} = +1.6 \times 10^{-19} \text{ C}$, $mass_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, $\mu_0 = 1.26 \times 10^{-6} \text{ H/m}$, $B_{\text{wire}} = \frac{\mu_0 i}{2\pi r}$, $A_{\text{circle}} = \pi r^2$, $C_{\text{circle}} = 2\pi r$, $A_{\text{triangle}} = \frac{1}{2}bh$

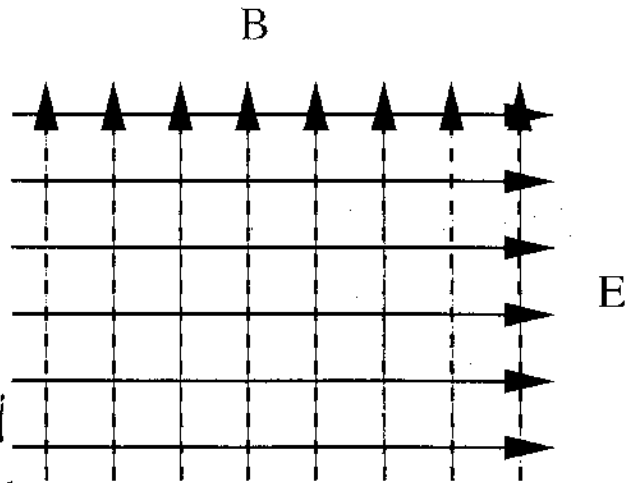
1. [17 pts] We have crossed electric and magnetic fields as shown. A proton enters this region.

(A) What would be the initial velocity **direction** if you wanted the proton to feel a total force to the right?

(B) What would be the initial velocity **magnitude and direction** if you wanted the proton to feel a total force of exactly zero?

a) $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

$\underbrace{\hspace{2em}}_{\text{Electric}} \quad \underbrace{\hspace{2em}}_{\text{Magnetic}}$



↓
always
to right
for "+" charges

↓
right hand
rule says
velocity has to be into page

b) F_{Electric} to right so F_B has to be to left

← F_B means velocity out of page

$$F_B = F_E$$

$$qvB = qE$$

$$\text{so } v = E/B$$

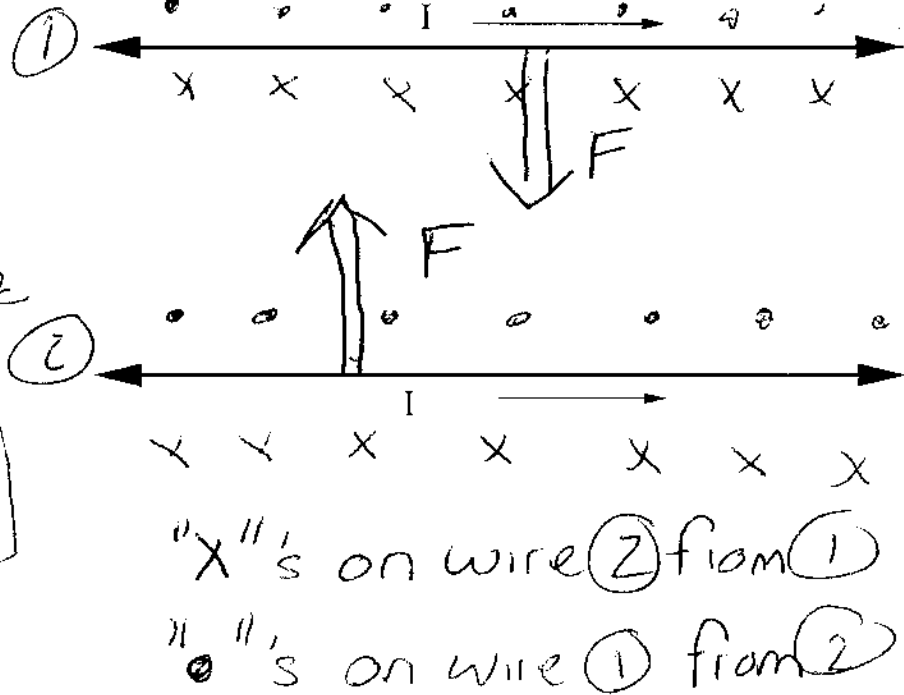
2. [20 pts] There are two wires 4.0 cm apart, both carrying 15 amps. What is the force per meter on each wire due to the other wire. Please label direction on the figure.

$$F = \vec{I} l \times \vec{B}_{Ext}$$

$$F = I l B_{Ext}$$

$$F = I_1 l \frac{\mu_0 I_2}{2\pi r}$$

$$\frac{F}{l} = \frac{I_1 I_2 \mu_0}{2\pi r}$$

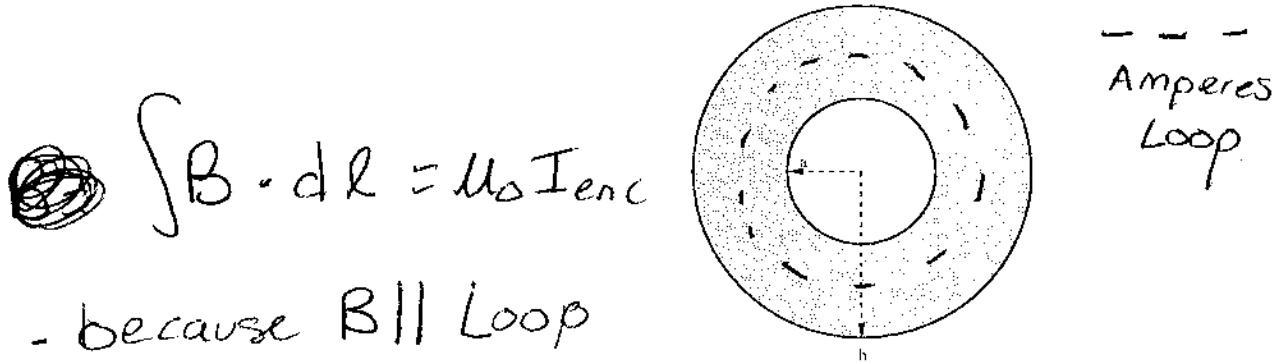


$$\frac{F}{\text{meter}} = \frac{(15A)(15A)(4\pi \times 10^{-7} \frac{Nm}{A^2})}{2\pi (.04m)} = 1.1 \times 10^{-3} N$$

3. [20 pts] The figure below shows a cross section of a hollow cylindrical conductor of inner radii a and outer radii b , carrying a uniformly distributed current i .

(A) Show that $B(r)$ for the range $b < r < a$ is given by (using Ampere's Law) $B = \frac{\mu_0 i}{2\pi r} \frac{r^2 - a^2}{b^2 - a^2}$ (use Ampere's Law, show all work).

(B) Show that when $r = b$, this equation gives the magnetic field magnitude B for a long straight wire; and when $r = a$, it gives zero magnetic field.



- because $B \parallel$ Loop

$$- B \cdot dl = B dl$$

if B has same shape around loop then B is constant

$$\text{so } Bl = \mu_0 I_{enc}$$

$$I_{enc} = i \left(\frac{\pi r^2 - \pi a^2}{\pi b^2 - \pi a^2} \right)$$

↑ hole
← part
← total

$$\text{so } I_{enc} = i \left(\frac{r^2 - a^2}{b^2 - a^2} \right)$$

↑ hole

$$B 2\pi r = \mu_0 i \left(\frac{r^2 - a^2}{b^2 - a^2} \right)$$

$$\text{so } B = \frac{\mu_0 i}{2\pi r} \left(\frac{r^2 - a^2}{b^2 - a^2} \right)$$

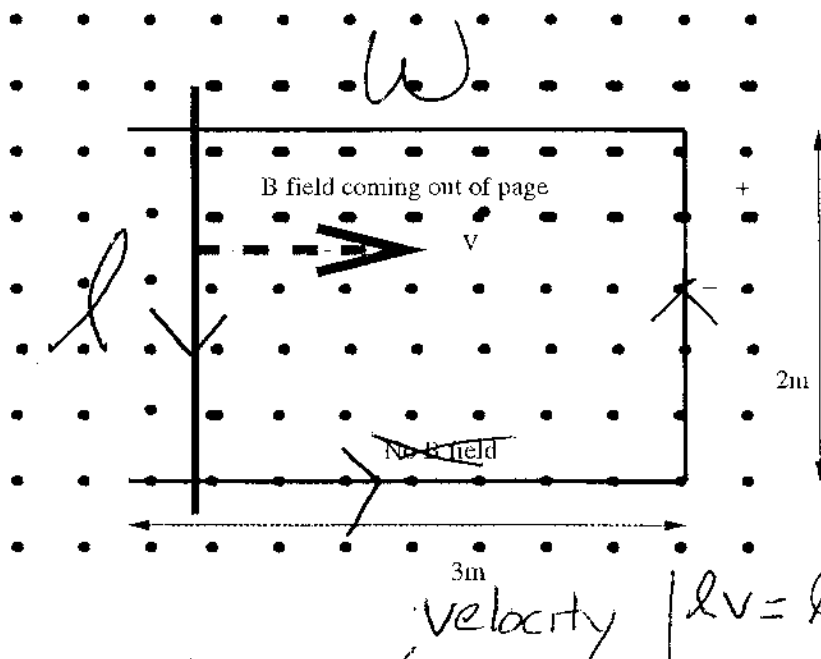
b) $r = b$
then $i_{enc} = i(1)$

if $r = a$

then $i_{enc} = i(0)$

4. [15pts] A rectangular wire loop with a 2.00 m side and a 3.00 m side (see figure) is perpendicular to a uniform magnetic field. The left side of the loop is free to move and is moving to the right at a constant velocity of 1.5 cm/s as shown below.

- (A) What is the induced emf in the circuit?
- (B) What is the direction of the induced emf?
- (C) Assume the rod is now still and the area of the loop is 4.0 m^2 . What would have to be the change in the magnetic field to produce the same effect as parts (A) and (B)?



$$\mathcal{E} = \frac{d\Phi_B}{dt}$$

$$\Phi_B = BA \cos\theta$$

so here

$$\frac{d\Phi_B}{dt} = B \frac{dA}{dt}$$

$$\frac{dA}{dt} = \frac{d(lw)}{dt} = l \frac{dw}{dt} = lv$$

$$lv = l \frac{dw}{dt} = l \frac{dB}{dt}$$

so $\mathcal{E} = Blv$

(B) Less dots so more are needed CCW

(C) if B changes $\mathcal{E} = A \frac{dB}{dt} = lw \frac{dB}{dt}$

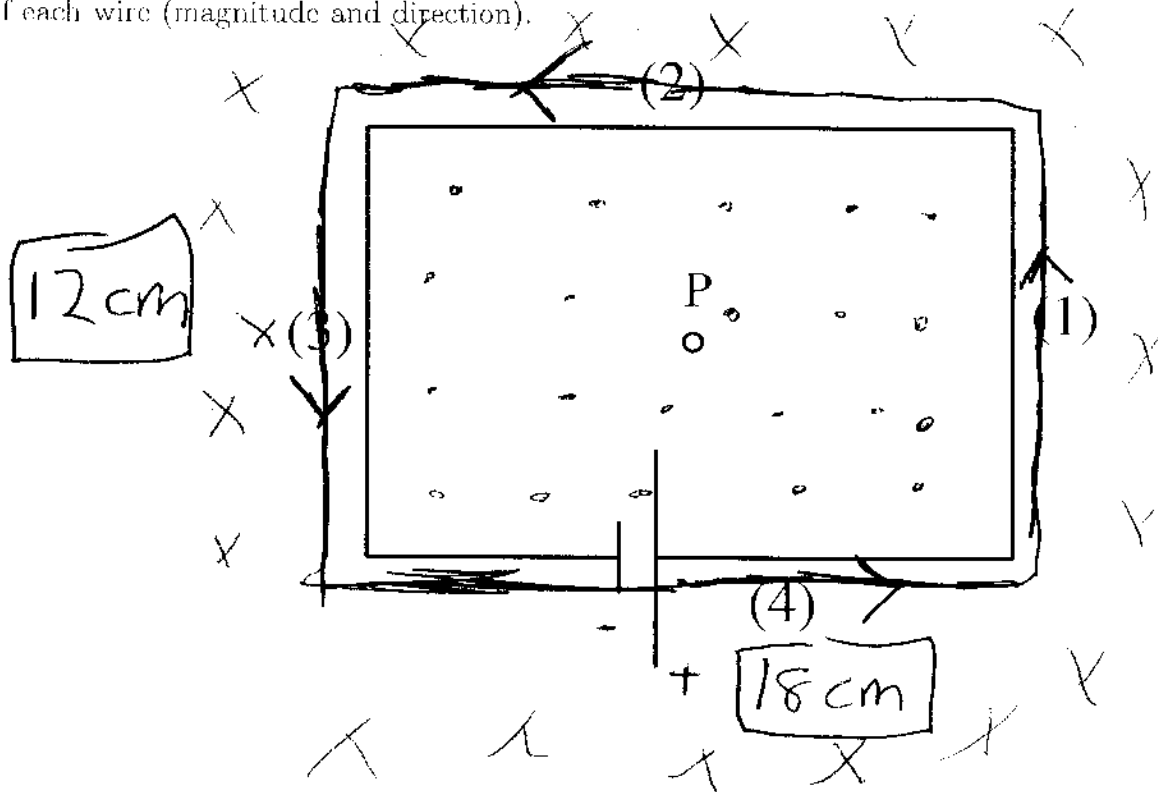
5. [8 pts] What is the role of an inductor in a circuit which is in series with a resistor (RL circuit)?

The inductor resists or tames the charges. Makes them less shocking.....

2!

6. [20 pts] Using the Biot-Savart Law we calculate that the magnetic field for a short thin straight wire is $B = \frac{\mu_0 I}{2\pi R} \frac{w}{\sqrt{w^2 + 4R^2}}$ where R is the distance from the wire as measured from the perpendicular bisector (wire center) and w is the length of the wire.

Using this information calculate B for the rectangular loop below at point P assuming it is made out of 4 thin straight wires and it is carrying a current of 2.5 A. Wires (2) and (4) have length 18 cm and wires (1) and (3) have length 12 cm. Please show explicitly the contribution of each wire (magnitude and direction).



All add constructively (all contribute dots in center)

$$B = B_1 + B_2 + B_3 + B_4$$

$$= \frac{\mu_0 I}{2\pi} \left(\frac{0.12 \text{ m} (2)}{0.09 \text{ m} \sqrt{(0.12 \text{ m})^2 + 4(0.09)^2}} + \frac{0.18 \text{ m} (2)}{0.06 \text{ m} \sqrt{(0.18 \text{ m})^2 + 4(0.06)^2}} \right)$$

wire (1)+(3) wire (2)+(4)

$$= 5 \times 10^{-7} (12.3 + 27.7) = 2.0 \times 10^{-5} \text{ T}$$