Mathematics/Physics Seminar
Spring 2017

Hwk #6: Higher order boundary value theory, boundary layers of variable thickness

Due: March 15, 2017

Higher order asymptotic matching involved in higher order approximations of boundary layer problems (problem 2 below). Example of a problem with a boundary layer thickness other than ε (problem 1 below). The material relevant to this homework is in sections 9.3 and 9.4 in Bender.

1) Given the boundary value problem:

$$\varepsilon y''(x) + y'(x) - x^{-3/2} y(x) = 0,\ y(0) = 1,\ y(1) = 1,\ 0 \le x \le 1,$$

is there is a boundary layer at $x = 0$, as $\varepsilon \to 0^+$? If so, what is its thickness? Hint: look at examples in section 9.4 in Bender.

2) Given the boundary value problem:

$$\varepsilon y''(x) + (1+x)^2 y'(x) + y(x) = 0,\ y(0) = 1,\ y(1) = 1,\ 0 \le x \le 1,$$

a) Obtain a uniform approximation to zeroth order.
b) Obtain a uniform approximation to first order in ε.
c) Use *Mathematica* to plot the uniform approximation obtained in part a) above, along with the numerical solution obtained by *Mathematica* (on the same graph), for $\varepsilon = .05$. Repeat, this time using the approximation found in part b). Produce a third plot displaying the percent error for the zeroth and first order approximations as a function of x. How large does the error become (between the numerical solution and the approximate analytical solution) in the region of largest error? How would you characterize the accuracy of the boundary layer analysis? Repeat the third plot (dealing with the error), but this time for $\varepsilon = .025$. Comment on the results.