Mathematics/Physics Seminar
Spring 2017

Hwk #4: Introduction to asymptotic matching

Due: March 1, 2017

Using asymptotic matching in order to find the arbitrary constant in a local analysis calculation. Retracing/Filling in some of the steps in Example 4, section 7.4 in Bender.

Given the boundary value problem (BVP) below in which ε is a small parameter

$$y''+(\nu+\frac{1}{2}+\frac{x^2}{4}-\varepsilon x^4)y=0,\ y(0)=1,\ y(+\infty)=0$$

our task is to find the leading behavior to the solution for large "x". Bender gives the leading behavior in equation 7.4.10., but note that the result, not surprisingly, is determined up to an arbitrary constant "a". This is because the local analysis used to find the result for large "x" is insensitive to the condition y(0)=1. In this homework we use asymptotic matching in order to find the constant "a", retracing the steps done in Bender.

a) Using the techniques learned last semester, do a local analysis for large "x" of the equation above to find the controlling factor of the leading behavior (exponential term in 7.4.10).

b) Optional: do the next step in the local analysis to recover the first factor in 7.4.10, namely, $(\frac{x^2}{4}+\varepsilon x^4)^{-1/4}$. Continuing in the usual way, with patience and care, we would obtain the full leading behavior given in 7.4.10. Let's call the solution given in 7.4.10 y_{right}

c) Using zeroth order perturbation theory, verify that after specifying $y(0)=1$, one obtains the approximate solution for small "x" given by 7.4.9. No need to do a calculation here since the zeroth order solution is a known function, just convince yourself that 7.4.9 makes sense. Let's call this solution y_{left}.

d) Verify that, as shown in Bender but filling in the missing steps, for large x y_{left} has the same asymptotic behavior as y_{right} for small enough x (this is the intermediate region where the two solutions overlap). By equating the two asymptotic solutions in the overlap region find the constant "a".