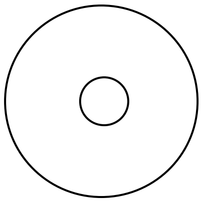
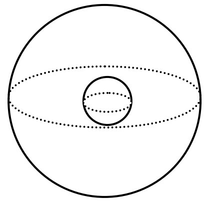
Lab: Gauss' Law and Equipotentials

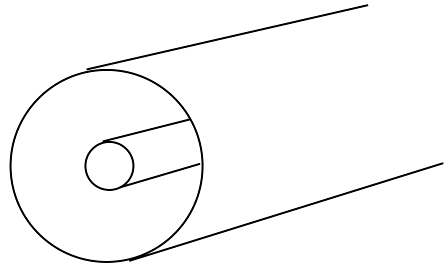
A two-dimensional representation of co-centric spheres and coaxial cables looks the same as below (a). Is this 2-D drawing (a) below for a set of spheres (b) or cylinders (c)?



2D representation



(a)



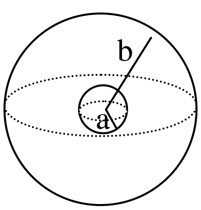
(b)

What is your initial guess/prediction?

You will measure the electrostatic potential for the 2-D drawing and then compare the data to the predicted potential for the sphere and cylinder configurations above. There are two sections to the lab: calculating/predicting the electric fields in the two configurations (and thus the electrostatic potentials) and making the measurements.

Calculation: Use Gauss' law to calculate the electric field in between co-centric spheres with inner radius *a* and outer radius *b*. Assume the inner sphere has a charge of +Q and the outer shell has a charge of -Q (so the configuration is electrically neutral).

Gauss' Law: $\oint \vec{E} \bullet d\vec{A} = {Q_{enclosed}}/{\varepsilon_0}$



Gaussian surface: Draw a spherical (mimic the symmetry) Gaussian surface inside the outer sphere and label the radius *r*.

What is the charge enclosed in the Gaussian surface?
$Q_{enclosed}$= _________

What is the direction of $\vec{E}$ on the Gaussian surface?

What is the direction of $d\vec{A}$ on the Gaussian surface?

What is $\vec{E} \bullet d\vec{A}$? What is the cosine of the angle between $\vec{E}$ and $d\vec{A}$?

Why can you re-write $\oint \vec{E} \bullet d\vec{A}$ as $|E|\oint dA = |E|A$?

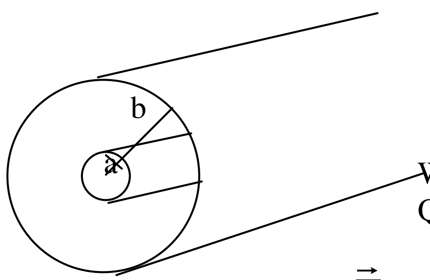
What is the value of A for the Gaussian surface you drew?

Therefore, plugging all the pieces back into Gauss' Law, what is the electric field inside the co-centric spheres?

$E =$ (sphere)

Now, repeat for the cylinder of inner radius *a* and outer radius *b*. Assume the inner cylinder has a charge per unit length of +λ and the outer cylinder has a charge per unit length of -λ (so the configuration is electrically neutral).

Gauss' Law: $\oint \vec{E} \bullet d\vec{A} = Q_{enclosed}/\varepsilon_0$



Gaussian surface: Draw a cylindrical (mimic the symmetry) Gaussian surface inside the outer cylinder of length *h* and radius *r*.

What is the charge enclosed in the Gaussian surface?
$Q_{enclosed}$= _________

What is the direction of $\vec{E}$ on the Gaussian surface(ends, curved walls)?

What is the direction of $d\vec{A}$ on the Gaussian surface (ends, curved walls)?

What is $\vec{E} \bullet d\vec{A}$? What is the cosine of the angle between $\vec{E}$ and $d\vec{A}$ (ends, curved walls)?

Why can you re-write $\oint \vec{E} \bullet d\vec{A}$ as $|E|\oint dA = |E|A$ for the curved-wall part of the cylinder?

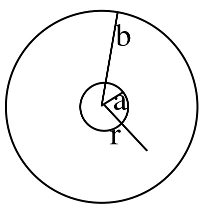
What is the value of A for the curved wall of the Gaussian cylindrical surface you drew?

Therefore, plugging all the pieces back into Gauss' Law, what is the electric field inside the coaxial cylinders?

$E =$ (cylinder)

As a function of *r*, what is the difference between the two values of the electric field (i.e. does one vary as $1/r$, while the other varies as $1/r^2$ or $1/r^3$)?

In the laboratory, you are **NOT** going to measure the electric field directly; instead you will measure the electrostatic **potential** (in volts). So, we need to calculate the **potential** inside the two configurations. For these cases, you can calculate the potential (as a function of the distance, *r*, from the center of the set-up) is given by using the following equation:



$$V(r) = -\int_b^r E(r')dr'$$

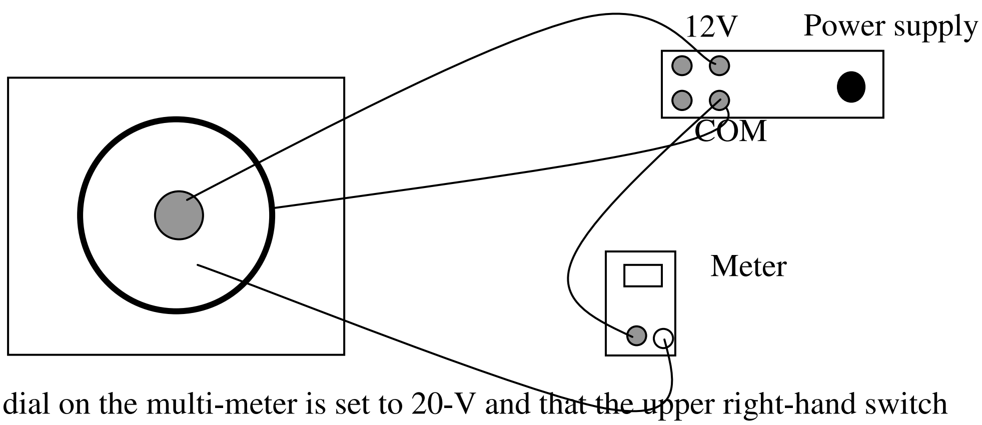
Find the potential as a function of r for the spherical case

V(r) = (sphere)

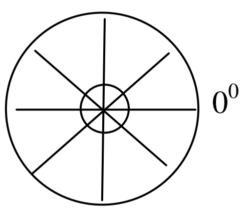
And the cylindrical case:

V(r) = (cylinder)

Measurement: You will take measurements of the potential for the 2-D pattern painted on the conductive black paper. Put a flag pin in the center conducting point and one on the outer shell. Connect the inner one to a 12-V supply and connect the outer pin to the ground (COM for "common ground") of the supply as shown below. Connect the black lead of the multi-meter (COM lead) to the COM of the power supply. You will use the red lead (V) to measure the potential at the points indicated on the grid.



Make sure the dial on the multi-meter is set to 20-V and that the upper right-hand switch is on DC and then turn the power to the multi-meter on. The grid points that you should measure voltages for are the grid points marked along the axes below (use only crosshairs marked on the conducting paper).



Notice that the points at the crosshairs on the 0^0, 90^0, 180^0, and 270^0 axes should have essentially the same value (are equipotential surfaces). Similarly, the points at the cross-hairs on the 45^0, 135^0, 225^0, and 315^0 will also have about the same value so you will average those points together. Record your data on the table below.

The distance to the points (which are on a 1cm grid) on the 45^0 axes are actually the number of blocks away times $\sqrt{2}$. Why?

Data Table: Measurement of the potential at the center: _______

a (inner radius)= ______ b (outer radius) = _____

Potential at other points:

Distance from center (m)	Axis 0^0	Axis 90^0	Axis 180^0	Axis 270^0	Average
.02					
.03					
.04					
.05					
.06					
.07					
.08					
.09					
	Axis 45^0	Axis 135^0	Axis 225^0	Axis 315^0	
.01*√2 =					
.02*√2 =					
.03*√2 =					
.04*√2 =					
.05*√2 =					
.06*√2 =					

To compare this with your theoretical calculation, you need to find values of Q and λ.

Write down your equation for the potential as a function of r for both cases below:

Sphere
V(r)=

Cylinder
V(r)=

For both of them, V at r = a is the value of the potential at the center. So, substitute the value of a and the value of b in your equation as well as the measured value of the potential in order to find the value of $Q/4\pi\varepsilon_0$ (for the sphere) or $\lambda/2\pi\varepsilon_0$ (for the cylinder). Find these values for the two cases:

<u>Sphere:</u>
V(r = a) = __________

which equals the potential in the center. From this, what is the value of

$Q/4\pi\varepsilon_0$ =

Using this value for $Q/4\pi\varepsilon_0$ what is the equation of V(r) (i.e. write the potential as a function of r only without any other unknown variables)?

V(r) =

<u>Cylinder:</u>
V(r = a)= __________

which equals the potential in the center. From this, what is the value of

$\lambda/2\pi\varepsilon_0$ =

Using this value for $\lambda/2\pi\varepsilon_0$ what is the equation of V(r) (i.e. write the potential as a function of r only without any other unknown variables)?

V(r) =

Combining the calculation with the measurements:
Using Logger Pro (or some other graphing program), record your distance values in the first column and your voltages in the second. Now, to check which configuration the 2-D pattern matches better, you need the equations above for each.

If you use Logger Pro, use a manual curve fit and type in the appropriate equation. For example if the equation you predict (this is not the correct one, by the way) is V(r)=10*(1/r-1/2), then type in "=a*(1/x-1/2)" and then in the curve fit box, try your value for a. Do this for both the spherical and cylindrical case and determine which best matches the data. [Note: In Logger Pro, to fit to a natural log, you need to type in "ln(x)".]

Best match? ________________